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LEFT AND RIGHT-MULTIPLIER HOMOMORPHISMS ON ALMOST DISTRIBUTIVE LATTICES

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ABSTRACT. In this paper, we define left and right multiplier homomorphism on ADLs and we characterize elements of the given ADL using the corresponding multiplier homomorphism. Moreover, we have proved that the class of all left multiplier homomorphisms forms a distributive lattice and therefore we characterize the given ADL in terms of the class of its multiplier homomorphism.

1. INTRODUCTION

The concept of an Almost Distributive Lattice (ADL) was first introduced by U. M. Swamy and G. C. Rao [4] in 1980. An ADL is an algebra with two binary operations $\vee$ and $\wedge$ which satisfies most of the properties of a distributive lattice with smallest element 0 except the commutativity of the binary operations $\vee$ and $\wedge$. The study of ideals, principal ideals, prime ideals, maximal ideals and congruence relations on ADLs have been initiated in [4] and latter studied by many authors (see [2], [3] and [5]).

In this paper we define right and left multiplier homomorphism on an ADL, and we characterize elements of the given ADL using their corresponding multiplier homomorphism. In addition, we consider the class $D_r(R)$ of all right multiplier homomorphisms and the class $D_L(R)$ of all left multiplier homomorphisms on ADL to get a new approach in the study of the properties of an ADL. More specifically, we prove that the class $D_r(R)$ of all right multiplier homomorphisms on an ADL forms an ADL and it is an isomorphic image of R. In this view, we give necessary and sufficient conditions for an ADL to be a distributive lattice, relatively-complemented [4], pseudo-complemented [6], weak-pseudo complemented [7] in terms of the class $D_r(R)$. On the other hand, we also prove that the class $D_L(R)$ of all left multiplier homomorphisms on an ADL forms a distributive lattice and this gives us an opportunity to extend those existing theories in a distributive lattice to the class of ADLs. Moreover, we give a characterization of the class of all principal filters (and hence all principal ideals) in terms of some class of congruence relations. Furthermore, If R is a relatively complemented ADL with a maximal element say m, then we prove that the class $D_L(R)$ forms a Boolean algebra.

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2. Preliminary Notes

In this section we recall some definitions and basic results on Almost Distributive Lattices.

Definition 2.1. [4] *An algebra $(R, \vee, \wedge, 0)$ of type $(2, 2, 0)$ is called an Almost Distributive Lattice, noted as ADL, if it satisfies the following axioms;*

(1) $a \vee 0 = a$,
(2) $0 \wedge a = 0$
(3) $(a \vee b) \wedge c = (a \wedge c) \vee (b \wedge c)$
(4) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$
(5) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$
(6) $(a \vee b) \wedge b = b$

for all $a, b, c \in R$.

Throughout this paper R denotes an ADL unless it is specified.

Lemma 2.1. [4] *For any $a \in R$, we have*

(1) $a \wedge 0 = 0$
(2) $0 \vee a = a$
(3) $a \wedge a = a$
(4) $a \vee a = a$

Lemma 2.2. [4] *For any $a, b \in R$, we have*

(1) $(a \wedge b) \vee b = b$
(2) $a \vee (a \wedge b) = a = a \wedge (a \vee b)$
(3) $a \vee (b \wedge a) = a = (a \vee b) \wedge a$
(4) $\wedge$ *is associative and* $a \wedge b \wedge c = b \wedge a \wedge c$

Corollary 2.1. [4] *For any $a, b \in R$, we have*

(1) $a \vee b = a$ *if and only if* $a \wedge b = b$
(2) $a \vee b = b$ *if and only if* $a \wedge b = a$

Definition 2.2. [4] *For any $a, b \in R$, we say that a is less than or equals to b and we write $a \leq b$ if $a \wedge b = a$ or equivalently $a \vee b = b$.*

Theorem 2.1. [4] *For any $a, b \in R$, the following are equivalent:*

(1) $(a \wedge b) \vee a = a$
(2) $a \wedge (b \vee a) = a$
(3) $(b \wedge a) \vee b = b$
(4) $b \wedge (a \vee b) = b$
(5) $a \wedge b = b \wedge a$
(6) $a \vee b = b \vee a$
(7) *the supremum of a and b exists in R and equals to $a \vee b$*
(8) *there exists $x \in R$ such that $a \leq x$ and $b \leq x$*
(9) *the infimum of a and b exists in R and equals to $a \wedge b$*

