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COMPOSITIONS IN AFFINE SPACE WITH TWO MANIFOLDS

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ABSTRACT. Let us consider the compositions of two basic manifolds which will complete the condition of tangent that are parallelly with two manifolds lines. Compositions with two manifolds are completed with symmetric affinors, affine connections in nets spaces, with Chebyshevian and geodesic compositions.

Let us take A_{2n} symmetric affine space. In space A_{2n} the compositions $X_n \times \overline{X}_n$, $X_n \times Y_n$, $X_n \times Z_n$ and $X_n \times \overline{X}_n$, $Y_n \times \overline{X}_n$, $Z_n \times \overline{X}_n$ are defined. Each of these compositions have its basic manifolds. Spaces with these products are distinct. Some of the curve tensor components with non-symmetric connections are computed with coordinative compositions. Two special cases of compositions with basic manifolds affinors and tensor are discussed as they are Weyl connections and affine connection, see [2, 3, 4, 5, 6, 9, 10].

1. INTRODUCTION

The spaces A_{2n} of compositions of two basic manifolds are presented with symmetric affine connections, see [1, 3, 4, 7]. Types of compositions $X_n \times \overline{X}_n$, $X_n \times Y_n$, $X_n \times Z_n$ and $X_n \times \overline{X}_n$, $Y_n \times \overline{X}_n$, $Z_n \times \overline{X}_n$, are obtained by using the conditions of space connections translated parallelly and quasi-parallelly with positions $P(X)$ and $P(\overline{X}_n)$, as well as the space compositions of Weyl's are presented in [4, 5]. These spaces have been studied in [1], and last spaces we consider as spaces in net forms with symmetric affine connections, with two types of compositions with two basic manifolds. In the first case we know that composition is completely defined by affinor field a_α^β which complete the condition, see [2, 4, 5, 6, 10]:

$$a_\alpha^\sigma \cdot a_\sigma^\beta = \delta_\alpha^\beta.$$

The affinor a_α^β is called an affinor of the composition [2].

In the second case we compute some of the curve tensor components, with connections that are presented in a way as adapted with coordinated compositions. So, we denote the two special cases about symmetric connections which are mentioned above.

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2. PRELIMINARIES

Let take the space A_N with the symmetrical affine connections, which we define with $\Gamma^{\gamma}_{\alpha\beta}$, where α, β and γ are connection coefficient. In A_N we consider the compositions $X_n \times X_m$ $(n+m=N)$, of two basic variable manifolds. The positions (plans tangents) of the compositions are defined with $P(X_m)$ and $P(X_n)$, see [1, 2, 4, 7].

In the first case the composition is defined by the affinors field a^{β}_{α} which completes the condition given in [1, 2].

The projective affinors $\overset{1}{a}{}^{\beta}_{\alpha}$ and $\overset{2}{a}{}^{\beta}_{\alpha}$, are defined with $\overset{1}{a}{}^{\beta}_{\alpha} = \frac{1}{2}(\delta^{\beta}_{\alpha}+a^{\beta}_{\alpha})$ and $\overset{2}{a}{}^{\beta}_{\alpha} = \frac{1}{2}(\delta^{\beta}_{\alpha}-a^{\beta}_{\alpha})$ and satisfy the conditions $\overset{1}{a}{}^{\beta}_{\alpha} + \overset{2}{a}{}^{\beta}_{\alpha} = \delta^{\beta}_{\alpha}$ and $\overset{1}{a}{}^{\beta}_{\alpha} - \overset{2}{a}{}^{\beta}_{\alpha} = a^{\beta}_{\alpha}$.

In the second case for the usual coordinates u^{α} $(\alpha = 1,\ldots,n)$ in A_N we define $(u^i, u^{\bar{i}})$ where $(i=1,\ldots,n)$ and $(\bar{i} = n+1, n+2, \ldots, n+m)$ which fit the compositions $X_n \times X_m$, see [3, 6].

