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THRIAD GEODESIC COMPOSITION IN FOUR DIMENSIONAL SPACE WITH AN AFFINE CONNECTEDNESS WITHOUT A TORSION AND ADDITIONS

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ABSTRACT. Let A_4 be an affinity connected space without a torsion. Following [7] we introduce the affinors a_α^β, b_α^β and $\widetilde{c}_\alpha^\beta = ic_\alpha^\beta = -ia_\sigma^\beta b_\alpha^\sigma$ $(i^2=-1)$ which define the compositions $X_2 \times \overline{X}_2$, $Y_2 \times \overline{Y}_2$ and $Z_2 \times \overline{Z}_2$, respectively. The first two composition are conjugate. The composition $U_2 \times \overline{U}_2$ generated by the affinor $d_\alpha^\beta = a_\alpha^\beta + b_\alpha^\beta + c_\alpha^\beta$ is considered too. We have found necessary and sufficient condition for any of the above composition to be of the kind $(g-g)$.

Four dimensional spaces with a symmetric affine connection and additional structures p (paracontact, semi-cyclic) are investigated. The spaces which contain such structures are defined. Nonsymmetric affine connections so that the affinors of the structures continue to translate paralelly along the lines of the space are introduced and investigated.

1. PRELIMINARY

Let A_N be a space with a symmetric affine connectedness without a torsion, defined by $\Gamma_{\alpha\beta}^\gamma$. Let consider a composition $X_n \times X_m$ of two differentiable basic manifolds X_n and X_m $(n+m=N)$ in the space A_N. For every point of the space of compositions A_N $(X_n \times X_m)$ there are two position of the basic manifolds, which we denotes by $P(X_n)$ and $P(X_m)$ ([3]). The defining of composition in the space A_N is equivalent to defining of a field of an affinor a_α^β that satisfies the condition [2] and [3].

$$a_\sigma^\beta a_\alpha^\sigma = \delta_\alpha^\beta. \tag{1.1}$$

The affinor a_α^β is called an affinor of the composition [2]. According to [3] and [5] the condition for integrability of the structure is $a_\beta^\sigma \nabla_{[\alpha a_\sigma^\nu]} - a_\alpha^\sigma \nabla_{[\beta a_\sigma^\nu]} = 0$. The projective affinors $\overset{n}{a}{}_\alpha^\sigma$ and $\overset{m}{a}{}_\alpha^\sigma$ ([3],[4]), defined by the equations $\overset{n}{a}{}_\alpha^\sigma = \frac{1}{2}(\delta_\alpha^\beta + a_\alpha^\beta)$, $\overset{m}{a}{}_\alpha^\sigma = \frac{1}{2}(\delta_\alpha^\beta - a_\alpha^\beta)$ satisfy the condition $\overset{n}{a}{}_\alpha^\beta + \overset{m}{a}{}_\alpha^\beta = \delta_\alpha^\beta$, $\overset{n}{a}{}_\alpha^\beta - \overset{m}{a}{}_\alpha^\beta = a_\alpha^\beta$. For every vector $v^\alpha \in A_N$ $(X_n \times X_m)$ we have $v^\alpha = \overset{n}{a}{}_\beta^\alpha v^\beta + \overset{m}{a}{}_\beta^\alpha v^\beta = \overset{n}{V}{}^\alpha + \overset{m}{V}{}^\alpha$, where $\overset{n}{V}{}^\alpha = \overset{n}{a}{}_\beta^\alpha v^\beta \in P(X_n)$, $\overset{m}{V}{}^\alpha = \overset{m}{a}{}_\beta^\alpha v^\beta \in P(X_m)$ [4].

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The composition $X_n \times X_m \in A_N$ $(n+m=N)$ for which the position $P(X_n)$ and $P(X_m)$ are paralelly translated along any line X_n and X_m, respectively is called composition of kind $(g-g)$ ([3]) or geodesic composition [6]. According to [3] the geodesic composition is characterized with the equality

$$a^{\sigma}_{\alpha}\nabla_{\beta}a^{v}_{\sigma} + a^{\sigma}_{\beta}\nabla_{\sigma}a^{v}_{\alpha} = 0. \tag{1.2}$$

Let A_4 be a space with affine connectedness without a torsion, defined by $\Gamma_{\alpha}\beta^{\sigma}$ $(\alpha,\beta,\sigma = 1,2,3,4)$. Let v^{α}_{1}, v^{α}_{2}, v^{α}_{3}, v^{α}_{4} are independent vector fields in A_4. Following [7] we defined the convectors $\overset{\sigma}{v}_{\alpha}$ by the equalities

$$\underset{\alpha}{v}^{\beta}\overset{\alpha}{v}^{\sigma} = \delta^{\beta}_{\sigma} \quad \Leftrightarrow \quad \underset{\alpha}{v}^{\beta}\overset{\sigma}{v}_{\beta} = \delta^{\sigma}_{\alpha}. \tag{1.3}$$

According to [6] and [7] we can define the affinor

$$a^{\beta}_{\alpha} = \underset{1}{v}^{\beta}\overset{1}{v}_{\alpha} + \underset{2}{v}^{\beta}\overset{2}{v}_{\alpha} - \underset{3}{v}^{\beta}\overset{3}{v}_{\alpha} - \underset{4}{v}^{\beta}\overset{4}{v}_{\alpha}, \tag{1.4}$$

that satisfies the equations (1.1). The affinor (1.4) defines a composition $(X_n \times X_m)$ in A_4. The projective affinors of the composition $(X_n \times X_m)$ are ([7]):

$$\overset{1}{a}{}^{\beta}_{\alpha} = \underset{1}{v}^{\beta}\overset{1}{v}_{\alpha} + \underset{2}{v}^{\beta}\overset{2}{v}_{\alpha}, \quad \overset{2}{a}{}^{\beta}_{\alpha} = \underset{3}{v}^{\beta}\overset{3}{v}_{\alpha} + \underset{4}{v}^{\beta}\overset{4}{v}_{\alpha}.$$