Definition 2.3. [4] *A nonempty subset I of R is called an ideal (respectively a filter) of R, if $a \vee b, a \wedge x \in I$ (respectively if $a \wedge b, x \vee a \in I$) for all $a, b \in I$ and all $x \in R$.*

Definition 2.4. [1] *A proper ideal (respectively filter) I of R is called prime ideal (respectively prime filter) if, $a \wedge b \in I \Rightarrow a \in I$ or $b \in I$ (respectively $a \vee b \in I \Rightarrow a \in I$ or $b \in I$) for all $a, b \in R$*

Definition 2.5. [1] *For any $a \in R$, the set $(a] = \{a \wedge x : x \in R\}$ is called a principal ideal of R generated by a and the set $[a) = \{x \vee a : x \in R\}$ is called the principal filter of R generated by a.*

Definition 2.6. [1] *Let R and R' be ADLs. A mapping $f : R \to R'$ is called an ADL-homomorphism if:*

(1) $f(0) = 0$
(2) $f(a \vee b) = f(a) \vee f(b)$
(3) $f(a \wedge b) = f(a) \wedge f(b)$

Definition 2.7. [4] *An equivalence relation θ on R is called a congruence relation on R, if for any $a, b, c,$ and $d \in R$:*

$$(a, b), (c, d) \in \theta \Rightarrow (a \wedge c, b \wedge d) \textit{ and } (a \vee c, b \vee d) \in \theta$$

Let θ be a congruence relation on an ADL R. For any $a \in R$, we define the congruence class of θ containing a by $\theta(a) = \{b \in R : (a, b) \in \theta\}$ and we denote the set of all congruence classes of R by $\frac{R}{\theta}$. Now define binary operations $\wedge$ and $\vee$ on $\frac{R}{\theta}$ by:

$$\theta(a) \wedge \theta(b) = \theta(a \wedge b) \text{ and } \theta(a) \vee \theta(b) = \theta(a \vee b)$$

Then $\left(\frac{R}{\theta}, \vee, \wedge, \theta(0)\right)$ is an ADL with $\theta(0)$ as its zero element.

Definition 2.8. [1] *Let $f : R \to R'$ be an ADL- homomorphism. Then kernel of f denoted by $kerf$ is a relation on R defined by:*

$$kerf = \{(a, b) \in R \times R : f(a) = f(b)\}$$

Note that $kerf$ is a congruence relation on R and its congruence class determined by zero is an ideal of R.

Definition 2.9. [4] *An ADL R is called relatively complemented if for any a and b in R there exists an element y in R such that $a \wedge y = 0$ and $a \vee y = a \vee b$. In this case this, y is unique and denoted by a^b.*

Definition 2.10. [6] *A unary operation $a \longmapsto a$ on R is called a pseudo-complementation on R if, for any $a, b \in R$, it satisfies the following conditions:*

(1) $a \wedge b = 0 \Rightarrow a^* \wedge b = b$
(2) $a \wedge a^* = 0$
(3) $(a \vee b)^* = a^* \wedge b^*$

An ADL R in which every element has a pseudo-complement is called a pseudo-complemented ADL.

Definition 2.11. [7] *A mapping $a \longmapsto a$ of an ADL R into itself is called weak pseudo-complementation on R if:*

$$a \wedge b = 0 \Leftrightarrow a^* \wedge b = b \textit{ for all } a, b \in R$$

An ADL R in which every element has a weak pseudo-complement is called a weak pseudo-complemented ADL.

3. Right multiplier homomorphisms on ADLs

In this section we define right multiplier homomorphism on ADLs and give some characterizing theorems.

