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SPLIT TOTAL STRONG (WEAK) DOMINATION IN BIPOLAR FUZZY GRAPH

R. Muthuraj[1] & A. Kanimozhi[2]

[1]PG and Research Department of Mathematics,

H.H.The Rajah's College, Pudukottai,Affiliated to Bharathidasan University, Tiruchirappalli,

Tamilnadu, India.E-mail: rmr1973@yahoo.co.in

[2]Department of Mathematics, NPR College of Engineering and Technology,Dindigul,.

Research Scholar, PG and Research Department of Mathematics,

H.H.The Rajah's College, Pudukottai,Affiliated to Bharathidasan University,Tiruchirappalli,

Tamilnadu, India.E-mail: kanimozhi2k22@yahoo.com

Abstract: In this paper, we define Split Total strong (weak) domination in Bipolar Fuzzy Graph and its various classifications. Size, Order and Degree of Split Total strong (weak) domination in Bipolar Fuzzy Graph is derived with some examples. Some basic parametric conditions are introduced with suitable examples. The properties of total strong (weak) domination number and Split total strong (weak) domination number in Bipolar Fuzzy Graph are discussed.

Keywords: Bipolar Fuzzy Graph, dominating set in BFG, strong (weak) dominating set in BFG, total strong (weak) dominating set in BFG, split total strong (weak) dominating set in BFG.

1. Introduction

The concept of fuzzy graph was proposed by Kaufmann, from the fuzzy relations introduced by Zadeh[8]. Although, in 1975, Rosenfeld introduced another elaborated concept, including fuzzy vertex and fuzzy edges and several fuzzy analogues of graph theoretic concepts such as paths, cycles, connectedness and etc. In the year 1998, the concept of domination in fuzzy graphs was investigated by A. Somasundaram, S. Somasundaram.[2] In the year, 2004 A.Somasundaram investigated the concepts of domination in fuzzy graph - II. In the year 2003, A.NagoorGani and M. Basheer Ahamed [9] investigated Order and Size in fuzzy graph. In 2010, C.Natarajan and S.K.Ayyasamy[10] introduced on strong (weak) domination in fuzzy graph. In 2011, Muhammad Akram[1] introduced Bipolar fuzzy graphs. In the year 2012, Muhammad Akram was proposed regular bipolar fuzzy graphs. In 2012, P.J. Jayalakshmi[7] et.al introduced total strong (weak) domination in fuzzy graph.

2 Preliminaries

In this section, some definitions are given.

2.1 Definition [1]

A fuzzy subset μ on a set X is a map μ:X→[0,1]. A map v:X× X → [0,1] is called a **fuzzy relation** on X if v(x,y) ≤ min(μ(x),μ(y)) for all x,y∈X. A fuzzy relation v is symmetric if

v(x,y) = v(y,x) for x,y∈ X.

2.2 Definition [1]

Let X be a non-empty set. A **bipolar fuzzy set** B in X is an object having the form $B=\{(x,\mu_B^P(x),\mu_B^N(x))/x\in X\}$ where $\mu_B^P: X \to [0,1]$ *and* $\mu_B^N: X \to [-1,0]$ are mappings.

We use the positive membership degree $\mu_B^P(x)$ to denote the satisfaction degree of an element x to the property corresponding to a bipolar fuzzy set B, and the negative membership degree $\mu_B^N(x)$ to denote the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar fuzzy set B. If $\mu_B^P(x) \neq 0$ and $\mu_B^N(x) = 0$. It is the situation that is regarded as having only positive satisfaction for B. If $\mu_B^P(x) = 0$ and $\mu_B^N(x) \neq 0$, it is the situation that x does not satisfy the property of B but somewhat satisfies the counter property of B. It is possible for an element x to be such that $\mu^P(x) \neq 0$ and $\mu^N(x) \neq 0$. When the membership function of the property overlaps that of its counter property over some portion of X.