Following [7] we choose the net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}, \underset{4}{v})$ for a coordinate one. Then we have

$$\begin{aligned} &\underset{1}{v}^{\alpha}(1,0,0,0), \quad \underset{2}{v}^{\alpha}(0,1,0,0), \quad \underset{3}{v}^{\alpha}(0,0,1,0), \quad \underset{4}{v}^{\alpha}(0,0,0,1), \\ &\underset{1}{v}_{\alpha}(1,0,0,0), \quad \underset{2}{v}_{\alpha}(0,1,0,0), \quad \underset{3}{v}_{\alpha}(0,0,1,0), \quad \underset{4}{v}_{\alpha}(0,0,0,1). \end{aligned} \tag{1.5}$$

Let consider the vectors ([7]):

$$\underset{1}{w}^{\alpha} = \underset{1}{v}^{\alpha} + \underset{3}{v}^{\alpha}, \quad \underset{2}{w}^{\alpha} = \underset{2}{v}^{\alpha} + \underset{4}{v}^{\alpha}, \quad \underset{3}{w}^{\alpha} = \underset{1}{v}^{\alpha} - \underset{3}{v}^{\alpha}, \quad \underset{4}{w}^{\alpha} = \underset{2}{v}^{\alpha} - \underset{4}{v}^{\alpha}. \tag{1.6}$$

We define the convectors w_{σ} by the equalities

$$\underset{\alpha}{w}^{v}\overset{\alpha}{w}_{\sigma} = \delta^{v}_{\sigma} \quad \Leftrightarrow \quad \underset{\alpha}{w}^{\sigma}\overset{\beta}{w}_{\sigma} = \delta^{\beta}_{\alpha}. \tag{1.7}$$

From (1.3) and (1.7) follows

$$\overset{1}{w}_{\alpha} = \frac{1}{2}\left(\overset{1}{v}_{\alpha} + \overset{3}{v}_{\alpha}\right), \quad \overset{2}{w}_{\alpha} = \frac{1}{2}\left(\overset{2}{v}_{\alpha} + \overset{4}{v}_{\alpha}\right), \quad \overset{3}{w}_{\alpha} = \frac{1}{2}\left(\overset{1}{v}_{\alpha} - \overset{3}{v}_{\alpha}\right), \quad \overset{4}{w}_{\alpha} = \frac{1}{2}\left(\overset{2}{v}_{\alpha} - \overset{4}{v}_{\alpha}\right).$$

Let consider the affinor

$$b^{\beta}_{\alpha} = \underset{1}{w}^{\beta}\overset{1}{w}_{\alpha} + \underset{2}{w}^{\beta}\overset{2}{w}_{\alpha} - \underset{3}{w}^{\beta}\overset{3}{w}_{\alpha} - \underset{4}{w}^{\beta}\overset{4}{w}_{\alpha}, \tag{1.8}$$

which according to [7] satisfies the equality $b^{\beta}_{\alpha}b^{\alpha}_{\sigma} = \delta^{\beta}_{\sigma}$. Therefore the affinor (1.8) defines a composition $Y_2 \times \overline{Y}_2$ in A_4. According to [7] the composition $X_2 \times \overline{X}_2$ and $Y_2 \times \overline{Y}_2$ are conjugate. By (1.3), (1.6), (1.7) and (1.8) we obtained

$$b^{\beta}_{\alpha} = \underset{1}{v}^{\beta}\overset{3}{v}_{\alpha} + \underset{3}{v}^{\beta}\overset{1}{v}_{\alpha} - \underset{2}{v}^{\beta}\overset{4}{v}_{\alpha} - \underset{4}{v}^{\beta}\overset{2}{v}_{\alpha}. \tag{1.9}$$

Following [7] let consider the affinor $c^{\beta}_{\sigma} = -a^{\beta}_{\alpha}b^{\alpha}_{\sigma}$, which satisfies the equality $c^{\beta}_{\sigma}c^{\sigma}_{\alpha} = -\delta^{\beta}_{\alpha}$. With the help of (1.3), (1.4), (1.9) we establish

$$c^{\beta}_{\alpha} = \underset{3}{v}^{\beta}\overset{1}{v}_{\alpha} - \underset{1}{v}^{\beta}\overset{3}{v}_{\alpha} + \underset{4}{v}^{\beta}\overset{2}{v}_{\alpha} - \underset{2}{v}^{\beta}\overset{4}{v}_{\alpha}. \tag{1.10}$$

The affinor $\tilde{c}_{\alpha}^{\beta} = i c_{\alpha}^{\beta}$, where $i^2 = -1$, defines a composition $Z_2 \times \overline{Z}_2$ in A_4.

2. Geodesic composition in space A_4

According to [8] we have the following derivative equations

$$\nabla_{\sigma} \underset{\alpha}{v}^{\beta} = \underset{\alpha^{\sigma}}{\overset{\sigma}{T}} \underset{v}{v}^{\beta}, \quad \nabla_{\sigma} \overset{\sigma}{v}_{\beta} = -\underset{v^{\sigma}}{\overset{\sigma}{T}} \overset{v}{v}_{\beta}. \tag{2.1}$$

Let consider the composition $X_2 \times \overline{X}_2$ and accept: $\alpha, \beta, \gamma, \sigma, v, \tau \in \{1,2,3,4\}$; $i, j, k, s \in \{1,2\}$, $\bar{i}, \bar{j}, \bar{k}, \bar{s} \in \{3,4\}$.

