ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **5** (2016), no.2, 179–189
ISSN 1857-8365 printed version
ISSN 1857-8438 electronic version

EXTENDED TRAVELING WAVE SOLUTIONS FOR SOME NONLINEAR EQUATIONS

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Abstract. The extended Kudryashov method (EKM) is executed to find the traveling wave solutions for the Benjamin-Bona-Mahony (m,n) and Sharma-Tasso-Olver equations. The effciency of this method for finding exact solutions and traveling wave solutions has been demonstrated and compared with the solutions obtained by classical method. It has been shown that the proposed method is effective, direct and can be used for many other nonlinear evolution equations (NLEEs) in mathematical physics.

1. Introduction

The study of nonlinear evolution equations has an intense period over the last decades and has continued to attract attention in more recent years. These equations are mathematical models of various physical phenomena that arise in many fields such as engineering, applied mathematics, dynamics, electromagnetic theory, nonlinear physics and so on. Thus, it is very important to search for exact traveling wave solutions of nonlinear evolution equations. Furthermore, when an original nonlinear equation is directly solved, the solution will preserve the actual physical characters of the equations.

In the past several decades many useful methods and techniques have been developed for finding exact traveling wave solutions to nonlinear evolution equations such as G'/G expansion method [2, 14], sine-cosine method [3, 12], exp-fuction method [4, 15], sub-equation method [5], functional variable method [7], multiple exp-function method [8], trial equation method [9], modified simple equation method [17], extended tanh method [24] and others.

In this study, we present an extended method by inspring of modified Kudryashov method which was first introduced by Kudryashov [19] and other similar methods refining the initial idea [6, 10, 11, 18, 21]. The advantage of this method over the other classical methods is that gives more solutions with some parameters which effects both (either) speed and (or) amplitude of waves. By choosing convenient parameter, solutions can be turned into certain solutions obtained by existing methods. It originated from the well-known the homogeneous balance principle.

2010 *Mathematics Subject Classification.* 35Q79; 35Q35; 35Q40.
Key words and phrases. Extended Kudryashov method, nonlinear evolution equations, Benjamin-Bona-Mahony equation, Sharma-Tasso-Olver equation.

The paper is organized as follows: In section 2,we give the algorithm of the method. In section 3, we apply the method to the Benjamin-Bona-Mahony (m,n) and Sharma-Tasso-Olver equations. At the end, conclusions are given.

2. Algorithm of the method

The features of this method can be presented as follows. Let us consider the nonlinear evolution equation (NEE) in several independent variables as:

$$P\left(u, u_t, u_x, u_y, u_z, u_{xy}, u_{yz}, u_{xz}, ...\right) = 0. \tag{2.1}$$

where the subscript denotes partial derivative, P is some function and $u = u(t, x, y, z, \ldots)$ is called a dependent variable or unknown function to be determined.
Step 1. We investigate the traveling wave solutions of equation (2.1) of the form:

$$u(x, y, z, \ldots, t) = u(\xi), \quad \xi = k(x + ct) \quad \texttt{or} \quad \xi = x - ct\,,$$

where k and c are arbitrary constants. Then equation (2.1) reduces to a nonlinear ordinary differential equation of the form:

$$G(u, u_\xi, u_{\xi\xi}, u_{\xi\xi\xi}, \ldots) = 0. \tag{2.2}$$

Step 2. We suppose that the solution of equation (2.2) has the following form:

$$u(\xi) = \sum_{i=0}^{N} a_i Q^i(\xi) \tag{2.3}$$

where $Q = \pm\frac{1}{\sqrt{1 \pm a^{2\xi}}}$ and the function Q is the solution of equation

$$Q_\xi = \ln a(Q^3 - Q)\,.$$

Step 3. In the view of the method, we presume that the solution of equation (2.2) can be pointed out in the form:

$$u(\xi) = a_N Q^N + \cdots\,. \tag{2.4}$$

The positive integer N in formula (2.4) that is the pole order for the general solution of Eq. (2.2). In order to calculate the value of N we consider the homogenous balance between the highest order nonlinear terms in Eq. (2.2). Supposing $u^l(\xi)u^s(\xi)$ and $(u^l(\xi))^r$ are the highest order nonlinear terms of Eq. (2.2) and balancing the highest order nonlinear terms we have:

$$N = \frac{2(s - rp)}{r - l - 1}\,.$$

Step 4. Substituting equation (2.3) into equation (2.2) and equating the coefficients of Q^i to zero, we obtain a system of algebraic equations. By solving this system with the aid of Mathematica, we get the traveling wave solutions of equation (2.2).