For the sake of simplicity, we shall use the symbol B=($\mu_B{}^P, \mu_B{}^N$), for the bipolar fuzzy set $B=\{(x,\mu_B{}^P(x),\mu_B{}^N(x))/x\in X\}$.

2.3 Definition [1]

A **bipolar fuzzy graph**, we define a pair G = (A,B) where $A=(\mu_A^P,\mu_A^N)$ is a bipolar fuzzy set in V and $B=(\mu_B^P,\mu_B^N)$ is a bipolar relation on E⊆V×V such that $\mu_B^P(\{x,y\})\leq \min(\mu_A^P(x),\mu_A^P(y))$ and $\mu_B^N(\{x,y\})\geq \max(\mu_A^N(x),\mu_A^N(y))$ for all {x,y}∈ E. We call A the bipolar fuzzy vertex set of V, B the bipolar fuzzy edge set of E, respectively. Note that B is a symmetric bipolar fuzzy relation on A. We use the notation xy for an element of E. Thus, $G=(\mathrm{A},\mathrm{B})$ is a bipolar graph of $G^*=(\mathrm{V},\mathrm{E})$ if $\mu_B^P(\{x,y\})\leq \min(\mu_A^P(x),\mu_A^P(y))$ and $\mu_B^N(\{x,y\})\geq \max(\mu_A^N(x),\mu_A^N(y))$ $\forall\ xy\in E$

2.4 Definition [13]

Let G = (A,B) be a bipolar fuzzy graph where $A=(\mu_A^P,\mu_A^N)$ and $B=(\mu_B^P,\mu_B^N)$ be two bipolar fuzzy sets on a non-empty finite set V and E⊆V×V respectively. The **positive degree of a vertex** $x\in G$ is $\deg(\mu_A^P(x)) = \sum_{xy\in E}\mu_B^P(xy)$. Similarly, the **negative degree of a vertex** $x\in G$ is $\deg(\mu_A^N(x)) = \sum_{xy\in E}\mu_B^N(xy)$. The **degree of a vertex** $x\in G$ is defined as the set of positive degree of a vertex and the negative degree of a vertex.

2.5 Definition [12]

Let G = (A,B) be a bipolar fuzzy graph. The **order of a bipolar fuzzy graph G** is $O(G) = \left(\sum_{x \in V} \mu_A^P(x), \sum_{x \in V} \mu_A^N(x) \right)$. The **size of a bipolar fuzzy graph G** is $S(G) = \left(\sum_{xy \in E} \mu_B^P(xy), \sum_{xy \in E} \mu_B^N(xy) \right)$

2.6 Definition [12]

Let G = (A,B) be a bipolar fuzzy graph. If each vertex of G has same closed neighbourhood degree, then G is called a totally regular bipolar fuzzy graph. The closed neighbourhood degree of a vertex x is defined by $\deg[x] = (\deg^p[x] + \deg^N[x])$, where $\deg^P[x] = \deg^p(x) + \mu_A^P(x)$,

$\deg^N[x] = \deg^N(x) + \mu_A^N(x)$

2.7 Definition [11]

The number of vertices, the cardinality of V is called the order of the bipolar fuzzy graph G= (A,B) and denoted by $|V| = \sum \frac{1 + \mu_A^p(x) + \mu_A^N(x)}{2}$.

2.8 Definition [11]

The number of edges, the cardinality of E, is called the size of a bipolar fuzzy graph G= (A,B) and denoted by $|E| = \sum \frac{1 + \mu_B^P(xy) + \mu_B^N(xy)}{2}$.