Theorem 1. *The composition $X_2 \times \overline{X}_2$ is of the kind $(g-g)$ if, and only if, the coefficients of the derivative equations* (2.1) *satisfy the conditions*

$$\underset{k^{\alpha}}{\overset{\bar{i}}{T}} \underset{s}{v}^{\alpha} = 0 \quad \textit{and} \quad \underset{\bar{k}^{\alpha}}{\overset{i}{T}} \underset{\bar{s}}{v}^{\alpha} = 0. \tag{2.2}$$

Proof. According to (1.4) and (2.1) we have

$$\begin{aligned} \nabla_{\beta} a_{\sigma}^{v} = {} & \underset{1^{\beta}}{\overset{\tau}{T}} \underset{\tau}{v}^{v} \overset{1}{v}_{\sigma} - \underset{\tau^{\beta}}{\overset{1}{T}} \underset{1}{v}^{v} \overset{\tau}{v}_{\sigma} + \underset{2^{\beta}}{\overset{\tau}{T}} \underset{\tau}{v}^{v} \overset{2}{v}_{\sigma} - \underset{\tau^{\beta}}{\overset{2}{T}} \underset{2}{v}^{v} \overset{\tau}{v}_{\sigma} \\ & - \underset{3^{\beta}}{\overset{\tau}{T}} \underset{\tau}{v}^{v} \overset{3}{v}_{\sigma} + \underset{\tau^{\beta}}{\overset{3}{T}} \underset{3}{v}^{v} \overset{\tau}{v}_{\sigma} - \underset{4^{\beta}}{\overset{\tau}{T}} \underset{\tau}{v}^{v} \overset{4}{v}_{\sigma} + \underset{\tau^{\beta}}{\overset{4}{T}} \underset{4}{v}^{v} \overset{\tau}{v}_{\sigma}. \end{aligned} \tag{2.3}$$

Taking into account the independence of convector $\overset{\sigma}{v}_{\alpha}$ and using (1.2), (1.3), (1.4) and (2.3), we find the equalities

$$\begin{aligned} &(\delta_{\beta}^{\sigma} + \alpha_{\beta}^{\sigma}) \left(\underset{1^{\sigma}}{\overset{3}{T}} \underset{3}{v}^{v} + \underset{1^{\sigma}}{\overset{4}{T}} \underset{4}{v}^{v} \right) = 0, \quad (\delta_{\beta}^{\sigma} + \alpha_{\beta}^{\sigma}) \left(\underset{1^{\sigma}}{\overset{3}{T}} \underset{3}{v}^{v} + \underset{1^{\sigma}}{\overset{4}{T}} \underset{4}{v}^{v} \right) = 0 \\ &(\delta_{\beta}^{\sigma} - \alpha_{\beta}^{\sigma}) \left(\underset{3^{\sigma}}{\overset{1}{T}} \underset{3}{v}^{v} + \underset{3^{\sigma}}{\overset{2}{T}} \underset{4}{v}^{v} \right) = 0, \quad (\delta_{\beta}^{\sigma} - \alpha_{\beta}^{\sigma}) \left(\underset{4^{\sigma}}{\overset{1}{T}} \underset{3}{v}^{v} - \underset{4^{\sigma}}{\overset{2}{T}} \underset{4}{v}^{v} \right) = 0. \end{aligned} \tag{2.4}$$

Because of the independence of vectors $\underset{\alpha}{v}^{v}$ it follows an equivalence of (2.4) to the following equalities.

$$\begin{aligned} &\underset{1^{\beta}}{\overset{3}{T}} + \alpha_{\beta}^{\sigma} \underset{1^{\sigma}}{\overset{3}{T}} = 0, \quad \underset{1^{\beta}}{\overset{4}{T}} + \alpha_{\beta}^{\sigma} \underset{1^{\sigma}}{\overset{4}{T}} = 0, \quad \underset{2^{\beta}}{\overset{3}{T}} + \alpha_{\beta}^{\sigma} \underset{2^{\sigma}}{\overset{3}{T}} = 0, \quad \underset{2^{\beta}}{\overset{4}{T}} + \alpha_{\beta}^{\sigma} \underset{2^{\sigma}}{\overset{4}{T}} = 0, \\ &\underset{3^{\beta}}{\overset{1}{T}} - \alpha_{\beta}^{\sigma} \underset{3^{\sigma}}{\overset{1}{T}} = 0, \quad \underset{3^{\beta}}{\overset{2}{T}} - \alpha_{\beta}^{\sigma} \underset{3^{\sigma}}{\overset{2}{T}} = 0, \quad \underset{4^{\beta}}{\overset{1}{T}} - \alpha_{\beta}^{\sigma} \underset{4^{\sigma}}{\overset{1}{T}} = 0, \quad \underset{4^{\beta}}{\overset{2}{T}} - \alpha_{\beta}^{\sigma} \underset{4^{\sigma}}{\overset{2}{T}} = 0. \end{aligned} \tag{2.5}$$

Now it is easy to see that equalities (2.2) follow after contraction by $\underset{1}{v}^{\beta}$ and $\underset{2}{v}^{\beta}$ for the first four equalities of (2.5) and by $\underset{3}{v}^{\beta}$ and $\underset{4}{v}^{\beta}$ for the last four equalities of (2.5). Let's note that the equalities (2.5) are proved in [6] by another approach. ☐

Corollary 1. *If the net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}, \underset{4}{v})$ is chosen as a coordinate one then the composition $X_2 \times \overline{X}_2$ form the kind $(g-g)$ characterized by the following equalities.*

(*i*) *The coefficient of the derivative equations*

$$\underset{k^{s}}{\overset{\bar{i}}{T}} = 0, \quad \underset{\bar{k}^{\bar{s}}}{\overset{i}{T}} = 0. \tag{2.6}$$

(*ii*) *The coefficient of the connectedness*

$$\Gamma^{\bar i}_{sk} = 0, \quad \Gamma^{i}_{\bar s \bar k} = 0. \tag{2.7}$$

Proof. Let choose the net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}, \underset{4}{v})$ for a coordinate one. Then by (1.5) and (2.2) we find (2.6). According to [1] and (2.1) we can write $\partial_\sigma \underset{\alpha}{v}^\beta + \Gamma^\beta_{\sigma v} \underset{\alpha}{v}^\beta = \overset{v}{\underset{\alpha^\sigma}{T}} \underset{v}{v}^\beta$ from where using (1.5) we obtain

$$\Gamma^\beta_{\sigma\alpha} = \overset{\beta}{\underset{\alpha^\sigma}{T}}. \tag{2.8}$$

The equalities (2.7) follow from (2.6) and (2.8). Let's note that the equalities (2.7) are obtained in [3] when the coordinates are adaptive with composition $X_2 \times \overline{X}_2$. This happens so, because the chosen coordinate net raises adaptive with the composition coordinative. □

From (2.7) and $R^v_{\alpha\beta\sigma} = 2\partial_{[\alpha}\Gamma^v_{\beta]\sigma} - 2\Gamma^\tau_{\sigma[\alpha}\Gamma^v_{\beta]\tau}$ [1] we establish the validity of the following statement:

Fact 1. When the composition $X_2 \times \overline{X}_2$ is of the kind $(g - g)$ then the parameters of the coordinate net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}, \underset{4}{v})$ the tensor of curvature satisfy the conditions $R^{\bar s}_{\alpha\beta\sigma} = 0$ and $R^{\bar s}_{\bar i \bar j \bar k} = 0$.