3. Applications

3.1. **Benjamin Bona Mahony (m,n) Equation.** We first apply the method to Benjamin Bona Mahony (m.n) equation in the form:

$$(u^l)_t + \alpha(u^l)_x + k(u^m)_x - b(u^n)_{xxt} = 0\,. \tag{3.1}$$

where α, k, b are constants and u is the function of (x,t).

This equation was first derived to characterize an approximation for surface long waves in nonlinear dispersive media. It can also describe the hydromagnetic waves in cold plasma, acoustic gravity waves in compressible fluids and acoustic waves in harmonic crystals [7, 13, 16, 20, 24].

By considering the traveling wave transformation:

$$u(x,t) = u(\xi), \quad \xi = s(x - \omega t)\,,$$

where $s, \omega \neq 0$ are constants, equation (3.1) can be reduced to the following ordinary differential equation:

$$(\alpha - \omega)u^l + ku^m + bms^2(u^n)'' = 0\,. \tag{3.2}$$

For $l = n$ and $m \neq n$, we use the transformation $u(\xi) = v^{\frac{1}{m-n}}(\xi)$, which will convert equation (3.2) into

$$(\alpha-\omega)(m-n)^2v^2 + k(m-n)^2v^3 + \omega bs^2[n(2n-m)(v')^2 + n(m-n)vv''] = 0\,. \tag{3.3}$$

Also we take $v(\xi) = \sum_{i=0}^{N} a_iQ^i$, where $Q(\xi) = \pm\frac{1}{(1\pm a^{2\xi})^{1/2}}$. We note that the function Q is the solution of $Q_\xi = \ln a(Q^3 - Q)$. Balancing the linear term of the highest order with the highest order nonlinear term in equation (3.3), we compute $N = 4$. Thus, we have

$$v(\xi) = a_0 + a_1Q(\xi) + a_2Q^2(\xi) + a_3Q^3(\xi) + a_4Q^4(\xi) \tag{3.4}$$

and substituting derivatives of $v(\xi)$ with respect to ξ in equation (3.4) The required derivatives in equation (3.3) are obtained

$$\begin{aligned} v_\xi &= \ln a(Q^3 - Q)(a_1 + 2a_2Q + 3a_3Q^2 + 4a_4Q^3), \\ v_{\xi\xi} &= \ln a(Q^3 - Q)\left[24a_4Q^5 + 15a_3Q^4 + (8a_2 - 16a_4)Q^3\right. \\ &\quad \left. + (3a_1 - 9a_3)Q^2 - 4a_2 - a_1\right] \end{aligned} \tag{3.5}$$

Substituting equation (3.4) and equation (3.5) into equation (3.3) and collecting the coefficient of each power of Q^i, setting each of coefficient to zero, solving the resulting system of algebraic equations we obtain the following solutions:

Case 1:

$$\begin{gathered} a_0 = 0, \quad a_1 = 0, \quad a_2 = a_2, \quad a_3 = 0, \quad a_4 = -a_2, \\ \alpha = \frac{[(m-n)^2 - 4sn^2s^2(\ln a)^2]\omega}{(m-n)^2}, \quad k = \frac{8bn(m+n)s^2\omega(\ln a)^2}{(n-m)^2a_2}\,. \end{gathered} \tag{3.6}$$

Inserting equation (3.6) into equation (3.4), we obtain the following solutions of equation (3.3):

$$v_1(\xi) = \frac{a_2}{5cFs^2\xi}\,, \qquad v_2(\xi) = -\frac{a_2}{5sFs^2\xi}\,.$$