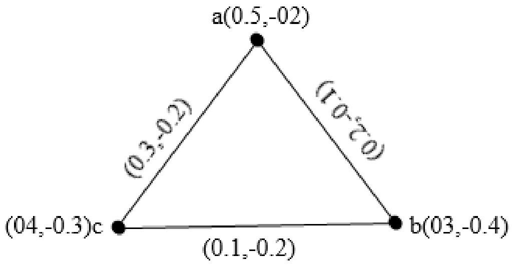
2.9 Definition [3]

Let G be a bipolar fuzzy graph. The neighbourhood of a vertex x in G is defined by N(x)= ($N_\mu(x), N_v(x)$). where $N_\mu(x) = \{y \in V : \mu_B^p(xy) \le \min(\mu_A^p(x), \mu_A^p(y))$ and $N_v(x) =$ $\{y \in V : \mu_B^N(xy) \ge \max(\mu_A^N(x), \mu_A^N(y))$

2.10 Example: Consider the Graph G have the vertices such that V={a,b,c} and edges set E={ab,bc,ac}contains the positive and negative membership values are given by

$\mu_A^P(a) = 0.5, \mu_A^P(\text{b}) = 0.3, \mu_A^P(\text{c}) = 0.4, \mu_A^N(a) = -0.2, \mu_A^N(\text{b}) = -0.4, \mu_A^N(\text{c}) = -0.3$

$\mu_B^P(ab) = 0.2, \mu_B^P(\text{bc}) = 0.1, \mu_B^P(ac) = 0.3, \mu_B^N(ab) = -0.1, \mu_B^N(ac) = -0.2, \mu_B^N(\text{bc}) = -0.2$



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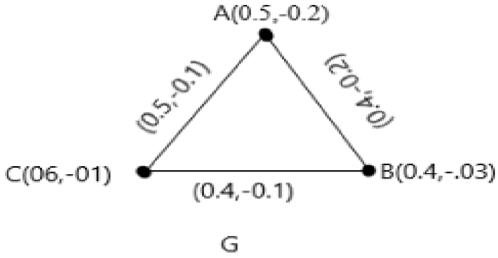
2.11 Definition

In a Bipolar Fuzzy Graph G, an edge is said to be an **effective edge(Strong arc)** if the positive membership value and negative membership value of the vertices (v_i, v_j) is equal to the minimum of the positive membership value of the vertices (v_i, v_j) and the maximum of the negative membership value of the vertices (v_i, v_j) .

2.12 Example:



2.13 Definition

In a Bipolar Fuzzy Graph G, the **weight** of the graph $W_{S_{T_b}}(G)$ is defined as the cardinality of the dominating set.

3. Total strong (weak) dominating bipolar set of G

3.1 Definition [11] A bipolar fuzzy graph G = (A,B) is called **strong bipolar fuzzy graph**, if $\mu_B^P(xy) = \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(xy) = \max(\mu_A^N(x), \mu_A^N(y))$ for all x,y$\in$ E.

3.2 Definition [14] A subset S of V is called a dominating set in G if for every $v \in V - S$, there exists $u \in S$ such that u dominates v.

3.3 Definition Let G be a bipolar fuzzy graph. Let A and B be any two vertices. Then **A totally strong dominates B (B totally weak dominates A)** if

i) $\mu_B^P(xy) = \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(xy) = \max(\mu_A^N(x), \mu_A^N(y))$ for all x,y$\in$ E.

ii) $\deg_N(A) \geq \deg_N(B)$ and

iii) every vertex in G dominates A.

3.4 Definition Let G be a bipolar Fuzzy Graph. T_b is said to be **total strong (weak) dominating bipolar set of G** if

i) $\deg_N(A) \geq \deg_N(B)$ for all A $\in T_b$, B $\in$ V - T_b and

ii) $\mu_B^P(xy) = \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(xy) = \max(\mu_A^N(x), \mu_A^N(y))$ for all x,y$\in$E.

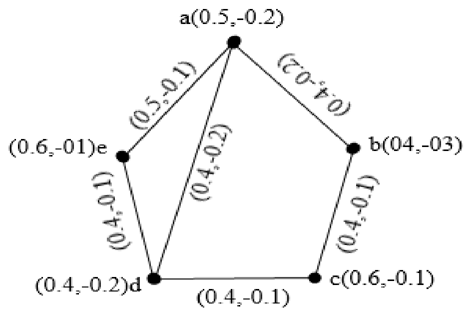
iii) T_b is the total dominating bipolar set.

