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TOTAL STRONG (WEAK) DOMINATION ON ANTI FUZZY GRAPH

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Abstract: In this paper, the concept of total strong(weak) domination on Anti fuzzy graphs are introduced. We discussed degree and distance in total strong (weak) domination on anti fuzzy graph with some suitable graphs with examples.

Key words: Anti fuzzy graph, dominating set, total dominating set, total strong (weak) dominating set and total strong(weak) domination number.

Mathematical classification: 05C72, 05C07, 05C12, 05C62.

1. Introduction

L.A. Zadeh [9] introduced the notion of a fuzzy set to describe vagueness mathematically and tried to solve such problems by giving a certain grade of membership to each member of a given set which laid the foundation of set theory. In 1975, Fuzzy graphs were introduced by A. Rosenfeld [6], the basic idea of fuzzy graph was introduced by Kauffmann [2] in 1973, fuzzy relation, represents the relationship between the objects of the given set. Domination in fuzzy graphs using effective edges was introduced by A. Somasundaram and S. Somasundaram [8]. Domination in fuzzy graphs using strong arc as discussed by A. Nagoorgani and V. T. Chandrasekaran [5]. R. Seethalaksmi and R. B. Gnanajothi [7] introduced the definition of Anti Fuzzy Graph. R. Muthuraj and A. Sasireka [3,4] defined some types of Anti Fuzzy Graph and illustrated the concepts of some types of regular Anti Fuzzy Graph, edge regular Anti Fuzzy Graph and some operations on Anti Fuzzy Graph. C.V.R. Harinarayanan, S. Revathi and P.J.Jayalakshmi [1] introduced Independent and Total strong (weak) domination in fuzzy graph. In this paper we introduce the concept of Total strong (weak) domination on Anti Fuzzy Graph. Some theorems are discussed and suitable examples are given.

2. An anti fuzzy graph

Definition 2.1 A fuzzy graph G = (σ,μ) is said to be an anti fuzzy graph with a pair of functions σ: V→[0,1] and μ: VxV→[0,1], where for all u,v∈V, we have μ(u,v)≥σ(u)∨σ(v) and it is denoted by G(σ,μ).

Note :μ is considered as reflexive and symmetric. In all examples σ is chosen suitably. i.e., only undirected anti fuzzy graph are considered.

Definition 2.2 The order p and size q of an anti fuzzy graph G=(V, σ, μ) are defined to be $p = \sum_{u\in V}\sigma(u)$ and $q = \sum_{uv\in E}\mu(u,v)$. It is denoted by O(G) and S(G).

Definition 2.3 The anti complement of anti fuzzy graph G(σ,μ) is an anti fuzzy graph $\overline{G} = (\overline{\sigma}, \overline{\mu})$ where $\overline{\sigma} = \sigma$ and $\overline{\mu}(u,v) = \mu(u,v) - (\sigma(\text{u}) \vee \sigma(\text{v}))$ for all u,v in V.

Definition 2.4 An anti fuzzy graph G(σ,μ) is a strong anti fuzzy graph μ(u,v) = σ(u)∨σ(v) for al (u,v)∈μ*and G is a complete anti fuzzy graph if μ(u,v) = σ(u)∨σ(v) for all (u,v) ∈μ* and u,v∈σ*. Two vertices u and v are said to be neighbours if μ(u,v) > 0.

Definition 2.5 An edge e = {u,v} of an anti fuzzy graph G is called an effective edge if μ(u,v) = σ(u)∨σ(v).

Definition 2.6 u is a vertex in an anti fuzzy graph G the N(u) = {v:(u,v) is an effective edge} is called the neighbourhood of u and N[u]U{u} is called closed neighbourhood of u.

Definition 2.7 Every vertex in an anti fuzzy graph G has unique fuzzy values the G is said to be v-nodal anti fuzzy graph. i.e.σ(u) = c for all u∈V(G),

Definition 2.8 Every edge in an anti fuzzy graph G has unique fuzzy values the G is said to be e-nodal anti fuzzy graph. i.eμ(u,v) = c for all u, v∈E(G).

Definition 2.9 Every vertices and edges in an anti fuzzy graph G have the unique fuzzy value then G is called as uninodal anti fuzzy graph.

Definition 2.10 The strong neighbourhood of an edge e_i in an anti fuzzy graph G is $N_s(e_i)$ = {e_j∈ E(G)/e_j is an effective edge with V N(e_i) in G and adjacent to e_j.

Theorem:2.11 On anti fuzzy graph, $\gamma_{T_A} \le p \le q$

Proof: Let T_A is a Total strong (weak) dominating set on anti fuzzy graph. We know that $p = \sum_{u\in V}\sigma(u)$ and $q = \sum_{u,v\in V}\mu(u,v)$. Also we know that, In a graph, $p \le q$, γ_{T_A} be a total strong (weak) domination number of on anti fuzzy graph. Clearly, $\gamma_{T_A} \le p$ (since $p \le q$)

Hence, $\gamma_{T_A} \le p \le q$.

