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# PERFECT DOMINATION ON ANTI FUZZY GRAPH

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**Abstract**: In this paper, we introduce the concept of perfect dominating set and perfect domination number of an anti-fuzzy graph. Some types of anti-fuzzy graphs are discussed with suitable examples.

**Key words:**Anti fuzzy graph, dominating set, perfect dominating set and perfect domination number.
**Mathematical Classification:** 05C72, 05C07, 05C12, 05C62.

## 1. Introduction

L.A. Zadeh [9] introduced the notion of a fuzzy set to describe vagueness mathematically and tried to solve such problems by giving a certain grade of membership to each member of a given set which laid the foundation of set theory. In 1975, Fuzzy graphs were introduced by A. Rosenfeld [6], the basic idea of fuzzy graph was introduced by Kauffmann [2] in 1973, fuzzy relation, represents the relationship between the objects of the given set. Domination in fuzzy graphs using effective edges was introduced by A. Somasundaram and S. Somasundaram [8]. Domination in fuzzy graphs using strong arc as discussed by A. Nagoorgani and V. T. Chandrasekaran [5]. R. Seethalaksmi and R. B. Gnanajothi [7] introduced the definition of Anti Fuzzy Graph. R. Muthuraj and A. Sasireka [3,4] defined some types of Anti Fuzzy Graph and illustrated the concepts of some types of regular Anti Fuzzy Graph, edge regular Anti Fuzzy Graph and some operations on the anti-Fuzzy Graph. C.V.R. Harinarayanan, S. Revathi and P.J. Jayalakshmi [1] introduced the perfect dominating set in fuzzy graph. In this paper, we introduce the concept of perfect domination on anti-Fuzzy Graph. Some theorems are discussed and suitable examples are given.

## 2. Preliminaries

***Definition 2.1:*** A fuzzy graph G = (σ,μ) is said to be an anti-fuzzy graph with a pair of functions σ: V→[0,1] and μ: VxV→[0,1], where for all u,v∈V, we have μ(u,v)≥σ(u)∨σ(v).

***Definition 2.2:*** The order p and size q of an anti-fuzzy graph G = (V, σ, μ) are defined to be $p = \sum_{u \in V} \sigma(u)$ and $q = \sum_{uv \in E} \mu(u,v)$. It is denoted by O(G) and S(G).

***Definition 2.3:*** The anti-complement of anti-fuzzy graph G = (σ,μ) is an anti-fuzzy graph $\overline{G} = (\overline{\sigma}, \overline{\mu})$ where $\overline{\sigma} = \sigma$ and $\overline{\mu}(u,v) = \mu(u,v) - (\sigma(\mathrm{u}) \vee \sigma(\mathrm{v}))$ for all u,v in V.

***Definition 2.4:*** An anti-fuzzy graph G(σ,μ) is a strong anti-fuzzy graph μ(u,v) = σ(u)∨σ(v) for all (u,v)∈μ* and G is a complete anti-fuzzy graph if μ(u,v) = σ(u)∨σ(v) for all (u,v) ∈μ* and u,v∈σ*. Two vertices u and v are said to be neighbours if μ(u,v) > 0.

***Definition 2.5:*** An edge e = {u,v} of an anti-fuzzy graph G is called an effective edge if μ(u,v) = σ(u)∨σ(v).

***Definition 2.6:*** u is a vertex in an anti-fuzzy graph G the N(u) = {v:(u,v) is an effective edge} is called the neighbourhood of u and N[u]U{u} is called closed neighbourhood of u.

***Definition 2.7:*** Every vertex in an anti-fuzzy graph G has unique fuzzy values the G is said to be v-nodal anti fuzzy graph. i.e.σ(u) = c for all u∈V(G),

***Definition 2.8:*** Every edge in an anti-fuzzy graph G has unique fuzzy values the G is said to be e-nodal anti fuzzy graph. i.e μ(u,v) = c for all uv∈E(G).

***Definition 2.9:*** Every vertices and edges in an anti-fuzzy graph G have the unique fuzzy value then G is called as uninodal anti fuzzy graph.