3.5 Definition Total strong (weak) dominating bipolar set T_b of a fuzzy graph G is called **minimal total strong (weak) dominating bipolar set** of G, if there does not exist any total strong (weak) dominating bipolar set of G, whose cardinality is less than the cardinality of T_b

3.6 Definition The minimum fuzzy cardinality among all minimal total strong (weak) dominating bipolarset G is called **total strong (weak) dominating bipolar set of G** and its total strong (weak) domination bipolar number is denoted by $\gamma_{T_b}(G)$

3.7 Example: Let the graph G with vertices V={a,b,c,d,e) and Edges E={ab,bc,cd,de,ad,ae) with values are

$\mu_A^p(\mathrm{a}) = 0.5, \mu_A^p(\mathrm{b}) = 0.4, \mu_A^p(\mathrm{c}) = 0.6, \mu_A^p(\mathrm{d}) = 0.4, \mu_A^p(\mathrm{e}) = 0.6,$
$\mu_A^N(\mathrm{a}) = -0.2, \mu_A^N(\mathrm{b}) = -0.3, \mu_A^N(\mathrm{c}) = -0.1, \mu_A^N(\mathrm{d}) = -0.2, \mu_A^N(\mathrm{e}) = -0.1,$
$\mu_B^p(\mathrm{ab}) = 0.4, \mu_B^p(\mathrm{ae}) = 0.5, \mu_B^p(\mathrm{ad}) = 0.4, \mu_B^p(\mathrm{bc}) = 0.4, \mu_B^p(\mathrm{cd}) = 0.4, \mu_B^p(\mathrm{de}) = 0.4,$
$\mu_B^N(\mathrm{ab}) = -0.2, \mu_B^N(\mathrm{ae}) = -0.1, \mu_B^N(\mathrm{ad}) = -0.2, \mu_B^N(\mathrm{bc}) = -0.1, \mu_B^N(\mathrm{cd}) = -0.1, \mu_B^N(\mathrm{de}) = -0.1,$



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Here the total strong (weak) dominating set is {a,d}.

4. Main Results

4.1 Definition: A Total strong (weak) dominating set of a Bipolar fuzzy graph G is a **split Total strong (weak) dominating set** if the precipitated subgraph < V- S_{T_b} > is not connected.

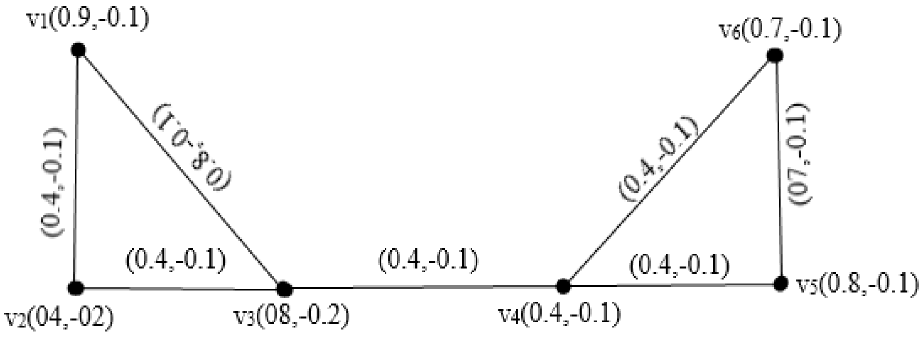
4.2 Definition: A Split total strong (weak) dominating set S_{T_b} of a fuzzy graph G is called minimal split total strong (weak) dominating set of G, of there does now not exist any total strong (weak) dominating set of G, Whose cardinality is less than the cardinality of S_{T_b} .

