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CARTESIAN COMPOSITIONS IN FOUR DIMENSIONAL SPACE WITH AFFINE CONNECTIONS, WITHOUT TORSION AND ADDITIONS

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ABSTRACT. The affine connection space A_4 , product spaces, product affinor $\left(a_\alpha^\beta, b_\alpha^\beta, c_\alpha^\beta\right)$ with symmetrical and additional connections (asymmetric P_α^β) where affine of structures continue to be transformed in parallel way along the lines in space, see [16].

1. INTRODUCTION

Let us take affine product on the four-dimensional space all along with symmetric connections and addition A_4 which have been studied, see [1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 16]. Let us take A_4, the affine symmetric space. In A_4 there have already been defined the products $X_2 \times \overline{X}_2$, $Y_2 \times \overline{Y}_2$, $Z_2 \times \overline{Z}_2$, and $X_3 \times X_1$, (addition) in such a way that each of them has a multiple base on A_4, and they have been analyzed in [1, 4, 6, 8, 14]. We have already discussed the space A_4 with the additional structure on the space of independent vectors in [4, 11, 12, 13, 14, 15].

2. PRELIMINARIES

Let A_4 be the space with affine symmetric connection. This will be presented with the formula $\Gamma_{\alpha\beta}^\sigma$ where the connection coefficients will be denoted $(\alpha, \beta, \sigma = 1, 2, 3, 4)$. In A_4 we consider the product $X_n \times X_m$ where $(n + m = 4)$. Both multipliers have differential bases. Let us take two transformation positions of $P(X_n)$ and $P(X_m)$, or $(P(X_n)$ and $P(\overline{X}_n))$ of the multipliers at any point A_4 , see [4, 6, 7, 9]. It s known that the product

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is in general defined based on affinors fields:

$$a_\alpha^\beta = \underset{1}{V}^\beta \overset{1}{V}_\alpha + \underset{2}{V}^\beta \overset{2}{V}_\alpha - \underset{3}{V}^\beta \overset{3}{V}_\alpha - \underset{4}{V}^\beta \overset{4}{V}_\alpha$$
$$b_\alpha^\beta = \underset{1}{V}^\beta \overset{3}{V}_\alpha + \underset{3}{V}^\beta \overset{1}{V}_\alpha + \underset{2}{V}^\beta \overset{4}{V}_\alpha + \underset{4}{V}^\beta \overset{2}{V}_\alpha \quad . \tag{2.1}$$
$$c_\alpha^\beta = \underset{3}{V}^\beta \overset{1}{V}_\alpha - \underset{1}{V}^\beta \overset{3}{V}_\alpha + \underset{4}{V}^\beta \overset{2}{V}_\alpha - \underset{2}{V}^\beta \overset{4}{V}_\alpha$$

Also

$$P_\alpha^\beta = \underset{1}{V}^\beta \overset{1}{V}_\alpha - \underset{3}{V}^\beta \overset{3}{V}_\alpha \tag{2.2}$$

and with affinor (2.2) we present the additional structure. Affinors (2.1) and affine (2.2) are called product affinors. We take them as affinors connected with the space A_4 in an integral structure of product. According to [1, 2, 3, 4, 9] and [8] the integral condition of structure is characterized with the equation:

$$\nabla_\sigma a_\alpha^\beta = 0\,. \tag{2.3}$$

Using [16] there is differential of the equation (2.3) for the field of vectors $\underset{\alpha}{\overset{\beta}{V}} = (V_1^\alpha, V_2^\alpha, V_3^\alpha, V_4^\alpha)$ and the result is:

$$\nabla_\sigma V_\alpha^\beta = \underset{\alpha}{\overset{\sigma}{T}} \sigma \underset{v}{V}^\beta\,, \quad \nabla_\sigma \overset{\alpha}{V}_\beta = -\overset{\alpha}{T}_v \sigma \overset{v}{V}_\beta\,. \tag{2.4}$$