***Definition 2.10:*** The strong neighbourhood of an edge $e_i$ in an anti-fuzzy graph G is $N_s(e_i)$ = {$e_j$∈ E(G)/$e_j$ is an effective edge with V N($e_i$) in G and adjacent to $e_j$.

***Definition 2.11:*** An edge e = {u,v}of an anti-fuzzy graph G is called a strong edge if $\mu^{\infty}(u,v) = \mu(u,v)$ where μ(u,v) is an effective edge.

***Definition 2.12:*** An edge e = {u,v} of an anti-fuzzy graph G is called an weak edge if μ(u,v) ≠σ(u)∨σ(v).

***Definition 2.13:*** A set D⊆V(G) is said to be a dominating set of an anti-fuzzy graph G is for every vertex v∈V(G)/D there exists u in D such that v is a strong neighbourhood of u with μ(u,v) = σ(u)∨σ(v) otherwise it dominates itself.

***Definition 2.14:*** A dominating se D with minimum number of vertices is called a minimal dominating set if no proper subset of D is a dominating set.

***Definition 2.15:*** The maximum fuzzy cardinality taken overall minimal dominating set in G is called a domination number of anti-fuzzy graph G and is denoted by γ(G). i.e $|D|_f = \sum_{v \in D} \sigma(v)$.

## 3. Main results

***Definition 3.1:*** Let $G=(\sigma,\mu)$ be an anti-fuzzy graph. Let $u,v\in V$. The vertex u dominates the vertex v in G is a strong arc. A subset P of V is called a perfect dominating set of G if each vertex v not in P is dominated by exactly one vertex of P.

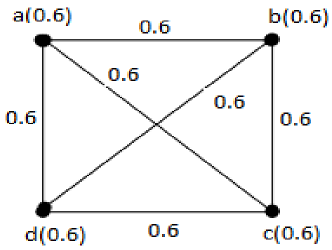
***Definition 3.2:*** A Perfect dominating set P of an anti-fuzzy graph G is said to be a minimal perfect dominating set, if for $v\in P,\ P-\{v\}$ is not a perfect dominating set of G.

**Definition 3.3:** The maximum fuzzy cardinality taken over all minimal perfect dominating set of G is called a perfect domination number of an anti-fuzzy graph and is denoted by $\gamma_{paf}(G)$ .

***Theorem 3.4:*** For any complete uni nodal anti-fuzzy graph G, perfect domination number $\gamma_{paf}(G)=\sigma(u)$ for all $u\in V(G)$ where $\sigma(u)$ is the membership value of a vertex u.

**Proof:** By the definition of uni nodal anti fuzzy graph, every vertices and edges in an anti-fuzzy graph G have the unique fuzzy values. Let G be a complete unimodal anti-fuzzy graph and P be the perfect dominating set of G. In this complete anti fuzzy graph every arc in G is a strong arc and each vertex dominates all other vertices and each vertex in V-P is dominated by exactly one vertex in P which is the perfect dominating set and perfect domination number $\gamma_{paf}(G)=\sigma(u)$ for all $u\in V(G)$.

***Example 3.5:*** Let G be a complete uninodal anti fuzzy graph

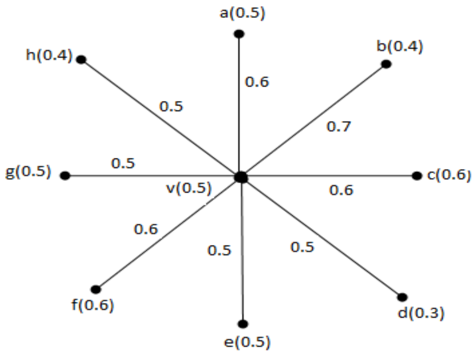



Perfect dominat set P = { a or b or c or d}
Perfect domination number $\gamma_{paf}(G)=0.6$

***Theorem 3.6:*** If G is a star anti fuzzy graph then $\gamma_{paf}(G)=\sigma(v)$ where v is the centre vertex of G.

**Proof:** If G is a star anti fuzzy graph and P is a minimal perfect dominating set of G. The vertex set of $G=\{v,v_1,v_2,..............v_n\}$, v should be dominates to $v_i$, for i = 1, 2, 3, ……………….n, where v is the centre vertex of G which satisfies every vertex $v_i\in V-P$ is dominated by exactly one vertex in P. So, P = {v} is a minimal perfect dominating set and therefore $\gamma_{paf}(G)=\sigma(v)$ .