4.3 Definition: The Minimum fuzzy cardinality among all minimal split total strong(weak) dominating set is called split total strong(weak) dominating set and its split total strong(weak) domination number and it is denoted by $\gamma_{s_b}(G)$

4.4 Example: Consider a Graph G such that the set of vertices be V=$\{v_1, v_2, v_3, v_4, v_5, v_6\}$ and set of edges be E=$\{v_1 v_2, v_1v_3, v_2v_3, v_3v_4, v_4v_5, v_4v_6, v_5v_6\}$.

Here $\mu_A^P(\mathrm{v}_1) = 0.9, \mu_A^P(\mathrm{v}_2) = 0.4, \mu_A^P(\mathrm{v}_3) = 0.8, \mu_A^P(\mathrm{v}_4) = 0.4, \mu_A^P(\mathrm{v}_5) = 0.8, \mu_A^P(\mathrm{v}_6) = 0.7$;
$\mu_A^N(\mathrm{v}_1) = -0.1, \mu_A^N(\mathrm{v}_2) = -0.2, \mu_A^N(\mathrm{v}_3) = -0.2, \mu_A^N(\mathrm{v}_4) = -0.1, \mu_A^N(\mathrm{v}_5) = -0.1, \mu_A^N(\mathrm{v}_6) = -0.1$;
$\mu_B^P(v_1v_2) = 0.4, \mu_B^P(v_1v_3) = 0.8, \mu_B^P(v_2v_3) = 0.4, \mu_B^P(v_3v_4) = 0.4, \mu_B^P(v_4v_5) = 0.4, \mu_B^P(v_5v_6) = 0.7, \mu_B^P(v_4v_6) = 0.4$
$\mu_B^N(v_1v_2) = -0.1, \mu_B^N(v_1v_3) = -0.1, \mu_B^N(v_2v_3) = -0.1, \mu_B^N(v_3v_4) = -0.1, \mu_B^N(v_4v_5) = -0.1, \mu_B^N(v_5v_6) = -0.1, \mu_B^N(v_4v_6) = -0.1$

The Split total strong(weak) dominating set $S_{T_b} = \{v_3, v_4\}, V - S_{T_b} = \{v_1, v_2, v_5, v_6\}$ by the usual computations we have the vertex cardinality ,Edge cardinality, weight of a dominating set, maximum degree, minimum degree and the cardinality of dominating set respectively as
$|V| = 4.6, |E| = 4.9, w_{s_{T_b}} = (1.2, -0.3), \Delta_{s_{T_b}} = (1.6, -0.2), \delta_{s_{T_b}} = (0.8, -0.3)\ \&\ \gamma_{s_{T_b}} = 1.45$



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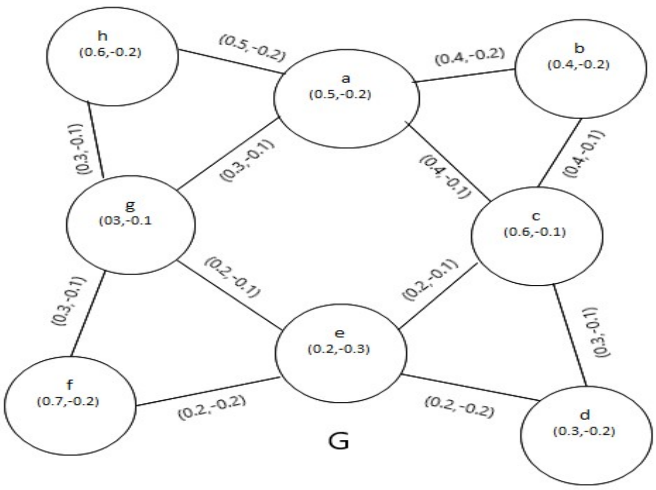
4.5 Example:



In this graph G, The split total strong(weak) dominating set $S_{T_b} = \{a, e, g\}$, V-$S_{T_b} = \{b, c, d, f, h\}$,

$$p = 5.05, q = 7.1, w_{s_{T_b}} = (1, -0.6), \Delta_{s_{T_b}} = (1.6, -0.3), \delta_{s_{T_b}} = (0.5, -0.6) \,\&\, \gamma_{s_{T_b}} = 1.6$$

4.6 Theorem Every split minimal Total strong(weak) dominating set in a Bipolar fuzzy graph has at least one pendent vertex.