For each vector V^{α}, we have

$$V^{\alpha} = \overset{1}{a}{}^{\beta}_{\sigma} V^{\sigma} + \overset{2}{a}{}^{\beta}_{\sigma} V^{\sigma} = \overset{1}{V}{}^{\alpha} + \overset{2}{V}{}^{\alpha},$$

where: $\overset{1}{V}{}^{\alpha} = \overset{1}{a}{}^{\beta}_{\sigma} V^{\sigma} \in P(X_m)$ and $\overset{2}{V}{}^{\alpha} = \overset{2}{a}{}^{\beta}_{\sigma} V^{\sigma} \in P(X_n)$.

In the coordinates of the adopted compositions, the affinor matrices a^{β}_{α}, $\overset{m}{a}{}^{\beta}_{\alpha}$ and $\overset{n}{a}{}^{\beta}_{\alpha}$ have the form (2.1):

$$a^{\beta}_{\alpha} = \begin{bmatrix} \delta^i_j & 0 \\ 0 & -\delta^{\bar{i}}_{\bar{j}} \end{bmatrix}, \quad \overset{m}{a}{}^{\beta}_{\alpha} = \begin{bmatrix} \delta^i_j & 0 \\ 0 & 0 \end{bmatrix}, \quad \overset{n}{a}{}^{\beta}_{\alpha} = \begin{bmatrix} 0 & 0 \\ 0 & -\delta^{\bar{i}}_{\bar{j}} \end{bmatrix}. \tag{2.1}$$

The compositions of the type $(Chebyshev, -)$ or $(ch, -)$ or the compositions $(-, ch)$ for which the positions $P(X_m)$ and $P(X_n)$ are parallelly translated along any basic manifolds X_n and X_m are characterized by the following conditions:

$$\overset{2}{a}{}^{\sigma}_{\alpha} \overset{1}{a}{}^{\nu}_{\sigma} \nabla_{\sigma} \overset{1}{a}{}^{\beta}_{\nu} = 0 \quad \text{or} \quad \overset{1}{a}{}^{\sigma}_{\alpha} \overset{2}{a}{}^{\nu}_{\delta} \nabla_{\sigma} \overset{2}{a}{}^{\beta}_{\nu} = 0. \tag{2.2}$$

The composition of type (ch, ch) for which the positions $P(X_m)$ and $P(X_n)$ are translated parallel every line of manifolds X_n and X_m respectively are characterized by condition (2.2) and as a result we have:

$$\nabla_{[\alpha} a^{\sigma}_{\beta]} = 0.$$

The compositions of the type $(G, -)$ or compositions $(-, G)$ for which the compositions of positions $P(X_m)$ and $P(X_n)$ are parallelly translated along every line of manifolds X_m and X_n are characterized by the condition:

$$\overset{1}{a}{}^{\sigma}_{\alpha} \overset{2}{a}{}^{\nu}_{\delta} \nabla_{\sigma} \overset{1}{a}{}^{\beta}_{\nu} = 0 \quad \text{or} \quad \overset{2}{a}{}^{\sigma}_{\alpha} \overset{2}{a}{}^{\nu}_{\delta} \nabla_{\sigma} \overset{2}{a}{}^{\beta}_{\nu} = 0. \tag{2.3}$$

The compositions of the type (G, G) for which the positions $P(X_m)$ and $P(X_n)$ are parallelly translated in the basic manifolds X_m and X_n are characterized by two conditions of the relation (2.3) or by:

$$a^{\sigma}_{\nu} \nabla_{\sigma} a^{\beta}_{\sigma} + a^{\sigma}_{\alpha} \nabla_{\sigma} a^{\beta}_{\nu} = 0. \tag{2.4}$$