Thus, we obtain new exact solutions to equation (3.1):

$$u_1(x,t) = \left(\frac{a_2}{5cFs^2(s(x-\omega t))}\right)^{\frac{1}{m-n}},$$
$$u_2(x,t) = \left(\frac{a_2}{5sFs^2(s(x-\omega t))}\right)^{\frac{1}{m-n}}.$$

Case 2:

$$(3.7)\quad a_0 = 0, \quad a_1 = 0, \quad a_2 = \frac{8bn\alpha s^2(m+n)(\ln a)^2}{k[(m-n)^2 - 4bn^2s^2(\ln a)^2]}, \quad a_3 = 0,$$
$$a_4 = -\frac{8bn\alpha s^2(m+n)(\ln a)^2}{k[(m-n)^2 - 4bn^2s^2(\ln a)^2]}, \quad \omega = \frac{(m-n)^2\alpha}{(m-n)^2 - 4bn^2s^2(\ln a)^2}.$$

Inserting equation (3.7) into equation (3.4), we get the following solutions of equation (3.3):

$$v_3(\xi) = \frac{8bn\alpha s^2(m+n)(\ln a)^2}{5k[(m-n)^2 - 4bn^2s^2(\ln a)^2]}\frac{1}{cFs^2\xi},$$
$$v_4(\xi) = -\frac{8bn\alpha s^2(m+n)(\ln a)^2}{5k[(m-n)^2 - 4bn^2s^2(\ln a)^2]}\frac{1}{cFs^2\xi}.$$

Traveling wave solutions to equation (3.1) are in the form:

$$u_3(x,t) = \left(\frac{8bn\alpha s^2(m+n)(\ln a)^2}{5k[(m-n)^2 - 4bn^2s^2(\ln a)^2]}\frac{1}{cFs^2\left(s(x-\left(\frac{(m-n)^2\alpha}{(m-n)^2-4bn^2s^2(\ln a)^2}\right)t\right)}\right)^{\frac{1}{m-n}},$$
$$u_4(x,t) = \left(\frac{8bn\alpha s^2(m+n)(\ln a)^2}{5k[(m-n)^2 - 4bn^2s^2(\ln a)^2]}\frac{1}{sFs^2\left(s(x-\left(\frac{(m-n)^2\alpha}{(m-n)^2-4bn^2s^2(\ln a)^2}\right)t\right)}\right)^{\frac{1}{m-n}}.$$

Case 3:

$$(3.8)\quad a_0 = 0, \quad a_1 = 0, \quad a_2 = a_2, \quad a_3 = 0, \quad a_4 = -a_2,$$
$$\alpha = \frac{2\omega(m+n) - kna_2}{2(m+n)}, \quad b = \frac{k(m-n)^2a_2}{8n\omega s^2(m+n)(\ln a)^2}.$$

Inserting equation (3.8) into equation (3.4), we obtain the following solutions of equation (3.3)

$$v_5(\xi) = \frac{a_2}{5cFs^2\xi}, \quad v_6(\xi) = -\frac{a_2}{5sFs^2\xi}.$$

We get the traveling wave solutions of equation (3.1):

$$u_5(x,t) = \left(\frac{a_2}{5cFs^2(s(x-\omega t))}\right)^{\frac{1}{m-n}},$$
$$u_6(x,t) = \left(\frac{a_2}{5sFs^2(s(x-\omega t))}\right)^{\frac{1}{m-n}}.$$

Case 4:

$$(3.9)\quad a_0 = 0, \quad a_1 = 0, \quad a_2 = a_2, \quad a_3 = 0, \quad a_4 = -a_2,$$
$$k = -\frac{2(m+n)(\alpha-\omega)}{na_2}, \quad b = -\frac{(m-n)^2(\alpha-\omega)}{4n^2s^2\omega(\ln a)^2}.$$

Inserting equation (3.9) into equation (3.4), we obtain the following solutions of equation (3.3):

$$v_7(\xi) = \frac{a_2}{5cFs^2\xi}\,, \quad v_8(\xi) = -\frac{a_2}{5sFs^2\xi}\,.$$