With $\{V_\alpha\}$ we mark the net of vectors. Let us take an independent vector in their field V_α^β. If we take $\{V_\alpha\}$ the widen net of the coordinates where we have affine, projected affinores $\overset{n}{a}{}_\alpha^\beta$ and $\overset{m}{a}{}_\alpha^\beta$ are defined by equation:

$$\overset{n}{a}{}_\alpha^\beta = \frac{1}{2}(\delta_\alpha^\beta + a_\alpha^\beta)\,, \quad \overset{m}{a}{}_\alpha^\beta = \frac{1}{2}(\delta_\alpha^\beta - a_\alpha^\beta)\,.$$

This equations meets the conditions:

$$\begin{aligned}
&\overset{n}{a}{}_\alpha^\beta\, \overset{n}{a}{}_\beta^\sigma = \overset{n}{a}{}_\alpha^\sigma\,, \quad \overset{n}{a}{}_\alpha^\beta\, \overset{n}{a}{}_\beta^\sigma = \overset{m}{a}{}_\alpha^\sigma\,,\\
&\overset{n}{a}{}_\alpha^\beta\, \overset{m}{a}{}_\beta^\sigma = \overset{m}{a}{}_\alpha^\beta\, \overset{n}{a}{}_\beta^\sigma = 0\,,\\
&\overset{n}{a}{}_\alpha^\beta + \overset{m}{a}{}_\alpha^\beta = \delta_\alpha^\beta\,,\\
&\overset{n}{a}{}_\alpha^\beta - \overset{m}{a}{}_\alpha^\beta = a_\alpha^\beta\,,
\end{aligned}$$

see [1, 2, 5, 6, 8].

For each vector $V^\alpha \in A_4$, in $(X_2 \times \overline{X}_2)$, $(Y_2 \times \overline{Y}_2)$, $(Z_2 \times \overline{Z}_2)$, $(X_2 \times Y_2)$, $(X_2 \times Z_2)$, $(Y_2 \times \overline{X}_2)$, $(Z_2 \times \overline{X}_2)$, and $(X_3 \times X_1)$, we have:

$$V^\alpha = \overset{n}{a}{}_\alpha^\beta\, V^\beta + \overset{m}{a}{}_\beta^\alpha\, V^\beta = \overset{n\alpha}{V} + \overset{m\alpha}{V}\,.$$

and the following equations hold:

$$\overset{n\alpha}{V} = \overset{n}{a}{}_\alpha^\beta\, V^\beta \in P(X_2), \overset{m}{V}{}^\alpha = \overset{m}{a}{}_\beta^\alpha\, V^\beta \in P(\overline{X}_2)\,.$$

These products have been studied in [1, 2, 3, 6, 5, 8].

The product (C, C) (Cartesian, Cartesian) is called of type Cartesian if the positions of $P(X_2)$ and $P(\overline{X}_2)$ are put parallel along the lines of A_4 and are characterized with (2.3). Let us see the vectors:

$$\begin{aligned} \underset{i}{W}^{\alpha} &= \underset{i}{V}^{\alpha} \\ \underset{i}{W}^{\alpha} &= \tfrac{1}{\sqrt{2}}\left(\underset{i-4}{V}^{\alpha} + \underset{i}{V}^{\alpha}\right). \end{aligned} \tag{2.5}$$

From (2.5) and the condition

$$\overset{\alpha}{W}_{\sigma}\underset{\beta}{V}^{\sigma} = \delta^{\alpha}_{\beta} \Leftrightarrow \overset{\sigma}{W}_{\beta}\underset{\sigma}{W}^{\alpha} = \delta^{\alpha}_{\beta}\,, \tag{2.6}$$

we have that

$$\overset{i}{W}_{\alpha} = \overset{i}{V}_{\alpha} - \overset{4+i}{V}_{\alpha}\,, \quad \overset{i}{W}_{\alpha} = \sqrt{2}\overset{i}{V}_{\alpha}\,, \tag{2.7}$$

where

$$\alpha, \beta, \sigma = (1,2,3,4)\,, \quad i = 1,2 \;\; \bar{i} = 3,4\,.$$

Let us see affine

$$a^{\beta}_{\alpha} = \underset{i}{W}^{\beta}\overset{i}{W}_{\alpha} - \underset{\bar{i}}{W}\underset{\alpha}{\overset{\bar{i}}{W}}\,. \tag{2.8}$$

From (2.6) and (2.8) we have $a^{\beta}_{\sigma}a^{\sigma}_{\alpha} = \delta^{\beta}_{\alpha}$ and we say that affine a^{β}_{α} satisfies the condition of production.