Theorem 2. *The composition $Y_2 \times \overline{Y}_2$ is of the kind $(g - g)$ if, and only if, the coefficient od the derivative equations satisfy the condition:*

$$\begin{array}{ll}
\left(\overset{1}{\underset{1^\sigma}{T}} - \overset{3}{\underset{3^\sigma}{T}}\right)\underset{1}{v}^\sigma + \left(\overset{1}{\underset{3^\sigma}{T}} - \overset{3}{\underset{1}{T}}\right)\underset{3}{v}^\sigma = 0, & \left(\overset{1}{\underset{1^\sigma}{T}} - \overset{3}{\underset{3^\sigma}{T}}\right)\underset{3}{v}^\sigma + \left(\overset{1}{\underset{3^\sigma}{T}} - \overset{3}{\underset{1}{T}}\right)\underset{1}{v}^\sigma = 0, \\
\left(\overset{1}{\underset{1^\sigma}{T}} - \overset{3}{\underset{3^\sigma}{T}}\right)\underset{2}{v}^\sigma + \left(\overset{1}{\underset{3^\sigma}{T}} - \overset{3}{\underset{1}{T}}\right)\underset{4}{v}^\sigma = 0, & \left(\overset{1}{\underset{1^\sigma}{T}} - \overset{3}{\underset{3^\sigma}{T}}\right)\underset{1}{v}^\sigma + \left(\overset{1}{\underset{3^\sigma}{T}} - \overset{3}{\underset{1}{T}}\right)\underset{2}{v}^\sigma = 0, \\
\left(\overset{2}{\underset{1^\sigma}{T}} - \overset{4}{\underset{3^\sigma}{T}}\right)\underset{1}{v}^\sigma + \left(\overset{2}{\underset{3^\sigma}{T}} - \overset{4}{\underset{1}{T}}\right)\underset{3}{v}^\sigma = 0, & \left(\overset{2}{\underset{1^\sigma}{T}} - \overset{4}{\underset{3^\sigma}{T}}\right)\underset{3}{v}^\sigma + \left(\overset{2}{\underset{3^\sigma}{T}} - \overset{4}{\underset{1}{T}}\right)\underset{1}{v}^\sigma = 0, \\
\left(\overset{2}{\underset{1^\sigma}{T}} - \overset{4}{\underset{3^\sigma}{T}}\right)\underset{2}{v}^\sigma + \left(\overset{2}{\underset{3^\sigma}{T}} - \overset{4}{\underset{1}{T}}\right)\underset{4}{v}^\sigma = 0, & \left(\overset{2}{\underset{1^\sigma}{T}} - \overset{4}{\underset{3^\sigma}{T}}\right)\underset{4}{v}^\sigma + \left(\overset{2}{\underset{3^\sigma}{T}} - \overset{4}{\underset{1}{T}}\right)\underset{2}{v}^\sigma = 0.
\end{array} \tag{2.9}$$

Proof. Because of equalities (1.9) and (2.2) we have

$$\begin{aligned}
\nabla_\sigma b^\beta_\alpha = {} & \overset{v}{\underset{1^\sigma}{T}}\underset{v}{v}^\beta \overset{3}{v}_\alpha - \overset{3}{\underset{v^\sigma}{T}}\underset{1}{v}^\beta \overset{v}{v}_\alpha + \overset{v}{\underset{3^\sigma}{T}}\underset{v}{v}^\beta \overset{1}{v}_\alpha - \overset{1}{\underset{v^\sigma}{T}}\underset{3}{v}^\beta \overset{v}{v}_\alpha \\
& + \overset{v}{\underset{3^\sigma}{T}}\underset{v}{v}^\beta \overset{4}{v}_\alpha - \overset{4}{\underset{v^\sigma}{T}}\underset{2}{v}^\beta \overset{v}{v}_\alpha + \overset{v}{\underset{4^\sigma}{T}}\underset{v}{v}^\beta \overset{2}{v}_\alpha - \overset{2}{\underset{v^\sigma}{T}}\underset{4}{v}^\beta \overset{v}{v}_\alpha.
\end{aligned} \tag{2.10}$$

Transforming the condition $b^\sigma_\alpha \nabla_\beta b^v_\sigma + b^\sigma_\beta \nabla_\sigma b^v_\alpha = 0$ with the help of (1.3), (1.9), (2.10) and using the independence of convector $\underset{\sigma}{v}^\alpha$ we obtain the following equalities:

$$\begin{array}{ll}
\overset{1}{\underset{1^\beta}{T}} - \overset{3}{\underset{3^\beta}{T}} + b^\sigma_\beta(\overset{1}{\underset{3^\beta}{T}}{}_\sigma - \overset{3}{\underset{1^\beta}{T}}{}_\sigma) = 0, & \overset{2}{\underset{1^\beta}{T}} - \overset{4}{\underset{3^\beta}{T}} + b^\sigma_\beta(\overset{2}{\underset{3^\beta}{T}}\sigma - \overset{4}{\underset{1^\beta}{T}}{}_\sigma) = 0 \\
\overset{1}{\underset{1^\beta}{T}} - \overset{3}{\underset{4^\beta}{T}} + b^\sigma_\beta(\overset{1}{\underset{4^\beta}{T}}{}_\sigma - \overset{3}{\underset{2^\beta}{T}}{}_\sigma) = 0, & \overset{2}{\underset{2^\beta}{T}} - \overset{4}{\underset{4^\beta}{T}} + b^\sigma_\beta(\overset{2}{\underset{4^\beta}{T}}{}_\sigma - \overset{4}{\underset{2^\beta}{T}}{}_\sigma) = 0.
\end{array} \tag{2.11}$$