Hence we obtain new exact solutions to equation (3.1):

$$u_7(x,t) = \left(\frac{a_2}{5cFs^2(s(x-\omega t))}\right)^{\frac{1}{m-n}}, \quad u_8(x,t) = \left(-\frac{a_2}{5sFs^2(s(x-\omega t))}\right)^{\frac{1}{m-n}}.$$

Case 5:

$$\begin{aligned}(3.10)\qquad a_0 &= 0, \quad a_1 = 0, \quad a_2 = \frac{8bn(m+n)s^2w(\ln a)^2}{k(n-m)^2}, \quad a_3 = 0,\\ a_4 &= -\frac{8bn(m+n)s^2w(\ln a)^2}{k(n-m)^2}, \quad \alpha = \frac{w\left((m-n)^2 - 4bn^2s^2(\ln a)^2\right)}{(m-n)^2}\,.\end{aligned}$$

Inserting equaion (3.10) into equation (3.4), we obtain the following solutions of equation (3.3):

$$v_9(\xi) = \frac{8bn(m+n)s^2w(\ln a)^2}{5k(n-m)^2}\frac{1}{cFs^2\xi}\,, \quad v_{10}(\xi) = -\frac{8bn(m+n)s^2w(\ln a)^2}{5k(n-m)^2}\frac{1}{sFs^2\xi}\,.$$

Thus, we obtain the solutions to equation (3.1)

$$\begin{aligned}u_9(x,t) &= \left(\frac{8bn(m+n)s^2w(\ln a)^2}{5k(n-m)^2}\frac{1}{cFs^2(s(x-\omega t))}\right)^{\frac{1}{m-n}},\\ u_{10}(x,t) &= \left(-\frac{8bn(m+n)s^2w(\ln a)^2}{5k(n-m)^2}\frac{1}{cFs^2(s(x-\omega t))}\right)^{\frac{1}{m-n}}.\end{aligned}$$

Case 6:

$$\begin{aligned}(3.11)\quad a_0 &= 0, \quad a_1 = 0, \quad a_2 = a_2, \quad a_3 = 0, \quad a_4 = -a_2\\ \omega &= \frac{(m-n)^2\alpha}{(m-n)^2 - 4bn^2s^2(\ln a)^2}, \quad k = -\frac{8bn(m+n)s^2\alpha(\ln a)^2}{\left((m-n)^2 - 4bn^2s^2(\ln a)^2\right)a_2}\,.\end{aligned}$$

Inserting equation (3.11) into equation (3.4), we obtain the following solutions of equation (3.3):

$$v_{11}(\xi) = \frac{a_2}{5cFs^2\xi}\,, \quad v_{12}(\xi) = -\frac{a_2}{5sFs^2\xi}\,.$$

Hence we get the traveling wave solutions to equation (3.1)

$$\begin{aligned}u_{11}(x,t) &= \left(\frac{a_2}{5cFs^2\left(s\left(x - \left(\frac{(m-n)^2\alpha}{(m-n)^2-4bn^2s^2(\ln a)^2}\right)t\right)\right)}\right)^{\frac{1}{m-n}},\\ u_{12}(x,t) &= \left(-\frac{a_2}{5sFs^2\left(s\left(x - \left(\frac{(m-n)^2\alpha}{(m-n)^2-4bn^2s^2(\ln a)^2}\right)t\right)\right)}\right)^{\frac{1}{m-n}}.\end{aligned}$$

For $l \neq n$ and $m = n$, we use the transformation

$$u(\xi) = v^{\frac{1}{l-n}}(\xi)$$

which will convert equation (3.2) into

$$(3.12)\qquad (\alpha-\omega)(l-n)^2v^3 + k(l-n)^2v^2 + \omega bs^2[n(2n-l)(v')^2 + n(l-n)vv''] = 0\,.$$