Theorem 2.1. *The product $X_2 \times \overline{X}_2$ is of the type (C, C) (Cartesian, Cartesian) if it satisfies the condition $\nabla_{\sigma}\underset{1}{a}^{\beta}_{\alpha} = 0$.*

Proof. Let us consider the condition

$$\nabla_{\sigma}a^{\beta}_{\alpha} = 0, \nabla_{\sigma}\delta^{\beta}_{\alpha} = 0\,. \tag{2.9}$$

Based on (2.3) and (2.9) the condition for the product $X_2 \times \overline{X}_2$ is satisfied, and the product is of the type (C, C). Further, based on the relations (2.7) and (2.8) we have that:

$$\nabla_{\sigma}a^{\beta}_{\alpha} = 0, \nabla_{\sigma}\underset{1}{d}^{\beta}_{\alpha} = 0\,, \tag{2.10}$$

where

$$d^{\,\beta}_{\,\alpha} = \underset{i}{V}^{\beta}\underset{\alpha}{\overset{n+i}{V}}\,, \quad \underset{1}{d}^{\,\beta}_{\,\alpha} = \underset{n+1}{V}^{\beta}\overset{i}{V}_{\alpha}\,.$$

Affine $d^{\,\beta}_{\,\alpha}$ and $\underset{1}{d}^{\,\beta}_{\,\alpha}$ are nilpotent because

$$d^{\,\beta}_{\,\alpha}d^{\,\sigma}_{\,\beta} = 0 \quad \textit{and} \quad \underset{1}{d}^{\,\beta}_{\,\alpha}\underset{1}{d}^{\,\sigma}_{\,\beta} = 0\,. \tag{2.11}$$

Finally, according to (2.11) and (2.10) even the products $(Y_2 \times \overline{Y}_2)$, $(Z_2 \times \overline{Z}_2)$ are of the type (C, C) by using relation (2.1). So, according to (2.9), (2.10) and (2.11) it holds $\nabla_{\sigma}d^{\,\beta}_{\,\alpha} = 0$. □

Theorem 2.2. *If the products $X_2 \times \overline{X}_2$, $X_2 \times Y_2$, $X_2 \times Z_2$, $Y_2 \times \overline{X}_2$, and $Z_2 \times \overline{X}_2$, are of the type (C, C), then the space A_4 is affine.*

Proof. According to the theorem 2.1 the products $X_2 \times \overline{X}_2$, $X_2 \times Y_2$, $X_2 \times Z_2$, $Y_2 \times \overline{X}_2$, and $Z_2 \times \overline{X}_2$ are of the type (C, C) if the condition (2.9) hold. Based on equation (2.8) and (2.11), equation (2.9) will be as the following:

$$
\begin{aligned}
&\nabla_\sigma \left(\underset{i}{V}^{\beta} \overset{i}{V}_{\alpha} - \underset{j}{V}^{\beta} \overset{j}{V}_{\alpha} \right) = 0 \\
&\nabla_\sigma \left(\underset{i}{V}^{\beta} \overset{n+i}{V}_{\alpha} \right) = 0 \,, \\
&i, j = 1, 2 \quad \bar{i}, \bar{j} = 3, 4 \quad n = 2 \,.
\end{aligned}
$$