***Example 3.7:*** Let G be a star anti fuzzy graph



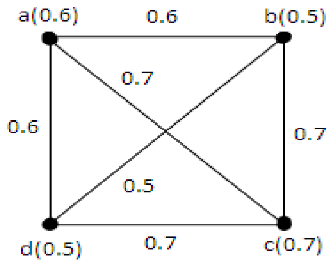

Perfect dominating set P = {v} which is the centre vertex of G.

$\gamma_{paf}(G) = \sigma(v)$

***Theorem 3.8:*** If G is a complete Anti fuzzy graph, then $\gamma_{paf}(G) = \vee\, \sigma(u)$ for all $u \in V(G)$ where $\sigma(u)$ is the membership value of a vertex u.

**Proof:** Let G be a complete anti fuzzy graph and let P be a perfect dominating set. In this complete anti fuzzy graph, every arc in G is a strong arc. So, every vertex dominates all other vertices and every vertex v not in P, v is dominated by exactly one vertex with maximum cardinality for u of P which is perfect dominating set. Therefore, $\gamma_{paf}(G) = \vee\, \sigma(u)$ .

***Example 3.9:*** Let G be an anti fuzzy graph

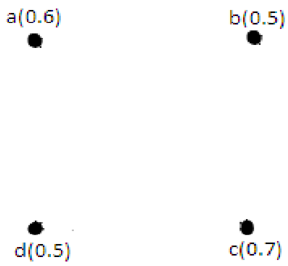



Perfect dominating set = {a or b or c or d}

$\gamma_{paf}(G) = \vee\, \sigma(u)$ = 0.7

***Proposition 3.10:*** Perfect dominating set does not exist for the complement of a complete anti fuzzy graph G.

***Example 3.11:*** F rom the above graph, the complement of a complete anti fuzzy graph $\overline{G}$

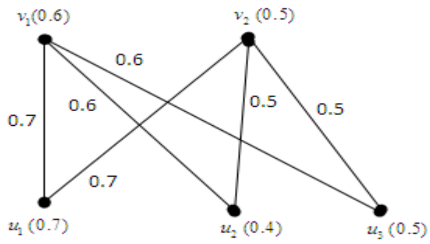



Here perfect dominating set does not exist. Since $\overline{G}$ has isolated vertices. It does not satisfy every vertex v not in P, v is dominated by exactly one vertex with maximum cardinality of P. So $\overline{G}$ is not the perfect dominating set .

***Theorem 3.12:*** Let G be a complete bipartite anti fuzzy graph G, then $\gamma_{paf}(G) = (\vee\sigma(u_i) + \vee\sigma(v_i))$ for all $u_i \in V_2(G)\, and\, v_i \in V_1(G)$.

**Proof:** Let G be a complete bipartite anti fuzzy graph and let P be a perfect dominating set. In this graph G, every arc in G is a strong arc. So, every vertex in $u_i \in V_2(G)$ is dominates all other vertices and every vertex $v_i \in V_1(G)$ and also satisfies every vertex v not in P, v is dominated by exactly one vertex with maximum cardinality for $u_i \in V_2(G)\, and\, v_i \in V_1(G)$ which is perfect dominating set. Therefore, $\gamma_{paf}(G) = (\vee\sigma(u_i) + \vee\sigma(v_i))$ for all $u_i \in V_2(G)\, and\, v_i \in V_1(G)$.