Also we take

$$v(\xi) = \sum_{i=0}^{N} a_i Q^i \,,$$

where $Q(\xi) = \pm \dfrac{1}{(1 \pm a^{2\xi})^{1/2}}$. We note that the function Q is the solution of the equation $Q_\xi = \ln a(Q^3 - Q)$. Balancing the linear term of the highest order with the highest order nonlinear term in equation (3.12), we compute $N = 4$. Thus, we have

$$v(\xi) = a_0 + a_1 Q(\xi) + a_2 Q^2(\xi) + a_3 Q^3(\xi) + a_4 Q^4(\xi)\,, \tag{3.13}$$

and substituting derivatives of $v(\xi)$ with respect to ξ in equation (3.13). The required derivatives in equation (3.12) are obtained

$$\begin{aligned} v_\xi &= \ln a(Q^3 - Q)(a_1 + 2a_2 Q + 3a_3 Q^2 + 4a_4 Q^3), \\ v_{\xi\xi} &= \ln a(Q^3 - Q)\left[24 a_4 Q^5 + 15 a_3 Q^4 + (8a_2 - 16a_4)Q^3 \right. \\ &\quad \left. + (3a_1 - 9a_3)Q^2 - 4a_2 - a_1\right] . \end{aligned} \tag{3.14}$$

Substituting equation (3.13) and equation (3.14) into equation (3.12) and collecting the coefficient of each power of Q^i, setting each of coefficient to zero, solving the resulting system of algebraic equations we obtain the following solutions:

Case 1:

$$\begin{aligned} a_0 &= 0, \quad a_1 = 0, \quad a_2 = -\frac{8bn(l+n)s^2\omega^2(\ln a)^2}{(n-l)^2(\omega-\alpha)}, \quad a_3 = 0\,, \\ a_4 &= \frac{8bn(l+n)s^2\omega^2(\ln a)^2}{(n-l)^2(\omega-\alpha)}, \quad k - \frac{4bn^2s^2\omega(\ln a)^2}{(l-n)^2} . \end{aligned} \tag{3.15}$$

Inserting equation (3.15) into equation (3.13), we obtain the following solutions of equation (3.1):

$$\begin{aligned} u_1(x,t) &= \left(-\frac{8bn(l+n)s^2\omega^2(\ln a)^2}{5(n-l)^2(\omega-\alpha)cFs^2\left(s(x-\omega t)\right))}\right)^{\frac{1}{l-n}}, \\ u_2(x,t) &= \left(\frac{8bn(l+n)s^2\omega^2(\ln a)^2}{5(n-l)^2(\omega-\alpha)cFs^2\left(s(x-\omega t)\right))}\right)^{\frac{1}{l-n}}. \end{aligned}$$

Case 2:

$$\begin{aligned} a_0 &= 0, \quad a_1 = 0, \quad a_2 = -\frac{2k(l+n)}{n(\alpha-\omega)}, \quad a_3 = 0, \\ a_4 &= \frac{2k(l+n)}{n(\alpha-\omega)}, \quad s = -\frac{i\sqrt{k}}{2n\sqrt{b\omega}(\ln a)} \end{aligned} \tag{3.16}$$

Inserting equation (3.16) into equation (3.13), we get the following solutions of equation (3.1)

$$\begin{aligned} u_3(x,t) &= \left(-\frac{2k(l+n)}{5n(\alpha-\omega)cFs^2\left(-\frac{i\sqrt{k}}{2n\sqrt{b\omega}(\ln a)}(x-\omega t)\right)}\right)^{\frac{1}{l-n}}, \\ u_4(x,t) &= \left(\frac{2k(l+n)}{5n(\alpha-\omega)sFs^2\left(-\frac{i\sqrt{k}}{2n\sqrt{b\omega}(\ln a)}(x-\omega t)\right)}\right)^{\frac{1}{l-n}}. \end{aligned}$$

Case 3:

(3.17)
$$a_0 = 0, \quad a_1 = 0, \quad a_2 = -\frac{2k(l+n)}{n(\alpha-\omega)}, \quad a_3 = 0,$$
$$a_4 = \frac{2k(l+n)}{n(\alpha-\omega)}, \quad s = \frac{i\sqrt{k}}{2n\sqrt{b\omega}(\ln a)}.$$