Now, after contraction by $\underset{\sigma}{v}^\alpha$ it is easy to see the equivalence of (2.11) to (2.9). □

Corollary 2. *If the net* $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}, \underset{4}{v})$ *is chosen as a condition one then the composition* $Y \times \overline{Y}_2$ *form the kind* $(g-g)$ *characterizes by the following equalities for:*

(i) *the coefficients of the derivative equations*

$$(2.12)\qquad \begin{aligned} &\underset{1^\alpha}{\overset{1}{T}} - \underset{3^\alpha}{\overset{3}{T}} = \underset{1^{\overline{\alpha}}}{\overset{3}{T}} - \underset{3^{\overline{\alpha}}}{\overset{1}{T}}, \qquad \underset{1^\alpha}{\overset{2}{T}} - \underset{3^\alpha}{\overset{4}{T}} = \underset{1^\alpha}{\overset{4}{T}} - \underset{3^{\overline{\alpha}}}{\overset{2}{T}},\\ &\underset{2^\alpha}{\overset{1}{T}} - \underset{4^\alpha}{\overset{3}{T}} = \underset{2^{\overline{\alpha}}}{\overset{3}{T}} - \underset{4^{\overline{\alpha}}}{\overset{1}{T}}, \qquad \underset{2^\alpha}{\overset{2}{T}} - \underset{4^\alpha}{\overset{4}{T}} = \underset{2^{\overline{\alpha}}}{\overset{4}{T}} - \underset{4^{\overline{\alpha}}}{\overset{2}{T}}; \end{aligned}$$

(ii) *the coefficient of the connectedness*

$$(2.13)\qquad \Gamma^\alpha_{11} + \Gamma^\alpha_{33} = 2\Gamma^{\overline{\alpha}}_{13}, \quad \Gamma^\alpha_{22} + \Gamma^\alpha_{44} = 2\Gamma^{\overline{\alpha}}_{24}, \quad \Gamma^\alpha_{12} + \Gamma^\alpha_{34} = 2\Gamma^{\overline{\alpha}}_{14} + \Gamma^{\overline{\alpha}}_{23},$$

as when α *accepst consecutively the values* $1, 2, 3, 4$, *then* $\overline{\alpha}$ *accepts the values* $3, 4, 1, 2$, *respectively.*

Proof. Let choose the net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}, \underset{4}{v})$ for a coordinate net. With the help of (1.5) and (2.9) we find (2.12). Then by (2.8) and (2.12) we obtain (2.13). □

Theorem 3. *The composition* $Z_2 \times \overline{Z}_2$ *is of the kind* $(g-g)$ *if, and only if, the coefficients of the derivative equations* (2.1) *satisfy the conditions*

$$(2.14)\qquad \begin{aligned} &(\underset{1^\sigma}{\overset{1}{T}} - \underset{3^\sigma}{\overset{3}{T}})\underset{1}{v}^\sigma = (\underset{3^\sigma}{\overset{1}{T}} + \underset{1^\sigma}{\overset{3}{T}})\underset{3}{v}^\sigma, \quad (\underset{1^\sigma}{\overset{1}{T}} - \underset{3^\sigma}{\overset{3}{T}})\underset{2}{v}^\sigma = (\underset{3^\sigma}{\overset{1}{T}} + \underset{1^\sigma}{\overset{3}{T}})\underset{4}{v}^\sigma,\\ &(\underset{3^\sigma}{\overset{3}{T}} - \underset{1^\sigma}{\overset{1}{T}})\underset{3}{v}^\sigma = (\underset{3^\sigma}{\overset{1}{T}} + \underset{1^\sigma}{\overset{3}{T}})\underset{1}{v}^\sigma, \quad (\underset{3^\sigma}{\overset{3}{T}} - \underset{1^\sigma}{\overset{1}{T}})\underset{4}{v}^\sigma = (\underset{3^\sigma}{\overset{1}{T}} + \underset{1^\sigma}{\overset{3}{T}})\underset{2}{v}^\sigma,\\ &(\underset{1^\sigma}{\overset{2}{T}} - \underset{3^\sigma}{\overset{4}{T}})\underset{1}{v}^\sigma = (\underset{3^\sigma}{\overset{2}{T}} + \underset{1^\sigma}{\overset{4}{T}})\underset{3}{v}^\sigma, \quad (\underset{1^\sigma}{\overset{2}{T}} - \underset{3^\sigma}{\overset{4}{T}})\underset{2}{v}^\sigma = (\underset{3^\sigma}{\overset{2}{T}} + \underset{1^\sigma}{\overset{1}{T}})\underset{4}{v}^\sigma,\\ &(\underset{3^\sigma}{\overset{4}{T}} - \underset{1^\sigma}{\overset{2}{T}})\underset{3}{v}^\sigma = (\underset{3^\sigma}{\overset{2}{T}} + \underset{1^\sigma}{\overset{1}{T}})\underset{1}{v}^\sigma, \quad (\underset{4^\sigma}{\overset{3}{T}} - \underset{1^\sigma}{\overset{4}{T}})\underset{4}{v}^\sigma = (\underset{3^\sigma}{\overset{2}{T}} + \underset{1^\sigma}{\overset{1}{T}})\underset{2}{v}^\sigma. \end{aligned}$$