This has been studied in [3, 4, 6, 7, 8]. From equation (2.4) we have:

$$
\begin{aligned}
&\underset{i}{\overset{v}{T}}_{\sigma} \underset{v}{\overset{\beta}{v}} \overset{i}{v}_{\alpha} - \underset{v}{\overset{i}{T}}_{\sigma} \underset{i}{\overset{\beta}{v}} \overset{v}{v}_{\alpha} - \underset{i}{\overset{v}{T}}_{\sigma} \underset{v}{\overset{\beta}{v}} \overset{\bar{i}}{v}_{\alpha} + \underset{i}{\overset{\bar{i}}{T}}_{\sigma} \underset{\bar{i}}{\overset{\beta}{v}} \overset{v}{v}_{\alpha} = 0 \\
&\underset{i}{\overset{v}{T}}_{\sigma} \underset{v}{\overset{\beta}{v}} \overset{n+i}{v}_{\alpha} - \underset{v}{\overset{n+i}{T}}_{\sigma} \underset{i}{\overset{\beta}{v}} \overset{v}{v}_{\alpha} = 0.
\end{aligned}
\tag{2.12}
$$

From the equation (2.12) we have the following:

$$
\begin{aligned}
&\underset{k}{\overset{v}{T}}_{\sigma} \underset{v}{v}^{\beta} - \underset{k}{\overset{i}{T}}_{\sigma} \underset{i}{v}^{\beta} + \underset{k}{\overset{\bar{i}}{T}}_{\sigma} \underset{\bar{i}}{v}^{\beta} = 0 \,, \\
&\underset{s}{\overset{i}{T}}_{\sigma} \underset{i}{v}^{\beta} - \underset{\bar{s}}{\overset{v}{T}}_{\sigma} \underset{v}{v}^{\beta} + \underset{\bar{s}}{\overset{\bar{i}}{T}}_{\sigma} \underset{\bar{i}}{v}^{\beta} = 0 \,, \\
&\underset{s}{\overset{2+i}{T}}_{\sigma} \underset{i}{v}^{\beta} = 0 \,, \quad \underset{s}{\overset{v}{T}}_{\sigma} \underset{v}{v}^{\beta} - \underset{2+s}{\overset{2+i}{T}}_{\sigma} \underset{\bar{i}}{v}^{\beta} = 0 \,.
\end{aligned}
$$

If we work with independent vectors $\left\{ \underset{\alpha}{V}^{\beta} \right\}$ we will get the equation:

$$
\underset{s}{\overset{\bar{i}}{T}}_{\sigma} = 0 \,, \quad \underset{\bar{s}}{\overset{i}{T}}_{\sigma} = 0 \,, \quad \underset{s}{\overset{i}{T}}_{\sigma} - \underset{2+s}{\overset{2+i}{T}}_{\sigma} = 0 \,. \tag{2.13}
$$

If we use the net $\{V_\alpha\}$ of coordinate $\left(\underset{1}{V}^{\alpha}, \underset{2}{V}^{\alpha}, \underset{3}{V}^{\alpha}, \underset{4}{V}^{\alpha} \right)$ then the equation (2.13) would appear like the following:

$$
\begin{aligned}
&\Gamma_{\sigma}{}^{\bar{i}}_{s} = 0 \,, \quad \Gamma_{\sigma}{}^{i}_{\bar{s}} = 0 \,, \\
&\Gamma_{\sigma}{}^{i}_{s} - \Gamma_{\sigma}{}^{2+i}_{2+s} = 0 \,.
\end{aligned}
$$

Then $\Gamma_{\alpha}{}^{\sigma}_{\beta} = 0$ and we have that A_4 is affine. □

3. Cartesian products with additional structure

Let P^{β}_{α} be the affine in the relation (2.2). Then it is called paracontact affine and it holds:

$$
P^{\beta}_{\alpha} = \underset{i}{V}^{\beta} \overset{i}{V}_{\alpha} - \underset{3}{V}^{\beta} \overset{3}{V}_{\alpha} \,. \tag{3.1}
$$

We know that:

$$
\underset{i}{V}^{\alpha} \overset{i}{V}_{\beta} = \delta^{\alpha}_{\beta} \quad \Leftrightarrow \quad \underset{\sigma}{V}^{i} \underset{i}{V}^{v} = \delta^{v}_{\sigma} \,. \tag{3.2}
$$