According [3] positions $P(X_m)$ and $P(X_n)$ are translated in queasy parallelly along the lines of basic manifolds X_m and X_n, respectively, if the projected affinors complete the condition:

$$\overset{2}{a}{}^{\sigma}_{\alpha} \overset{1}{a}{}^{\nu}_{\delta} \nabla_{\sigma} \overset{1}{a}{}^{\beta}_{\nu} - \psi_{\sigma} \overset{1}{a}{}^{\sigma}_{\delta} \overset{2}{a}{}^{\beta}_{\alpha} = 0 \quad \text{or} \quad \overset{1}{a}{}^{\sigma}_{\alpha} \overset{2}{a}{}^{\nu}_{\delta} \nabla_{\sigma} \overset{2}{a}{}^{\beta}_{\nu} - \psi_{\sigma} \overset{2}{a}{}^{\sigma}_{\delta} \overset{1}{a}{}^{\beta}_{\alpha} = 0. \tag{2.5}$$

In [3] it is proved that by the compositions coordinates which are denoted below with components φ_K and $\varphi_{\overline{K}}$ for the vector $\varphi_\nu = \frac{1}{2} a^\sigma_\alpha \nabla_\sigma a^\alpha_\nu$ differ from vectors ψ_K and $\psi_{\overline{K}}$ with only one constant.

Let the space n-dimensional be A_{2n} formed with the symmetric affine connection. Let $\underset{1}{V}^\beta$ $(\alpha = 1, 2, \ldots, 2n)$ be the independent vector field, whereas the reciprocal convector $\overset{\alpha}{V}_\beta$ is defined by:

$$\underset{\sigma}{V}^\alpha \overset{\sigma}{V}_\beta = \delta^\beta_\alpha \quad \Leftrightarrow \quad \underset{\sigma}{V}^\alpha \overset{\nu}{V}_\alpha = \delta^\nu_\sigma.$$

The coefficients of connection in the following are:

$$\begin{aligned} \alpha, \beta, \gamma, \delta, \nu, \ldots &= 1, 2, \ldots, 2n \\ i, j, s, k, \ldots &= 1, 2, \ldots, n \\ \bar{i}, \bar{j}, \bar{s}, \bar{k}, \ldots &= n+1, n+2, \ldots, 2n. \end{aligned}$$

The projected affinors of composition $X_n \times \overline{X}_n$ are in the form (see [2, 3, 4, 5, 6, 8]):

$$\overset{1}{a}{}^\beta_\alpha = \underset{i}{V}^\alpha \overset{i}{V}_\alpha \quad \text{and} \quad \overset{2}{a}{}^\beta_\alpha = \underset{\bar{i}}{V}^\beta \overset{\bar{i}}{V}_\alpha.$$

According to [2] the variable equations:

$$\nabla_\sigma \underset{\alpha}{V}^\beta = \underset{\alpha}{\overset{\nu}{T}}_\sigma \underset{\nu}{V}^\beta \quad \text{and} \quad \nabla_\sigma \overset{\alpha}{V}_\beta = -\underset{\nu}{\overset{\alpha}{T}}_\sigma \underset{\nu}{V}^\beta. \tag{2.6}$$

hold. If we take the vectors $\underset{1}{V}, \underset{2}{V}, \ldots, \underset{2n}{V}$ to be a coordinating net, we have :

$$\begin{aligned} &\underset{1}{V}^\alpha(1,0,0,\ldots,0,0); \underset{2}{V}^\alpha(0,1,0,\ldots,0,0); \ldots; \underset{2n}{V}^\alpha(0,0,0,\ldots,0,1) \\ &\overset{1}{V}_\alpha(1,0,0,\ldots,0,0); \overset{2}{V}_\alpha(0,1,0,\ldots,0,0); \ldots; \overset{2n}{V}_\alpha(0,0,0,\ldots,0,1). \end{aligned} \tag{2.7}$$

In this case $\underset{1}{V}, \underset{2}{V}, \ldots, \underset{2n}{V}$ defines the system with adopted coordinates compositions in space $X_n \times \overline{X}_n$.