***Example 3.13:*** Let G be a complete bipartite anti fuzzy graph

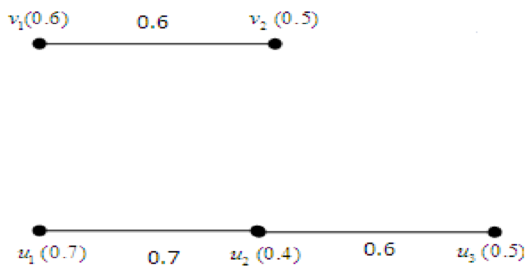



Perfect dominating set P = {($v_1$, $u_1$) or ($v_1$, $u_2$) or ($v_1$, $u_3$) or ($v_2$, $u_1$) or ($v_2$, $u_2$)or ($v_2$, $u_3$)}

$\gamma_{paf}(\overline{G}) = (\vee\sigma(u_i) + \vee\sigma(v_i))$ = 0.6 + 0.7 = 1.3

***Proposition 3.14:*** Perfect dominating set exists for the complement of a complete bipartite anti fuzzy graph G.

***Example 3.15:*** From the above graph, the complement of a complete bipartite anti fuzzy graph $\overline{G}$




Here Perfect dominating set P = {$v_1$ , $u_2$} . Since $\overline{G}$ has disconnected graph. It satisfy every vertex v not in P, v is dominated by exactly one vertex with maximum cardinality of P. So $\overline{G}$ is the perfect dominating set and $\gamma_{paf}(\overline{G})$ = 1

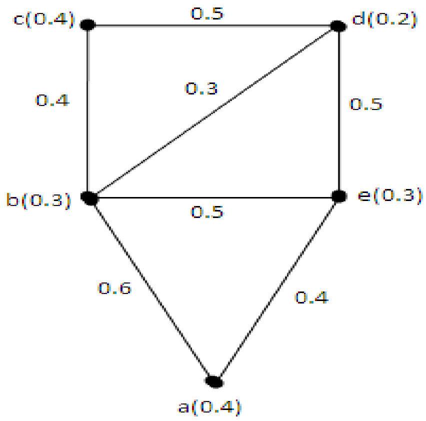
***Theorem 3.16:*** Every maximum fuzzy cardinality of a perfect dominating set of an anti fuzzy graph is minimal perfect dominating set.

***Theorem 3.17:*** A perfect dominating set P of an anti fuzzy graph G is minimal perfect dominating set if and only if for each $v \in P$ at least one of the following conditions are satisfied (i) $N(v) \cap P = \phi$ (ii) there exists a vertex $w \in V - P$ such that $N(w) \cap P = \{v\}$

**Proof:** Let G be an anti fuzzy graph and P be a perfect dominating set of G. Let $v \in P$ and $w \in V - P$. Then to prove that at least one of the conditions (i) and (ii) are satisfied. Suppose P-{v} is a perfect dominating set of G. Then w is not dominated by exactly one vertex in P which is contradiction to our assumption. Hence the vertex v is strong neighbourhood to w and w is dominated by exactly one vertex v. $N(w) \cap P = \{v\}$ and P is a minimal perfect dominating set of G and also there is no common vertex in N(v) and P. Therefore, $N(v) \cap P = \phi$.

Conversely, If at least one of the conditions are holds. Then to prove that P is a minimal perfect dominating set of anti fuzzy graph G. Suppose p is not a minimal perfect dominating set then there exists a vertex $v \in P$ such that P-{v} is a perfect dominating set. But, v is a strong neighbourhood to exactly one vertex in P-{v}. Therefore (i) and (ii) conditions are fails. It is clear that P-{v} is a perfect dominating set which is contradiction to our assumption. Therefore, P is a minimal perfect dominating set of G.

***Example 3.18:*** Let G be an anti fuzzy graph




Here Perfect dominating set P = {b,c} $\gamma_{paf}(G)$ =0.7

## 4. Conclusion

In this paper, perfect domination number is defined on anti fuzzy graphs and also applied for the various types of anti fuzzy graph and suitable examples are given.

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