Proof:

Let S_{T_b} =(x,y)Be the Split minimal total strong(weak) dominating set in a Bipolar fuzzy graph G, by way of the definition of Split Total Strong (weak)) dominating set, the triggered subgraph (V- S_{T_b}) is not connected. Therefore, there is no path between the subgraphs and there exist at least one pendent vertex may be arise. Hence the proof.

4.7 Theorem

For the Split Total strong(weak) bipolar fuzzy graph, $\gamma_{s_{T_b}} \leq |V| \leq |E|$

Proof: Let G be a Total robust(vulnerable) bipolar fuzzy graph. Let (G) be a split general robust (susceptible) domination quantity of G and its miles minimal dominating set. p and q be the sum of all vertex cardinality and edge cardinality of split total Strong (Weak) domination of G respectively.

Which now not at least split Total strong(Weak) domination quantity of G then the cardinality of dominating set will now not exceed. Hence $\gamma_{s_{T_b}} \leq |V| \leq |E|$.

4.8 Theorem: For the Split Total strong(weak) bipolar fuzzy graph, the domination number is at most half of the sum of vertex cardinality and edge cardinality. (i.e) $\gamma_{s_{T_b}} \leq \frac{1}{2}(\mathrm{p}+\mathrm{q})$.

Proof: Proof is true from the above theorem and it is perceivable.

4.9 Theorem: For the Split total strong(weak) BFG,

i) $\delta^{P}{}_{s_{T_b}} \leq w^{P}{}_{s_{T_b}} \leq \Delta^{P}{}_{s_{T_b}}$

ii) $\delta^{N}{}_{s_{T_b}} \leq w^{N}{}_{s_{T_b}} \leq \Delta^{P}{}_{s_{T_b}}$

Proof: Consider the Example 4.5, The minimum and maximum degree of total strong(weak) bipolar fuzzy graph is $\delta_{\mu}(G) = \wedge(d_{\mu}(v_i)/v_i \in V) = 0.5$, $\delta_{\gamma}(G) = \wedge(d_{\gamma}(v_i)/v_i \in V) = -0.6$. The minimum degree of a STS(W)BFG is $\delta_{s_{T_b}}$ is the minimum of degrees of the all the positive membership values and the negative membership values that is $(0.5, -0.6)$, The maximum degree of the total strong (weak) bipolar fuzzy graph is $\Delta_{\mu}(G) = \vee(d_{\mu}(v_i)/v_i \in V) = 1.6$, $\Delta_{\gamma}(G) = \vee(d_{\gamma}(v_i)/v_i \in V) = -0.3$ then the maximum degree of a STS(W)BFG is $\Delta_{S_{T_b}}(G)$ is the maximum of degrees of all the positive membership values and the negative membership values that is $(1.6, -0.3)$. The weight of the STS(W)BFG is the cardinality of the dominating set and it is by the usual calculation we have $w_{s_{T_b}} = (1, -0.6)$. Hence the proof.