From (3.1) and (3.2) we get that $P_{\alpha}^{\beta}P_{\beta}^{\sigma} = \delta_{\alpha}^{\beta} - \underset{4}{V}^{\beta}\overset{4}{V}_{\alpha}$. The affine (3.1) defines the paracontact structure in the space A_4, see [12, 13, 14, 16]. Using (3.2) and equations

$$\underset{1}{V}^{\alpha}(1,0,0,0)\,, \underset{2}{V}^{\alpha}(0,1,0,0)\,, \underset{3}{V}^{\alpha}(0,0,1,0)\,, \underset{4}{V}^{\alpha}(0,0,0,4)\,,$$

$$\overset{1}{V}_{\alpha}(1,0,0,0), \overset{2}{V}_{\alpha}(0,1,0,0), \overset{3}{V}_{\alpha}(0,0,1,0), \overset{4}{V}_{\alpha}(0,0,0,1)\,,$$

with parameters of coordinated net $\{V_{\alpha}\}$ the matrix (P_{α}^{β}) would look like the following:

$$P_{\alpha}^{\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Theorem 3.1. *The equality* $\nabla_{\sigma}P_{\alpha}^{\beta} = 0$ *is fulfilled if and only if it holds*

$$\overset{\bar{i}}{\underset{j}{T}}_{\sigma} = \overset{i}{\underset{\bar{j}}{T}}_{\sigma} = \overset{3}{\underset{4}{T}}_{\sigma} = \overset{4}{\underset{3}{T}}_{\sigma} = 0\,. \tag{3.3}$$

Proof. From relations (2.4) and (3.1) we can write the equation

$$\nabla_{\sigma}P_{\alpha}^{\beta} = 0 \tag{3.4}$$

like:

$$\overset{v}{\underset{i}{T}}_{\sigma}\underset{v}{v}^{\beta}\overset{i}{v}_{\sigma} - \overset{i}{\underset{v}{T}}_{\sigma}\underset{i}{v}^{\beta}\overset{v}{v}_{\alpha} - \overset{v}{\underset{3}{T}}_{\sigma}\underset{v}{v}^{s}\overset{3}{v}_{\alpha} + \overset{3}{\underset{v}{T}}_{\sigma}\underset{3}{v}^{s}\overset{v}{v}_{\alpha} = 0. \tag{3.5}$$

Using simple operation the equation (3.5) with $\underset{i}{V}^{\alpha}$ and$\underset{\bar{i}}{V}^{\alpha}$, and reading independence of vector fields $\underset{\alpha}{V}^{\beta}$ we get that the equation (3.3) and (3.4) are equivalent, proving the theorem. □

Next, using theorem 3.1 and equation

$$\Gamma_{\alpha\beta}^{\sigma} = \overset{\sigma}{\underset{\beta}{T}}_{\alpha}\,.$$

we can write the tensor of the curve $R_{\alpha,\beta,\sigma}^{v}$ in the space A_4 like the following:

$$R_{\alpha\beta\sigma}^{v} = \partial_{\alpha}\Gamma_{\beta\sigma}^{v} - \partial_{\beta}\Gamma_{\alpha\sigma}^{v} + \Gamma_{\alpha\gamma}^{v}\Gamma_{\beta\sigma}^{v} - \Gamma_{\beta\gamma}^{v}\Gamma_{\alpha\sigma}^{v}\,. \tag{3.6}$$

Corollary 3.1. *In parameters of coordinative net* $\{\underset{\alpha}{V}\}$ *and equation* (3.3) *we get the following equation*

$$\Gamma_{\sigma j}^{\bar{i}} = \Gamma_{\sigma\bar{j}}^{i} = \Gamma_{\sigma\bar{j}}^{3} = \Gamma_{\sigma 3}^{4} = 0. \tag{3.7}$$

Based on continuity and the relation (3.6), see [4, 13, 14] we have the following:

Corollary 3.2. *If affine* P_{α}^{β} *satisfies the condition* $\nabla_{\sigma}P_{\alpha}^{\beta} = 0$, *then the product* $X_2 \times \overline{X}_2$ *and* $X_3 \times X_1$ *are of the type* (C, C) *(Cartesian, Cartesian).*