Definition 3.1. *For any $a \in R$ define a mapping $f_a : R \to R$ by*

$$f_a(x) = x \wedge a \text{ for all } x \in R$$

Lemma 3.1. *For any $a \in R$, the map f_a is a homomorphism on R.*

Definition 3.2. *For each $a \in R$ we call f_a a right multiplier homomorphism on R.*

Lemma 3.2. *For any $a \in R$, and f_a on R, we have the following*

(1) $f_a(0) = 0$
(2) $f_0(x) = 0$ *for all* $x \in R$
(3) $f_a(x) = x$ *if and only if* $x \leq a$

Lemma 3.3. *For any $a \in R$, and f_a on R, we have the following*

(1) ${f_a}^{-1}(\{a\}) = [a)$*; the principal filter of R generated by* a
(2) $f_a^{-1}(\{0\}) = (a)^*$*; the annihilated ideal of R generated by* a.

Theorem 3.1. *Let $a, b \in R$ and f_a on R, then the following are equivalent:*

(1) *For any $x, y \in R$ we have* $f_a(x) = f_a(y) \Rightarrow f_b(x) = f_b(y)$
(2) $a \wedge b = b$
(3) *For any prime ideal P of R,* $a \in P \Rightarrow b \in P$

Proof. $(1) \Rightarrow (2)$ For any x and y in R Assume that,
$f_a(x) = f_a(y) \Rightarrow f_b(x) = f_b(y)$. That is, $x \wedge a = y \wedge b \Rightarrow x \wedge b = y \wedge b$. Now we have

$$\begin{aligned} b \wedge a = (b \wedge a) \wedge a &\Rightarrow f_a(b) = f_a(b \wedge a) \\ &\Rightarrow f_b(b) = f_b(b \wedge a) \quad (\texttt{by(1)}) \\ &\Rightarrow b \wedge b = (b \wedge a) \wedge b \\ &\Rightarrow b = a \wedge b \wedge b \\ &\Rightarrow b = a \wedge b \end{aligned}$$

$(2) \Rightarrow (3)$ Suppose that $b = a \wedge b$ and let P be any prime ideal of R. Then

$$\begin{aligned} a \in P &\Rightarrow a \wedge x \in P, \forall x \in R \quad \textit{(Since P is an ideal)} \\ &\Rightarrow \textit{In particular } \; a \wedge b \in P \\ &\Rightarrow b = a \wedge b \in P \end{aligned}$$

Thus $a \in P \Rightarrow b \in P$
$(3) \Rightarrow (1)$ For any prime ideal P of R, suppose that $a \in P \Rightarrow b \in P$. Then we get that $b \in (a]$. Otherwise; if $b \notin (a]$, then there exists a prime ideal P of R containing $(a]$ and not containing b. This implies that $a \in (a] \subseteq P$, and $b \notin P$, for some prime ideal P, which is a contradiction to our assumption. Therefore, $b \in (a]$ which is equivalently saying that $b = a \wedge b$. Now for any $x, y \in R$,

$$\begin{aligned} f_a(x) = f_a(y) &\Rightarrow x \wedge a = y \wedge a \\ &\Rightarrow (x \wedge a) \wedge b = (y \wedge a) \wedge b \\ &\Rightarrow x \wedge (a \wedge b) = y \wedge (a \wedge b) \\ &\Rightarrow x \wedge b = y \wedge b \\ &\Rightarrow f_b(x) = f_b(y) \end{aligned}$$

□

Theorem 3.2. *For any $a \in R$ let us put θ_a for $kerf_a$ (kernel of f_a) for simplicity. Then the quotient ADL $\dfrac{R}{\theta_a}$ forms a distributive lattice.*

Proof. For any $a \in R$, let $\theta_a = kerf_a$. Then we have that θ_a is a congruence relation on R and $\dfrac{R}{\theta_a}$ is a quotient ADL. To show that $\dfrac{R}{\theta_a}$ is a distributive lattice it suffices to prove that $\wedge$ is commutative in $\dfrac{R}{\theta_a}$. For; for any $x, y \in R$. Since $(x \wedge y) \wedge a = (y \wedge x) \wedge a$, then

$$\begin{aligned} f_a(x \wedge y) = f_a(y \wedge x) &\Rightarrow (x \wedge y, y \wedge x) \in \theta_a \\ &\Rightarrow \theta_a(x \wedge y) = \theta_a(y \wedge x) \\ &\Rightarrow \theta_a(x) \wedge \theta_a(y) = \theta_a(y) \wedge \theta_a(x) \end{aligned}$$