4.10 Theorem: In a Split total strong(weak) bipolar fuzzy graph, G=(V,E, μ, ρ), the following inequalities holds:

(i) $O^{P}_{s_{T_b}}(G) \geq S^{P}_{s_{T_b}}(G)$

(ii) $O^{N}_{s_{T_b}}(G) \leq S^{N}_{s_{T_b}}(G)$

Proof: Let G be a Split Total Strong (weak) bipolar fuzzy graph. By the definition 2.5 we have $O^{P}_{s_{T_b}}(G) = \sum_{x \in V} \mu^{P}_{A}(x) \geq \sum_{xy \in E} \mu^{P}_{B}(xy) = S^{P}_{s_{T_b}}(G)$. This implies the inequality (i). Similarly, we have by the same definition, $O^{N}_{s_{T_b}}(G) = \sum_{x \in V} \mu^{N}_{A}(x) \leq \sum_{xy \in E} \mu^{N}_{B}(xy) = S^{N}_{s_{T_b}}(G)$. Hence the inequality (ii) holds.

4.11 Theorem: In a Split total strong(weak) bipolar fuzzy graph, G=(V,E, μ, ρ), the following inequalities holds:

(i) $O^{P}_{s_{T_b}}(G) + \Delta^{P}_{s_{T_b}}(G) \geq S^{P}_{s_{T_b}}(G) + \delta^{P}_{s_{T_b}}(G)$

(ii) $O^{N}_{s_{T_b}}(G) + \Delta^{N}_{s_{T_b}}(G) \geq S^{N}_{s_{T_b}}(G) + \delta^{N}_{s_{T_b}}(G)$

5. Conclusions

In this paper, the split total strong(weak) domination in a bipolar fuzzy graph was introduced and some theorems are examined. Based on these ideas, we can extend our research work to other areas of bipolar fuzzy graph.

Reference

[1] Muhammad Akram“ Bipolar Fuzzy graphs” Information Sciences 181(2011)5548-5564.

[2] A. Somasundaram, S. Somasundaram “Domination in Fuzzy Graphs-I” Pattern Recognition Letters,19(1998), pp.787-791.

[3] BasheerAhamedMohideen,” Types of Degrees in Bipolar Fuzzy Graphs”. Applied Mathematical Sciences, Vol.7,2013,no.98,4857-4866.

[4] A. Prasanna, C. Gurubaran and S. Ismail Mohideen,” Split amd Non-Split domination number in Bipolar fuzzy graphs.

[5] BasheerAhamedMohideen,” Perfect Bipolar Fuzzy graphs. J. Math.ComputerSci.19(2019)120-128 ,Journal of Mathematics and Computer Science.

[6] P.J Jayalakshmi, Dr. C.V.R. Hari Narayanan, Dr. R. Muthuraj.,” TotalStrong(weak) Domination in Intuitionistic Fuzzy graphs. Advances in Theoretical and Applied Mathematics.ISSN 0973-4554 Volume 11, Number 3(2016), pp.203-212.

[7] S. Revathi, Dr. C.V.R. Hari Narayanan, Dr. R. Muthuraj “Regular Split Perfect Domination number in Fuzzy Graph. East journal of Scientific Research 25(2): 380-386,2017.

[8] L.A. Zadeh, Fuzzy Sets, Information Sciences.8(1965),338-353.

[9] A. NagoorGani and M Basheer Ahamed, Order and Size in Fuzzy Graphs Bulletin of pure and Applied Sciences, vol.22E(No1) 2003;p. 145-148.

[10] C. Natarajan and S.K. Ayyaswamy, On Strong(weak) Domination in Fuzzy graphs, World Academy of Science, Engineering and Technology,67,247-249,2010.

[11] Mohammad Akram, Bipolar fuzzy graphs with applications, Knowledge – Based systems 39 (2013) 1-8

[12] Mohammad Akram, Regular bipolar fuzzy graphs, Neural Comput&Applic (2012) 21 (Suppl 1): S197-S205

[13] Dr. M. Vijaya, V. Mekala, On Edge Degree Properties of Middle, Subdivision and Total Bipolar Fuzzy Graphs International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 18 (2018) pp. 13817-13825

[14] M.G. Karunambigai, M. Akram, K. Palanivel, Domination in Bipolar Fuzzy Graphs. 2013 Fuzz-IEEE International Conference on Fuzzy Systems.