According to equations (2.5), (2.6) and (2.7), we see the parameters of connected coordinative net and it holds:

$$\Gamma^\sigma_{\alpha\beta} = \overset{\sigma}{T}_{\beta^\alpha} \tag{2.8}$$

or

$$\Gamma^{\overline{k}}_{i,j} = \Gamma^k_{\bar{i},\bar{j}} = 0. \tag{2.9}$$

The curvature tensor $R^\nu_{\alpha\beta\sigma}$ in space A_N ([2]) is defined as usual:

$$R^\gamma_{\alpha\beta\sigma} = \partial_\alpha \Gamma^\nu_{\beta\sigma} - \partial_\beta \Gamma^\nu_{\alpha\sigma} + \Gamma^\nu_{\alpha\rho}\Gamma^\rho_{\beta\sigma} - \Gamma^\nu_{\beta\rho}\Gamma^\sigma_{\alpha\sigma}. \tag{2.10}$$

3. The creation of connections

Let the coefficients connections noted by ${}^1\Gamma^\sigma_{\alpha\beta}$ is defined with

$${}^1\Gamma^\sigma_{\alpha\beta} = \Gamma^\sigma_{\alpha\beta} + A^\sigma_{\alpha\beta}, \tag{3.1}$$

where $A^\sigma_{\alpha\beta}$ is called a deformed tensor.

In the following we have to consider ${}^1\nabla$ and ${}^1R^\sigma_{\alpha\beta\gamma}$, the deformed covariant as the curvative tensor and considering ${}^1\Gamma^\gamma_{\alpha\beta}$.

3.1. **The connections on the Chebyshev compositions.** Let the space A_{2n} be endowed (with a net) with compositions $X_n \times X_n$ of type (ch, ch). We have the following theorem:

Theorem 3.1. *Let the composition $X_m \times X_n$ be Chebyshev composition. Then ${}^1\nabla_{[\alpha} a^{\sigma}_{\beta]} = 0$ if and only if in the adopted to the compositions coordinates, the deformation tensor $A^{\nu}_{\alpha\beta}$ satisfies the condition:*

$$A^{k}_{i\bar{j}} = A^{\bar{k}}_{\bar{i}j} = A^{\bar{k}}_{[ij]} = A^{k}_{[\bar{i}\,\bar{j}]} = 0. \tag{3.2}$$

Proof. We know that by (2.10) we have:

$${}^1\nabla_{[\alpha} a^{\sigma}_{\beta]} = \nabla_{[\alpha} a^{\sigma}_{\beta]} + A^{\sigma}_{\alpha\beta}, \tag{3.3}$$

whereas:

$$A^{\sigma}_{\alpha\beta} = L^{\nu}_{\alpha\beta} a^{\nu}_{\sigma} - L^{\nu}_{\beta\sigma} a^{\sigma}_{\alpha} - L^{\sigma}_{\alpha\beta} a^{\nu}_{\sigma} - L^{\sigma}_{\beta\alpha} a^{\nu}_{\sigma}. \tag{3.4}$$

According to (2.1) and (3.3) in the adopted coordinates, for the components of the tensor $A^{\nu}_{\alpha\beta}$ we get:

$$\begin{gathered} A^{k}_{\bar{i}j} = 2L^{k}_{j\bar{i}}, \quad A^{k}_{i\bar{j}} = -2L^{k}_{i\bar{j}}, \quad A^{\bar{k}}_{i\bar{j}} = -2L^{k}_{\bar{j}i}, \\ A^{\bar{k}}_{\bar{i}j} = 2L^{\bar{k}}_{j\bar{i}}, \quad A^{\bar{k}}_{ij} = 4L^{k}_{[ji]}, \quad A^{k}_{\bar{i}\,\bar{j}} = -4L^{k}_{[\bar{i}\,\bar{j}]}, \\ A^{k}_{ij} = A^{\bar{k}}_{i\bar{j}} = 0. \end{gathered} \tag{3.5}$$