Thus $\wedge$ is commutative in $\dfrac{R}{\theta_a}$ and hence $\dfrac{R}{\theta_a}$ is a distributive lattice. □

Theorem 3.3. *There is s one-to-one correspondence between the class of all principal filters of R and the collection of all congruence relations θ_a of R where θ_a is the kernel of f_a for all $a \in R$.*

Proof. Put $RC(R) = \{\theta_a : a \in R\}$ and $PF(R) = \{[a) : a \in R\}$. Define $\alpha : RC(R) \to PF(R)$ by:

$$\alpha(\theta_a) = [a)$$

for all $a \in R$. Then this α is a one-to-one correspondence. □

Theorem 3.4. *Let R and R' be two ADLs, $a \in R$, $b \in R'$ and let $\alpha : R \to R'$ be an ADL-homomorphism such that $\alpha(a) = b$, then there exists a homomorphism $g : \dfrac{R}{\theta_a} \to \dfrac{R'}{\theta_b}$ such that $g \circ h = k \circ \alpha$ where $h : R \to \dfrac{R}{\theta_a}$ and $k : R' \to \dfrac{R'}{\theta_b}$ are the canonical epimorphisms.*

Theorem 3.5. *The set $D_r(R) = \{f_a : a \in R\}$ of all right multiplier homomorphisms on R forms an ADL.*

Proof. Let us define binary operations $\wedge$ and $\vee$ on $D_r(R)$ by:

$$(f_a \wedge f_b)(x) = f_a(x) \wedge f_b(x) \text{ and } (f_a \vee f_b)(x) = f_a(x) \vee f_b(x)$$

for all $a, b, x, y \in R$. Then for any $x \in R$:

$$\begin{aligned} (f_a \wedge f_a)(x) &= f_a(x) \wedge f_b(x) \\ &= (x \wedge a) \wedge (x \wedge b) \\ &= x \wedge a \wedge x \wedge b \\ &= x \wedge a \wedge b \\ &= f_{(a \wedge b)}(x) \end{aligned}$$

and

$$\begin{aligned} (f_a \vee f_b)(x) &= f_a(x) \vee f_b(x) \\ &= (x \wedge a) \vee (x \wedge b) \\ &= x \wedge (a \vee b) \\ &= f_{(a \vee b)}(x). \end{aligned}$$

Therefore $(D_r(R), \vee, \wedge, f_0)$ is an ADL having f_0 as its zero element. $\square$

Theorem 3.6. *Let R be an ADL and $D_r(R)$ the set of all right multiplier homomorphisms on R. Then $R \cong D_r(R)$.*

Proof. Define $\alpha : R \to D_r(R)$ by $\alpha(a) = fa$ for all $a \in R$. Since $f_{(a\wedge b)} = f_a \wedge f_b$ and $f_{(a\vee b)} = f_a \vee f_b$, it follows that α is an ADL-homomorphism and it is also clear that α is onto and hence an epimorphism. Now it suffices to prove that α is one-one. For any $a, b \in R$ consider:

$$\begin{aligned} \alpha(a) = \alpha(b) &\Rightarrow \quad f_a = f_b \\ &\Rightarrow \quad f_a(x) = f_b(x), \ \forall\, x \in R \\ &\Rightarrow \quad x \wedge a = x \wedge b, \ \forall\, x \in R \end{aligned}$$

In particular if $x = a$, then we have $a = a \wedge b$ which implies that $a \leq b$. Also if $x = b$, then we get $b \wedge a = b$ which implies that $b \leq a$ and hence we get $a = b$. So that α is one-one and therefore an isomorphism. $\square$

The following results can be deduced as an immediate consequence of the above theorem.