Proof. By the equalities (1.10) and (2.2) we obtain

$$(2.15)\qquad \begin{aligned} \nabla_\sigma c^\beta_\alpha = {}&\underset{3^\sigma v}{\overset{v}{T}} v^\beta \overset{1}{v}_\alpha - \underset{v^\sigma 3}{\overset{1}{T}} v^\beta \overset{v}{v}_\alpha - \underset{1^\sigma v}{\overset{v}{T}} v^\beta \overset{3}{v}_\alpha + \underset{v^\sigma 1}{\overset{3}{T}} v^\beta \overset{v}{v}_\alpha\\ &+ \underset{4^\sigma v}{\overset{v}{T}} v^\beta \overset{2}{v}_\alpha - \underset{v^\sigma 4}{\overset{2}{T}} v^\beta \overset{v}{v}_\alpha - \underset{2^\sigma v}{\overset{v}{T}} v^\beta \overset{2}{v}_\alpha - \underset{v^\sigma 2}{\overset{4}{T}} v^\beta \overset{v}{v}_\alpha. \end{aligned}$$

Transforming the condition $c^\sigma_\alpha \nabla_\beta c^v_\sigma + c^\sigma_\beta \nabla_\sigma c^v_\alpha = 0$ with the help of (1.3) , (1.10), (2.15) and using the independence of the convectors $v\sigma_\alpha$ we obtain the following equalities

$$(2.16)\qquad \begin{aligned} &\underset{3^\beta}{\overset{3}{T}} - \underset{1^\beta}{\overset{1}{T}} + c^\sigma_\beta(\underset{3^\sigma}{\overset{1}{T}} + \underset{1^\sigma}{\overset{3}{T}}) = 0, \qquad \underset{3^\beta}{\overset{4}{T}} - \underset{1^\beta}{\overset{2}{T}} + c^\sigma_\beta(\underset{3^\sigma}{\overset{2}{T}} + \underset{1^\sigma}{\overset{4}{T}}) = 0,\\ &\underset{4^\beta}{\overset{3}{T}} - \underset{2^\beta}{\overset{1}{T}} + c^\sigma_\beta(\underset{2^\sigma}{\overset{3}{T}} + \underset{4^\sigma}{\overset{1}{T}}) = 0, \qquad \underset{4^\beta}{\overset{4}{T}} - \underset{2^\beta}{\overset{2}{T}} + c^\sigma_\beta(\underset{4^\sigma}{\overset{2}{T}} + \underset{2^\sigma}{\overset{4}{T}}) = 0. \end{aligned}$$

Now, after contraction by $\underset{\sigma}{v}^\alpha$ it is easy to see the equivalence of (2.16) to (2.14). □

Fact 2: *If two of the compositions* $X_2 \times \overline{X}_2$, $Y_2 \times \overline{Y}_2$, $Z_2 \times \overline{Z}_2$ *are from the kind* $(g-g)$ *then the third composition is also of the kind* $(g-g)$.

Since from (2.7) and (2.13) follows

$$\Gamma^{\alpha}_{ij} = \Gamma^{\alpha}_{\bar{i}\bar{j}} = 0, \quad \Gamma^{\alpha}_{13} = \Gamma^{\alpha}_{24} = 0, \quad \Gamma^{\alpha}_{14} + \Gamma_2 3^{\alpha} = 0,$$

we can formulate

Fact 3: *When the compositions $X_2 \times \overline{X}_2$, $Y_2 \times \overline{Y}_2$, $Z_2 \times \overline{Z}$ are of the kind $(g-g)$ then in the parameters of the coordinate net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}, \underset{4}{v})$ the tensor of curvature satisfies the conditions*

$$R^{\overline{S}}_{ijk} = R^{S}_{\bar{i}\bar{j}\bar{k}}, \quad R^{\alpha}_{133} = R^{\alpha}_{244} = R^{\alpha}_{311} = R^{\alpha}_{422} = R^{\alpha}_{143} = R^{\alpha}_{234} = R^{\alpha}_{321} = R^{\alpha}_{412} = 0.$$

Let consider the affinor

$$d^{\beta}_{\alpha} = a^{\beta}_{\alpha} + b^{\beta}_{\alpha} + c^{\beta}_{\alpha}. \tag{2.17}$$

According to (1.3), (1.4), (1.8) and (1.10) we have

$$a^{\beta}_{\alpha} b^{\alpha}_{\sigma} + b^{\beta}_{\alpha} a^{\alpha}_{\sigma} = 0, \quad b^{\beta}_{\alpha} c^{\alpha}_{\sigma} + c^{\beta}_{\alpha} b^{\alpha}_{\sigma} = 0, \quad c^{\beta}_{\alpha} a^{\alpha}_{\sigma} + a^{\beta}_{\alpha} c^{\alpha}_{\sigma} = 0. \tag{2.18}$$

From (2.17) and (2.18) it follows $d^{\beta}_{\alpha} d^{\alpha}_{\sigma} = a^{\beta}_{\alpha} a^{\alpha}_{\sigma} + b^{\beta}_{\alpha} b^{\alpha}_{\sigma} + c^{\beta}_{\alpha} c^{\alpha}_{\sigma} = \delta^{\beta}_{\alpha} + \delta^{\beta}_{\alpha} - \delta^{\beta}_{\alpha} = \delta^{\beta}_{\alpha}$, which means that the affinor d^{β}_{α} defines a composition $U_2 \times \overline{U}_2$ with the positions $P(U_2)$ and $P(\overline{U}_2)$.