According to (3.3) we conclude that $\nabla_{[\alpha} a^{\sigma}_{\beta]} = 0$ and ${}^1\nabla_{[\alpha} a_{\beta^{\sigma}]} = 0$ if and only if $A^{\gamma}_{\alpha\beta} = 0$ by (3.4), we obtain that the last condition is equivalent to (3.2) which proves the theorem. In the case of $\nabla_{[\alpha} a^{\sigma}_{\beta]} =^1 \nabla_{[\alpha} a^{\sigma}_{\beta]} = 0$ by (2.8), (2.9) and (3.1) we obtain the components ${}^1\Gamma^{\sigma}_{\alpha\beta}$, as follows:

$$\begin{gathered} {}^1\Gamma^{k}_{ij} = \Gamma^{k}_{ij} + A^{k}_{ij}, \quad {}^1\Gamma^{\bar{k}}_{\bar{i}\,\bar{j}} = \Gamma^{k}_{\bar{i}\,\bar{j}} + A^{\bar{k}}_{\bar{i}\,\bar{j}}, \quad {}^1\Gamma^{k}_{ij} =^1 \Gamma^{\bar{k}}_{ji}, \quad {}^1\Gamma^{k}_{\bar{i}\,\bar{j}} =^1 \Gamma^{k}_{\bar{j}\,\bar{i}}, \\ {}^1\Gamma^{k}_{\bar{i}j} = A^{k}_{\bar{i}j}, \quad {}^1\Gamma^{\bar{k}}_{i\bar{j}} = A^{k}_{i\bar{j}}, \quad {}^1\Gamma^{k}_{i\bar{j}} =^1 \Gamma^{\bar{k}}_{\bar{i}j} = 0. \end{gathered}$$

We consider the curvature properties of the connections using (2.10) and (3.5) , ${}^1\Gamma^{\sigma}_{\alpha\beta}$ with the following adapted component ${}^1R^{s}_{\alpha\beta\sigma}$ and we have :

$${}^1R^{s}_{ij\bar{k}} = R^{\bar{s}}_{\bar{i}\,\bar{j}k} = 0 \quad \text{and} \quad {}^1R^{s}_{ij\bar{k}} = 2\partial_{[i} A^{k}_{\bar{j}]s} + 2A^{k}{}_{[i|p|} A^{p}_{\bar{j}]s}$$

Then we take the Weyl connections $\Gamma^{\sigma}_{\alpha\beta}$ with the fundamental tensor $g_{\alpha\beta}$ and the addition form of convector ω_{σ}, where ω_{σ} satisfies the condition $\nabla_{\sigma} g_{\alpha\beta} = 2\omega_{\sigma} g_{\alpha\beta}$. According [2, 4, 7, 8, 10] there is :

$$\Gamma^{\sigma}_{\alpha\beta} = \{\overset{\sigma}{\alpha\beta}\} - \left(\omega_{\alpha}\delta^{\nu}_{\beta} + \omega_{\beta}\delta^{\nu}_{\alpha} - \omega_{\sigma} g^{\sigma\nu}\right) \cdot g_{\alpha\beta}.$$

We know that $g_{\alpha\beta} \cdot g^{\beta\sigma} = \delta^{\sigma}_{\alpha} \{\overset{\sigma}{\alpha\beta}\}$ are Christophes symbols $g_{\alpha\beta}$.