Corollary 3.1. *The following holds for any ADL R:*

1. *R is a distributive lattice if and only if $D_r(R)$ is a distributive lattice*
2. *I is an ideal (respectively a filter) of R if and only if $\{f_a : a \in I\}$ is an ideal (respectively a filter) of $D_r(R)$*
3. *R has a maximal element if and only if $D_r(R)$ has a maximal element.*
4. *R is relatively complemented if and only if $D_r(R)$ is relatively complemented.*
5. *R is pseudo-complemented if and only if $D_r(R)$ is pseudo-complemented.*
6. *R is weak-pseudo complemented if and only if $D_r(R)$ is weak-pseudo complemented.*

4. Left-multiplier homomorphisms on ADLs

In this section we define and characterize left multiplier homomorphism on ADLs.

Definition 4.1. *For any $a \in R$ define a mapping ${}_af : R \to R$ by ${}_af(x) = a \wedge x$ for all $x \in R$. Then this ${}_af$ is a homomorphism on R. We call ${}_af$ a left multiplier homomorphism on R.*

Lemma 4.1. *For any $a \in R$ and ${}_af$ on R we have the following;*

1. ${}_af(0) = 0$
2. ${}_0f(x) = 0$ *for all* $x \in R$
3. ${}_af(x) = x$ *if and only if* $x \in (a]$

Lemma 4.2. *Let $a \in R$ and ${}_af$ on R, then*

1. ${}_af^{-1}(a) = \{x \in R : a \leq x\}$
2. ${}_af^{-1}(a) = (a)^*$*; the annihilated ideal generated by* a

Theorem 4.1. *The set $D_L(R) = \{{}_af : a \in R\}$ of all left-multiplier homomorphisms of R forms a distributive lattice with ${}_0f$ as its zero element.*

Proof. Let us define binary operations $\wedge$ and $\vee$ on $D_L(R)$ by:

$$({}_af \wedge {}_bf)(x) = {}_af(x) \wedge {}_bf(x) \text{ and } ({}_af \vee {}_bf)(x) = {}_af(x) \vee {}_bf(x), \ \forall a, b, x \in R$$

Then for any $a, b \in R$ we have ${}_af \wedge {}_bf = {}_{(a\wedge b)}f$ and ${}_af \vee {}_bf = {}_{(a\vee b)}f$. Since ${}_{(a\wedge b)}f = {}_{(b\wedge a)}f$ then it follows that $(D_L(R), \vee, \wedge, {}_0f)$ is a distributive lattice. □

Theorem 4.2. *Let R be an ADL and $D_L(R)$ the set of all left-multiplier homomorphisms on R. Then R and $D_L(R)$ are isomorphic up to associate to each other.*

Proof. Define $\alpha : R \to D_L(R)$ by $\alpha(a) ={}_a f$ for all $a \in R$, then this α is an ADL-isomorphism up to associate; in the sense that, $\alpha(a) = \alpha(b)$ if and only if $a \sim b$ (a and b are associate elements to each other). Where $a \sim b$ if and only if $(a] = (b]$ (See [8]) □

Theorem 4.3. *Let $\phi = ker\alpha = \{(a, b) \in R \times R : \alpha(a) = \alpha(b)\}$ where α is defined as in Theorem 4.2. Then for any $a \in R$, the congruence class $\phi(a)$ of ϕ in R determined by a is:*

$$\phi(a) = (a] \cap [a)\,.$$

Proof.

$$\begin{aligned} b \in \phi(a) &\Rightarrow (a, b) \in \phi = ker\alpha \\ &\Rightarrow \alpha(a) = \alpha(b) \\ &\Rightarrow {}_af = {}_bf \\ &\Rightarrow {}_af(x) = {}_bf(x),\ \forall x \in R \\ &\Rightarrow a \wedge x = b \wedge x, \forall x \in R \end{aligned}$$

In particular, $a \wedge x = b \wedge x$ for $x = a$ and for $x = b$. That is; $a = b \wedge a$ and $b = a \wedge b$ which implies that $b \in (a]$ and $b \in [a)$. So that $b \in (a] \cap [a)$ and hence $\phi(a) \subseteq (a] \cap [a)$. Conversely suppose that, $b \in (a] \cap [a)$. Then $a = b \wedge a$ and $a \wedge b = b$. Now for any $x \in R$ consider:

$$\begin{aligned} {}_af(x) &= a \wedge x \\ &= (b \wedge a) \wedge x \\ &= (a \wedge b) \wedge x \\ &= b \wedge x \\ &= {}_bf(x) \end{aligned}$$