Theorem 4. *The composition $U_2 \times \overline{U}_2$ is of the kind $(g-g)$ if, and only if, coefficients of the derivative equations* (2.1) *satisfy the conditions*

$$\underset{\bar{k}^{\beta}}{\overset{S}{T}} - d^{\sigma}_{\beta} \underset{\bar{k}^{\beta}}{\overset{S}{T}} = 0, \tag{2.19}$$

$$\underset{k^{\beta}}{\overset{\overline{S}}{T}} + \underset{k+2^{\beta}}{\overset{\overline{S}}{T}} - \underset{k^{\beta}}{\overset{\overline{S}-2}{T}} - 2 \underset{k+2^{\beta}}{\overset{\overline{S}-2}{T}} + d^{\sigma}_{\beta} \left(\underset{k^{\sigma}}{\overset{\overline{S}}{T}} + \underset{k+2^{\sigma}}{\overset{\overline{S}}{T}} - \underset{k^{\sigma}}{\overset{\overline{S}-2}{T}} \right) = 0. \tag{2.20}$$

Proof. According to (1.2) the composition $U_2 \times \overline{U}_2$ will be of the kind $(g-g)$ if, and only if,

$$d^{\sigma}_{\alpha} \nabla_{\beta} d^{v}_{\sigma} + d^{\sigma}_{\beta} \nabla_{\sigma} d^{v}_{\alpha} = 0. \tag{2.21}$$

With the help of (1.4), (1.8), (1.10) and (2.17) we find

$$d^{v}_{\sigma} = a^{v}_{\sigma} + 2 \left(\underset{3}{v}^{v} \overset{1}{v}_{\sigma} + \underset{4}{v}^{v} \overset{2}{v}_{\sigma} \right) = \underset{i}{v}^{v} \overset{i}{v}_{\sigma} - \underset{\bar{i}}{v}^{v} \overset{\bar{i}}{v}_{\sigma} + 2 \underset{2+i}{v}{}^{v} \overset{i}{v}_{\sigma}. \tag{2.22}$$

Then (2.21) can be written in the form

$$\begin{aligned} & d^{\sigma}_{\alpha} \left(\underset{i^{\beta}\,\delta}{\overset{\delta}{T}} v^{v} \overset{i}{v}_{\sigma} - \underset{\delta^{\beta}\,i}{\overset{i}{T}} v^{v} \overset{\delta}{v}_{\sigma} - \underset{\bar{i}^{\beta}\,\delta}{\overset{\delta}{T}} v^{v} \overset{\bar{i}}{v}_{\sigma} + \underset{\delta^{\beta}\,\bar{i}}{\overset{\bar{i}}{T}} v^{v} \overset{\delta}{v}_{\sigma} + 2 \underset{2+i^{\beta}\,\delta}{\overset{\delta}{T}} v^{v} \overset{i}{v}_{\sigma} - 2 \underset{\delta^{\beta}\,2+i}{\overset{i}{T}} v\, {}^{v} \overset{\delta}{v}_{\sigma} \right) \\ & + d^{\sigma}_{\beta} \left(\underset{i^{\beta}\,\delta}{\overset{\delta}{T}} v^{v} \overset{i}{v}_{\sigma} - \underset{\delta^{\beta}\,i}{\overset{i}{T}} v^{v} \overset{\delta}{v}_{\sigma} - \underset{\bar{i}^{\beta}\,\delta}{\overset{\delta}{T}} v^{v} \overset{\bar{i}}{v}_{\sigma} + \underset{\delta^{\beta}\,\bar{i}}{\overset{\bar{i}}{T}} v^{v} \overset{\delta}{v}_{\sigma} + 2 \underset{2+i^{\sigma}\,\delta}{\overset{\delta}{T}} v^{v} \overset{i}{v}_{\sigma} - 2 \underset{\delta^{\sigma}\,2+i}{\overset{i}{T}} v\, {}^{v} \overset{\delta}{v}_{\sigma} \right) = 0. \end{aligned} \tag{2.23}$$

The equalities received from (2.23) after contraction by $\underset{k}{v}$ and $\underset{\bar{k}}{v}^{\alpha}$ are contracted once again by $\overset{s}{v}_{v}$ and $\overset{\bar{s}}{v}_{v}$. As result of these operations we reach (2.19) and (2.20). □

Corollary 3. *If the net* $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}, \underset{4}{v})$ *is chosen as coordinate one then the composition* $U_2 \times \overline{U}_2$ *from the kind* $(g-g)$ *characterizes by the following equalities for:*

(i) *the coefficients of the derivative equations*

$$\underset{\overline{K}^{\bar{i}}}{\overset{s}{T}} = 0, \tag{2.24}$$

$$\underset{k^i}{\overset{\bar{s}}{T}} + \underset{k+2^i}{\overset{\bar{s}}{T}} - \underset{k^i}{\overset{\bar{s}}{T}} - \underset{k+2^i}{\overset{\bar{s}-2}{T}} + \underset{k^{i+2}}{\overset{\bar{s}}{T}} + \underset{k+2^{i+2}}{\overset{\bar{s}}{T}} - \underset{k^{i+2}}{\overset{\bar{s}-2}{T}} = 0;$$

(ii) *the coefficients of the connectedness*

$$\Gamma^{s}_{\overline{K}\,\bar{i}} = 0, \tag{2.25}$$

$$\Gamma^{\bar{s}}_{i\,k} + \Gamma^{\bar{s}}_{i\,k+2} - \Gamma^{\bar{s}-2}_{i\,k} - \Gamma^{\bar{s}-2}_{i\,k+2} + \Gamma^{\bar{s}-2}_{i+2\,k} + \Gamma^{\bar{s}-2}_{i+2\,k+2} - \Gamma^{\bar{s}-2}_{i+2\,k} = 0.$$

Proof. Let choose the net $(\underset{2}{v}, \underset{3}{v}, \underset{3}{v}, \underset{4}{v})$ as coordinate one. Then taking into account (1.4), (1.5) and (2.22) we find the following presentation of the affinor d_α^β,

$$(d_\alpha^\beta) = \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{2.26}$$

From (2.19), (2.20) and (2.26) we obtain the equalities (2.24), from where according to (2.8) follows (2.25). □

From [2] and the first equations of (2.24) it follows the validity of the statement:

Fact 4: If the composition $U \times \overline{U}_2$ is of the kind $(g-g)$, then the composition $X_2 \times \overline{X}_2$ is of the kind $(X_2 - g)$, i.e., the positions $P(\overline{X}_2)$ are parallel translated along any line of $\overline{X}_2$.