Inserting equation (3.17) into equation (3.13), we obtain the following solutions of equation (3.1):

$$u_3(x,t) = \left(-\frac{2k(l+n)}{5n(\alpha-\omega)cFs^2\left(\frac{i\sqrt{k}}{2n\sqrt{b\omega}(\ln a)}(x-\omega t)\right)}\right)^{\frac{1}{l-n}},$$
$$u_4(x,t) = \left(\frac{2k(l+n)}{5n(\alpha-\omega)sFs^2\left(\frac{i\sqrt{k}}{2n\sqrt{b\omega}(\ln a)}(x-\omega t)\right)}\right)^{\frac{1}{l-n}}.$$

Case 4:

(3.18)
$$a_0 = 0, \quad a_1 = 0, \quad a_2 = a_2, \quad a_3 = 0, \quad a_4 = -a_2,$$
$$s = \frac{i\sqrt{k}(l-n)}{2n\sqrt{b\omega}(\ln a)}, \quad \alpha = -\frac{2k(l+n+n\omega a_2)}{na_2}.$$

Inserting equation (3.18) into equation (3.13), we get the following solutions of equation (3.1):

$$u_7(x,t) = \left(\frac{a_2}{5cFs^2\left(\frac{i\sqrt{k}(l-n)}{2n\sqrt{b\omega}(\ln a)}(x-\omega t)\right)}\right)^{\frac{1}{l-n}},$$
$$u_8(x,t) = \left(-\frac{a_2}{5sFs^2\left(\frac{i\sqrt{k}(l-n)}{2n\sqrt{b\omega}(\ln a)}(x-\omega t)\right)}\right)^{\frac{1}{l-n}}.$$

Case 5:

(3.19)
$$a_0 = 0, \quad a_1 = 0, \quad a_2 = a_2, \quad a_3 = 0, \quad a_4 = -a_2,$$
$$\alpha = -\frac{\omega\left(8bn(l+n)s^2(\ln a)^2 + (l-n)^2 a_2\right)}{(l-n)^2 a_2}, \quad k = -\frac{4bn^2s^2\omega(\ln a)^2}{(n-l)^2}.$$

Inserting equation (3.19) into equation (3.13), we obtain the following solutions of equation (3.1):

$$u_9(x,t) = \left(\frac{a_2}{5cFs^2\left(s(x-\omega t)\right)}\right)^{\frac{1}{l-n}},$$
$$u_{10}(x,t) = \left(-\frac{a_2}{5sFs^2\left(s(x-\omega t)\right)}\right)^{\frac{1}{l-n}}.$$

Case 6:

(3.20)
$$a_0 = 0, \quad a_1 = 0, \quad a_2 = a_2, \quad a_3 = 0, \quad a_4 = -a_2,$$
$$\alpha = \frac{-2k(l+n)+n\omega a_2}{na_2}, \quad b = -\frac{k(l-n)^2}{4n^2s^2\omega}(\ln a)^2.$$

Inserting equation (3.20) into equation (3.13), we get the following solutions of equation (3.1):

$$\begin{aligned} u_{11}(x,t) &= \left(\frac{a_2}{5cFs^2\left(s(x-\omega t)\right)}\right)^{\frac{1}{l-n}}, \\ u_{12}(x,t) &= \left(-\frac{a_2}{5sFs^2\left(s(x-\omega t)\right)}\right)^{\frac{1}{l-n}}. \end{aligned}$$

3.2. **Sharma-Tasso-Olver Equation.** Next, we consider the following Sharma-Tasso-Olver equation:

$$u_t+\alpha(u^3)_x+\frac{3}{2}\alpha(u^2)_{xx}+\alpha u_{xxx}=0\,, \tag{3.21}$$

where α is a real parameter and $u(x,t)$ is the unknown function that depends on the temporal variable t and the spatial variable x . The STO equation contains both linear dispersive term αu_{xxx} and the double nonlinear terms $\alpha(u^3)_x$ and $\frac{3}{2}\alpha(u^2)_{xx}$. This equation is well known as a model equation describing the propagation of nonlinear dispersive waves in inhomogeneous media. The STO equation attracted great interest among mathematicians and physicists due to its appearance in scientific applications [1, 14, 22, 23].