For the curvature tensor $R^{\gamma}_{\alpha\beta\sigma}$ it stands the equation:

$$R^{\gamma}_{\alpha\beta\sigma} = -2P\nabla_{[\alpha\omega_{\beta}]}.$$

We have:

$${}^1R^{\nu}_{\alpha\beta\sigma} = R^{\gamma}_{\alpha\beta\sigma} + 2P\nabla_{\alpha A^{\sigma}_{\beta}}\sigma \tag{3.6}$$

We take the deforming tensor $A^{\sigma}_{\alpha\beta}$ which is :

$$A^{\sigma}_{\alpha\beta} = \omega_{\alpha} a^{\sigma}_{\beta}. \tag{3.7}$$

For the adopted coordinates we have:

$$A^k_{ij}=\omega_\alpha\delta^k_j,\quad A^{\overline{k}}_{\overline{i}\,\overline{j}}=\omega_{\overline{i}}\delta^{\overline{k}}_{\overline{j}},\quad A^k_{\overline{i}j}=\omega_{\overline{i}}\delta^k_j,\quad A^{\overline{k}}_{i\overline{j}}=\omega_i\delta^{\overline{k}}_{\overline{j}},$$
$$A^k_{i\overline{j}}=A^{\overline{k}}_{ij}=A^{\overline{k}}_{\overline{i}j}=A^k_{\overline{i}\,\overline{j}}=0. \tag{3.8}$$

The equations (3.8) with $A^k_{i\overline{j}}$ satisfy the condition as well as the Theorem 3.1, for the adopted coordinate holds (3.7) and (3.8),

$$^1R^\alpha_{ij\alpha}={}^1R^\alpha_{i\overline{j}\alpha}={}^1R^\alpha_{\overline{i}j\alpha}=0.$$

□

3.2. **The compositions with basic manifolds of geodesic compositions.** Let us take the space A_N of the type (G,G) with the compositions $X_m\times X_n$, we have to consider the space connections $\Gamma^\sigma_{\alpha\beta}$ defined in (2.8).

Theorem 3.2. *Let the composition $X_m\times X_n$ be geodesic $a^\sigma_\alpha\nabla_\beta a^\nu_\sigma+a^\nu_\beta\nabla_\sigma a^\nu_\alpha=0$ with conditions of the adoptive coordinates, then the deforming tensor $g^\nu_{\alpha\beta}$ satisfies the conditions:*

$$g^k_{\overline{i}\,\overline{j}}=g^{\overline{k}}_{ij}=0.$$

Proof. Considering $a^\sigma_\alpha\cdot a^\beta_\sigma=\delta^\beta_\alpha$ and defined connections $^1\Gamma^\nu_{\alpha\beta}$ as $^1\Gamma^\nu_{\alpha\beta}=\Gamma^\nu_{\alpha\beta}+g^\nu_{\alpha\beta}$ then we have:

$$a^{\sigma}_\alpha{}^1\nabla_\beta a^\nu_\sigma+a^{\nu}_\beta{}^1\nabla_\sigma a^\nu_\alpha=a^\sigma_\alpha\nabla_\beta a^\nu_\sigma+a^\nu_\sigma\nabla_\sigma a^\nu_\alpha+h^\nu_{\alpha\beta} \tag{3.9}$$

where

$$h^\nu_{\alpha\beta}=g^\nu_{\beta\alpha}-g^\rho_{\beta\sigma}a^\sigma_\alpha a^\nu_\rho+g^\nu_{\sigma\rho}a^\rho_\alpha a^\sigma_\beta-g^\rho_{\sigma\alpha}a^\nu_\rho a^\sigma_\beta, \tag{3.10}$$

with adopted coordinates according to (3.10) we evaluate the components of connection $h^\nu_{\alpha\beta}$ of the following

$$h^{\overline{k}}_{ij}=4g^k_{ji},\quad h^k_{\overline{i}\,\overline{j}}=4g^k_{\overline{j}\,\overline{i}},\quad h^k_{ij}=h^k_{\overline{i}j}=h^k_{i\overline{j}}=h^{\overline{k}}_{\overline{i}j}=h^{\overline{k}}_{i\overline{j}}=h^k_{\overline{i}\,\overline{j}}=0. \tag{3.11}$$

The equations in two relations (2.4) and (3.9) are adapted with compositions $X_m\times X_n$ which is geodetic, then holds:

$$a^{\sigma}_\alpha{}^1\nabla_\beta a^\nu_\sigma+a^{\sigma}_\beta{}^1\nabla_\sigma a^\nu_\alpha=0,$$

if and only if is $h^\nu_{\alpha\beta}=0$.