3. Spaces A_4 with additional structures.

Let us consider the following affinor

$$L_\alpha^\beta = \underset{1}{v}^\beta \overset{i}{v}_\alpha - \underset{3}{v}^\beta \overset{3}{v}_\alpha. \tag{3.1}$$

From (1.1) and Corollay 2(i) we obtain $L_\alpha^\beta L_\beta^\sigma = \delta_\alpha^\beta - \underset{4}{v}^\beta \overset{4}{v}_\alpha$, which means that the affinor (3.1) defines a paracontact structure in A_4. According to (1.5) and (3.1) in the parameters of the coordinate net $\{\underset{a}{v}\}$ the matrix (L_α^β) has the following presentation:

$$(L_\alpha^\beta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{3.2}$$

Theorem 5. *The equality $\nabla_\sigma L^\beta_\alpha = 0$ is fulfilled if, and only if, the coefficients from the derivative equations satisfy the conditions*

$$\underset{s^\sigma}{\overset{\bar{i}}{T}} = \underset{\bar{s}^\sigma}{\overset{i}{T}} = \underset{4^\sigma}{\overset{3}{T}} = \underset{3^\sigma}{\overset{4}{T}} = 0. \tag{3.3}$$

Proof. By (2.1) and (3.1) we write the equality

$$L^\sigma_\gamma \nabla_\sigma L^\beta_\alpha = 0 \tag{3.4}$$

in the following way

$$\underset{i^\sigma\, v}{\overset{v}{T}} v^\beta \overset{i}{v}_\alpha - \underset{v^\sigma\, i}{\overset{i}{T}} v^\beta \overset{i}{v}_\alpha - \underset{\bar{3}^\sigma\, v}{\overset{v}{T}} v^\beta \overset{3}{v}_\alpha + \underset{v^\sigma\, 3}{\overset{3}{T}} v^\beta \overset{v}{v}_\alpha = 0.$$

Using the contraction of the last equality with $\underset{s}{v}^\alpha$ and $\underset{\bar{s}}{v}^\alpha$ and reading the independence of the vector fields $\underset{\alpha}{v}^\beta$ we obtain the equivalence of (3.3) and (3.4). □

From the Theorem 5 and (2.7) it follows

Corollary 4. *In the parameters of the coordinate net $\{\underset{a}{v}\}$ the equalities* (3.3) *accept the presentation*

$$\Gamma^{\bar{i}}_{\sigma s} = \Gamma^{i}_{\sigma \bar{s}} = \Gamma^{3}_{\sigma 4} = \Gamma^{4}_{\sigma 3} = 0. \tag{3.5}$$

Corollary 5. *If the affinor L^β_α satisfies the condition $L^\sigma_\gamma \nabla_\sigma L^\beta_\alpha = 0$ then the compositions $X_2 \times \overline{X}_2$ and $X_3 \times \overline{X}_1$ are of the type $(g - g1)$.*

Let us consider in the space A_4 with an additional paracontact structure L^β_α the following new nonsymmetric connection

$$^1\Gamma^v_{\alpha\beta} = \Gamma^v_{\alpha\beta} +^1 S^v_{\alpha\beta}, \tag{3.6}$$

where $S^v_{[\alpha\beta]}$ is the tensor of the torsion in the new connection. Denote by $^1\nabla$ and $^1R^v_{\alpha\beta\sigma}$ the covariant derivative and the tensor of the curvature for the connection $^1\Gamma^v_{\alpha\beta}$, respectively.

Theorem 6. *If $\nabla_\sigma L^\beta_\alpha = 0$, then $^1\nabla_\sigma L^\beta_\alpha = 0$ if, and only if, the tensor $^1S^v_{\alpha\beta}$ satisfies the conditions*

$$^1S^{\bar{i}}_{\alpha s} =^1 S^{i}_{\alpha \bar{s}} =^1 S^{3}_{\alpha 4} =^1 S^{4}_{\alpha 3} = 0 \tag{3.7}$$

in the parameters of the coordinate net $\{\underset{a}{v}\}$.

Proof. Let the equalities (3.4) and

$$^1\nabla_\sigma L^\beta_\alpha = 0 \tag{3.8}$$

are fulfilled. From (3.4) and (3.6) it follows $^1\nabla_\sigma L^\beta_\alpha = K^\beta_{\sigma\alpha} = 0$ are equivalent. Let choose the net $\{\underset{a}{v}\}$ as a coordinate one. From (3.2) and (3.8) we find for the components of the tensor $K^\beta_{\sigma\alpha}$, which are different from zero, the following presentation

$$^1K^i_{\alpha\bar{j}} = \lambda^1 S^i_{\alpha\bar{j}}, \quad K^{\bar{i}}_{\alpha j} = \mu^1 S^{\bar{i}}_{\alpha j}, \quad K^3_{\alpha 4} = v^1 S^3_{\alpha 4}, \quad K^4_{\alpha 3} = t^1 S^4_{\alpha 3}, \tag{3.9}$$

where $\lambda, \mu, v, t = \pm 1, \pm 2$. Now from (3.9) it follows (3.7).

By (3.5), (3.6) and (3.7) we establish

$$(3.10) \qquad {}^1\Gamma^i_{\alpha\bar{j}} = \Gamma^i_{\alpha\bar{j}}, \quad {}^1\Gamma^{\bar{i}}_{\alpha\bar{j}} = \Gamma^{\bar{i}}_{\alpha\bar{j}} = 0, \quad {}^1\Gamma^3_{\alpha 4} = \Gamma^3_{\alpha 4} = 0, \quad {}^1\Gamma^4_{\alpha 3} = \Gamma^4_{\alpha 3} = 0.$$

By (3.7) and (3.10) we find for the components of the tensors $R_\alpha \beta \sigma^v$ and $R^v_{\alpha\beta\sigma}$:

$${}^1R^i_{\alpha\beta\bar{j}} = R^i_{\alpha\beta\bar{j}} = {}^1R^{\bar{i}}_{\alpha\beta j} = R^{\bar{i}}_{\alpha\beta j} = {}^1R^3_{\alpha\beta 4} = R^3_{\alpha\beta 4} = {}^1R^4_{\alpha\beta 3} = R^4_{\alpha\beta 3} = 0.$$

□

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