Proof. If we take in the space A_4 with additional paracontact structure P_α^β with a new asymmetric connection, we will get

$$ {}^1\Gamma^v_{\alpha\beta} = \Gamma^v_{\alpha\beta} + {}^1A^v_{\alpha\beta} \tag{3.8}$$

Where ${}^1A^v_{[\alpha,\beta]}$ is torsion tensor with a new connection, written with ${}^1\nabla$ and ${}^1R^v_{\alpha,\beta,\sigma}$ is the coo-variation of derivation and the tensor of curve in relation to the ${}^1\Gamma^v_{\alpha\beta}$, see [7, 8, 9, 14]. □

Theorem 3.2. *If $\nabla_\sigma P_\alpha^\beta = 0$ and ${}^1\nabla_\sigma P_\alpha^\beta = 0$ then the tensor ${}^1A_{\alpha}{}^{v}_{\beta}$ satisfy the condition*

$$ {}^1A^{\bar{i}}_{\alpha j} = {}^1A^i_{\alpha\bar{j}} = {}^1A^3_{\alpha 4} = {}^1A^4_{\alpha 3} = 0 \,. \tag{3.9}$$

Also, in the contracting net $\{V_\alpha\}$ the parameters are replaced.

Proof. The equation ${}^1\nabla_\sigma P_\alpha^\beta = 0$, hold. Based on (3.4) and (3.8) the line of the curve is ${}^1\nabla_\sigma P_\alpha^\beta = L_{\sigma}{}^{\beta}_{\alpha}$. Then

$$ L_{\sigma}{}^{\beta}_{\alpha} = {}^1A_{\sigma}{}^{\beta}_{v} P^v_\alpha - {}^1A_{\sigma}{}^{v}_{\alpha} P^\beta_v \,. \tag{3.10}$$

Now it follows that (3.4) and (3.10) are equivalent.

Next, let us take the net $\{V_\alpha\}$ as a single coordinate $L^\beta_{\sigma\alpha}$ which is changeable from zero. We introduce the following:

$$ L^i_{\alpha\bar{j}} = \eta \cdot {}^1A_{\alpha}{}^{i}_{\bar{j}}, \quad L^i_{\alpha j} = \chi \cdot {}^1A^{\bar{i}}_{\alpha j} \,, \tag{3.11}$$

and $L^3_{\alpha 4} = \pi \cdot {}^1A^3_{\alpha 4}$, $L^4_{\alpha 3} = \mu \cdot {}^1A^4_{\alpha 3}$ for $\eta, \chi, \pi, \mu = \pm 1 \pm 2 \pm ...$
Now, from (3.11) we have (3.9).

According to equations (3.7), (3.8) and (3.9) we have that:

$$\begin{aligned} &{}^1\Gamma_{\alpha}{}^{i}_{\bar{j}} = \Gamma_{\alpha}{}^{i}_{\bar{j}} = 0\,, \quad {}^1\Gamma_{\alpha}{}^{\bar{i}}_{j} = \Gamma_{\alpha}{}^{\bar{i}}_{j} = 0\,, \\ &{}^1\Gamma_{\alpha}{}^{3}_{4} = \Gamma_{\alpha}{}^{3}_{4} = 0\,, \quad {}^1\Gamma_{\alpha}{}^{4}_{3} = \Gamma_{\alpha}{}^{4}_{3} = 0\,. \end{aligned} \tag{3.12}$$

Finally from equations (3.6), (3.9) and (3.12) we get the components of the tensor $R^v_{\alpha\beta\sigma}$ and ${}^1R^v_{\alpha\beta\sigma}$:

$$ {}^1R_{\alpha\beta}{}^{i}_{\bar{j}} = R_{\alpha\beta}{}^{i}_{\bar{j}} = {}^1R_{\alpha\beta}{}^{\bar{i}}_{j} = R_{\alpha\beta}{}^{\bar{i}}_{j} = {}^1R_{\alpha\beta}{}^{3}_{4} = R_{\alpha\beta}{}^{3}_{4} = {}^1R_{\alpha\beta}{}^{4}_{3} = R_{\alpha\beta}{}^{4}_{3} = 0\,. $$

□

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