By the equation (3.11), the last condition holds if $g^k_{\overline{i}\,\overline{j}}=g^{\overline{k}}_{ij}=0$ which satisfies the condition of proofing the theorem.

By the conditions

$$a^\sigma_\alpha\nabla_\beta a^\nu_\sigma+a^\sigma_\beta\nabla_\sigma a^\nu_\alpha=0\quad\text{and}\quad a^{\sigma}_\alpha{}^1\nabla_\beta a^\nu_\sigma+a^{\sigma}_\beta{}^1\nabla_\sigma a^\nu_\alpha=0 \tag{3.12}$$

and by applying (3.1) we obtain the following equations:

$$^1\Gamma^k_{ij}=\Gamma^k_{ij}+g^k_{ij},\quad {}^1\Gamma^k_{\overline{i}j}=\Gamma^k_{ij}+g^k_{\overline{i}j},\quad {}^1\Gamma^k_{i\overline{j}}=\Gamma^k_{ij}+g^k_{i\overline{j}},\quad {}^1\Gamma^{\overline{k}}_{ij}=\Gamma^k_{ij}+g^{\overline{k}}_{\overline{i}j},$$
$$^1\Gamma^{\overline{k}}_{i\overline{j}}=\Gamma^k_{ij}+g^{\overline{k}}_{i\overline{j}},\quad {}^1\Gamma^k_{\overline{i}\,\overline{j}}=\Gamma^k_{ij}+g^k_{\overline{i}\,\overline{j}},\quad {}^1\Gamma^{\overline{k}}_{\overline{i}j}=\Gamma^{\overline{k}}_{\overline{i}j}+g^{\overline{k}}_{\overline{i}j},\quad {}^1\Gamma^k_{i\overline{j}}=\Gamma^k_{i\overline{j}}+g^{\overline{k}}_{ij}, \tag{3.13}$$
$$^1\Gamma^k_{\overline{i}\,\overline{j}}=\Gamma^k_{\overline{i}\,\overline{j}}+g^k_{\overline{i}\,\overline{j}},\quad {}^1\Gamma^k_{ij}={}^1\Gamma^k_{\overline{i}\,\overline{j}}=0.$$

By the equations (2.10) and (3.13) for the adapted components of tensor $^1R^\nu_{\alpha\beta\gamma}$ we have

$$^1R^s_{\overline{i}\,\overline{j}\,\overline{k}}={}^1R^{\overline{s}}_{ijk}=0.$$

We consider that $\Gamma^{\gamma}_{\alpha\beta}$ there is a connection that is called equafinne:

$$\Gamma^{S}_{KS} = \partial_K \ln e.$$

Equals affinors with density "e" is equivalent with:

$$\Gamma^{\nu}_{\alpha\beta} = \partial_K \ln e \quad \Leftrightarrow \quad R^{\nu}_{\alpha\beta\gamma} = 0.$$

Then: $X_m \times X_n$ is geodetic-geodetic (G, G) if

$$R^{\sigma\nu}\overline{R}_{\beta\nu}\nabla_{\alpha}(\overline{R}_{\alpha\beta}R^{\sigma\beta}) + \overline{R}_{\alpha\sigma}R^{\beta\nu}\nabla_{\sigma}(\overline{R}_{\beta\nu}R^{\gamma\nu}) = 0.$$

We conclude according (3.6) and (3.12) ${}^{1}\Gamma^{\nu}_{\alpha\beta} = grad$ if and only if:

$$g^{\nu}_{\alpha\beta} = grad \quad \text{and} \quad {}^{1}R^{\nu}_{\alpha\beta\gamma} = 0.$$

□

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