By using the traveling wave transformation:

$$u(x,t)=u(\xi),\quad \xi=x-ct\,,$$

where $c\neq 0$ is constant, equation (3.21) can be reduced to the following ordinary differential equation:

$$-cu+\alpha u^3+3\alpha uu'+\alpha u''=0. \tag{3.22}$$

Also we take

$$u(\xi)=\sum_{i=0}^{N}a_iQ^i\,,$$

where $Q(\xi)=\pm\dfrac{1}{(1\pm e^{2\xi})^{1/2}}$. Take in consideration that the function Q is the solution of $Q'=Q^3-Q$. Balancing the the linear term of the highest order with the highest order nonlinear term in equation (3.22), we compute $N=2$. Thus,

$$u(\xi)=a_0+a_1Q(\xi)+a_2Q^2(\xi)\,, \tag{3.23}$$

and substituting derivatives of $u(\xi)$ with respect to ξ in equation (3.23) we obtain

$$\begin{aligned} u'(\xi) &= 2a_2Q^4(\xi)+a_1Q^3(\xi)-2a_2Q^2(\xi)-a_1Q(\xi), \\ u''(\xi) &= 8a_2Q^6(\xi)+3a_1Q^5(\xi)-12a_2Q^4(\xi)-4a_1Q^3(\xi)+4a_2Q^2(\xi)+a_1Q(\xi). \end{aligned}$$

Substituting the derivatives into equation (3.22) and collecting the coefficient of each power of Q^i, setting each of coefficient to zero, solving the resulting system of algebraic equations we obtain the following solutions:

Case 1:

$$a_0=2,\quad a_1=0,\quad a_2=-4,\quad c=4\alpha\,. \tag{3.24}$$

Inserting equation (3.24) into equation (3.23), we obtain the following solutions of equation (3.21):

$$\begin{aligned} u_1(x,t) &= 2\tanh(x-4\alpha t)\,, \\ u_2(x,t) &= 2\coth(x-4\alpha t)\,. \end{aligned}$$

Case 2:

$$a_0 = 1, \quad a_1 = 0, \quad a_2 = -2, \quad c = \alpha\,. \tag{3.25}$$

Inserting equation (3.25) into equation (3.23), we get the following solutions of equation (3.21):

$$\begin{aligned} u_3(x,t) &= \tanh(x-\alpha t)\,, \\ u_4(x,t) &= \coth(x-\alpha t)\,. \end{aligned}$$

Case 3:

$$a_0 = 2, \quad a_1 = 0, \quad a_2 = -2, \quad c = 4\alpha\,. \tag{3.26}$$

Inserting equation (3.26) into equation (3.23), the following solutions of equation (3.21):

$$\begin{aligned} u_5(x,t) &= 2\left(1 - \frac{1}{1+\cosh(2x-8\alpha t)+\sinh(2x-8\alpha t)}\right)\,, \\ u_6(x,t) &= 2\left(1 - \frac{1}{1-(\cosh(2x-8\alpha t)+\sinh(2x-8\alpha t))}\right)\,, \end{aligned}$$

are obtained.

Case 4:

$$a_0 = 0, \quad a_1 = 0, \quad a_2 = -2, \quad c = 4\alpha\,. \tag{3.27}$$

Inserting equation (3.27) into equation (3.23), we obtain the following solutions of equation (3.21):

$$\begin{aligned} u_7(x,t) &= \tanh(x-4\alpha t) - 1\,, \\ u_8(x,t) &= \coth(x-4\alpha t) - 1\,. \end{aligned}$$

4. Conclusion

In this work, extended Kudryashov method is proposed to construct exact solutions of evolution equations with constant coefficients. By using the proposed method we have successfully obtained analytical solutions of the BBM(m,n) equation and the Sharma Tasso Olver (STO) equation.Besides the Kudryashov method, more traveling wave solution cases are obtained. In addition, change in the parameters effects both the wave length and speed of the wave. The obtained solutions may have importance for some special physical phenomena. It can be concluded that this method is standard, effective and also allows to solve complicated algebraic calculation.

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