Thus ${}_af = {}_bf$ which implies that $\alpha(a) = \alpha(b)$. That is, $(a, b) \in \phi$ so that $b \in \phi(a)$.
Therefore $\phi(a) = (a] \cap [a)$. □

Corollary 4.1. *R is a distributive lattice if and only if $(a] \cap [a) = \{a\}$ for all $a \in R$.*

Lemma 4.3. *If R has a maximal element say m, then $D_L(R)$ has a greatest element ${}_mf$. In this case if m_1 and m_2 are any two maximal elements in R then we have ${}_{(m_1)}f = {}_{(m_2)}f$.*

Theorem 4.4. *If R is relatively complemented ADL with a maximal elements say m, then $D_L(R)$ is a Boolean algebra.*

Proof. Suppose that R is relatively complemented ADL with a maximal element say m. Then for any $a, b \in R$, there is an element denoted by $a^b \in R$ such that:

$$\begin{aligned} a \wedge a^b = 0 \text{ and } a \vee a^b = a \vee b &\Rightarrow {}_{(a\wedge a^b)}f = {}_0f \text{ and } {}_{(a\vee a^b)}f = {}_mf \\ &\Rightarrow {}_af \wedge {}_{(a^b)}f = {}_0f \text{ and } {}_af \vee {}_{(a^b)}f = {}_mf \end{aligned}$$

Now for any $a \in R$, put ${}_af' = {}_{(a^m)}f$, where m is a maximal element in R. Then we have the following:

$$\begin{aligned} {}_af \wedge {}_af' &= {}_af \wedge {}_{(a^m)}f \\ &= {}_{(a\wedge a^m)}f \\ &= {}_0f, \quad (\textit{which is the zero element in } D_L(R)) \end{aligned}$$

Also,

$$\begin{aligned} {}_af \vee {}_af' &= {}_af \vee {}_{(a^m)}f \\ &= {}_{(a\vee a^m)}f \\ &= {}_{(a\vee m)}f \\ &= {}_{(m\vee a)}f \quad (\textit{since } \vee \textit{ is commutative in } D_L(R))) \\ &= {}_mf \quad (\textit{which is the largest element in } D_L(R)) \end{aligned}$$

Therefore $D_L(R)$ is complemented and hence a Boolean algebra. □

Finally we conclude this paper by giving a necessary and sufficient condition for an ADL to be a distributive lattice in the following theorem.

Theorem 4.5. *R is a distributive lattice if and only if $D_L(R) = D_r(R)$.*

References

[1] Y.S. Pawar, I.A.Shaikh: *Congruence Relations on Almost Distributive Lattices*, Southeast Asian Bull. Math., **36** (2012), 519-527.

[2] T.P. Speed: *Two congruences on distributive lattices*, Bulletin de la Société Royale des Sciences de Liège, 38e année, **3-4** (1969) 86-96.

[3] U.M. Swamy, S. Ramesh: *Birkhoff Center of an almost distributive lattice*, Int. Jour. Algebra, **11** (2009) 539-546.

[4] U.M. Swamy, G.C. Rao: *Almost Distributive Lattices*, Jour. Aust. Math. Soc., (Series A), **31** (1981) 77-91.

[5] U.M. Swamy, G.C. Rao, G.N. Rao: *On Characterizations of Stone Almost Distributive Lattices*, Southeast Asian Bull. Math., **32** (2008) 1167-1176.

[6] U.M. Swamy, G.C. Rao, G.N. Rao: *Pseudo-complementation on almost distributive lattices,* Southeast Asian Bull. Math.,0 **24** (2000), 95-104.

[7] R. Vasu Babu, Ch. Raj, S. Sundar, B. Venkateswarlu: *Weak pseudo-complementations on ADLs*, Archivum Mathematicum (BRNO), **50** (2014), 151-159.

[8] B. Venkateswarlu, R. Vasu Babu: *Associate elements in ADL's*, Asian-Eur. J. Math., **7** (2014).

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