3.Total strong (weak) domination on anti fuzzy graph

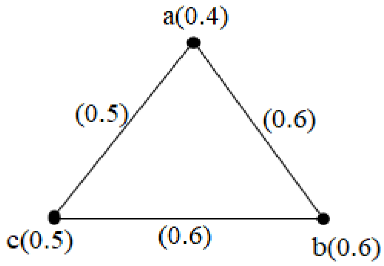
In this section, we discussed total strong (weak) domination on anti fuzzy graph and e-nodal anti fuzzy graph, uni nodal anti fuzzy graph were discussed.

Definition 3.1 A fuzzy graph G = (σ,μ) is said to be an Total strong (weak) domination on anti fuzzy graph if

$(i)\ \mu(u,v) \geq \mu^{\infty}(u,v)\quad (ii)\ d_N(v) \geq d_N(u)\ \ \forall u \in T_A,\ v \in v - T_A\quad (iii)\ T_A$

is a total dominating set.

Example:3.2 In an anti fuzzy graph,



$T_A = \{a,b,c,f,h,i\},\ V - T_A = \{d,e,g,j\},\ \gamma_{T_A} = 3.1,\ p = 5.4,\ q = 7.8,\ \Delta_N = 1.8,\ \delta_N = 0.8, \Delta_E = 2.4,\ \delta_E = 1.2$

$T_A = \{a,b\},\ V - T_A = \{c\},\ \gamma_{T_A} = 0.4 + 0.6 = 1,\ p = 1.5,\ q = 1.7,\ \Delta_N = 1.1,\ \delta_N = 0.9, \Delta_E = 1.2,\ \delta_E = 1.1$

$\mu(a,c) = \mu(a,b) + \mu(b,c) = 0.6 + 0.6 = 1.2,\ d(\mu(a,c)) = 1.2$

Theorem:3.3 On anti fuzzy graph, $\gamma_{T_A} \leq\ p\ \leq\ q$.

Proof: Let T_A is a Total strong (weak) dominating set on anti fuzzy graph. We know that $p = \sum_{u \in V} \sigma(u)$ and $q = \sum_{u,v \in V} \mu(u,v)$, Also we know that, in a graph, $p \leq q$, γ_{T_A} be a total strong (weak) domination number of on anti fuzzy graph. Clearly, $\gamma_{T_A} \leq p$ (since $p \leq q$).

Hence, $\gamma_{T_A} \leq\ p\ \leq\ q$

Theorem: 3.4 On anti fuzzy graph, $\begin{matrix} (i)\ \gamma_{T_A} \leq p - \Delta_N \leq q - \delta_N \\ (ii)\ \gamma_{T_A} \leq p - \Delta_E \leq q - \delta_E \end{matrix}$.

Proof: Let T_A is a Total strong (weak) dominating set on anti fuzzy graph. We know that $p = \sum_{u \in V} \sigma(u)$ and $q = \sum_{u,v \in V} \mu(u,v)$. Also we know that, In a graph, $p \leq q$ and γ_{T_A} be a total strong (weak) domination number of on anti fuzzy graph. If $\Delta_N \leq \gamma_{T_A} \leq p$

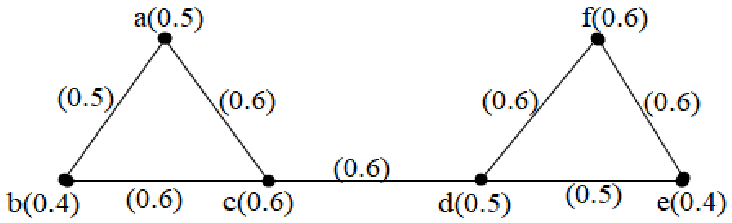
$\gamma_{T_A} \leq p - \Delta_N$ ----------(1), where p is a sum of vertices.

$\delta_N \leq \gamma_{T_A} \leq q$

$\gamma_{T_A} \leq q - \delta_N$ ----------(2) (since $p \leq q$)

From the equation (1) and (2), we get $\gamma_{T_A} \leq p - \Delta_N \leq q - \delta_N$. Similarly, $\gamma_{T_A} \leq p - \Delta_E \leq\ q - \delta_E$. Hence proved.

Example:3.5



$T_A = \{a,c,d\}, V - T_A = \{b,e,f\}, \gamma_{T_A} = 1.6, \ p = 3, \ q = 4, \ \Delta_N = 1.4, \ \delta_N = 0.6, \Delta_E = 1.8, \ \delta_E = 0.6$

Relations: An e- nodal anti fuzzy graph,

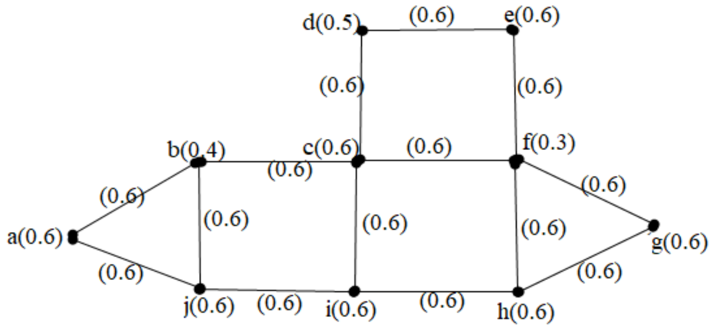
$(i) \ p - \Delta_N \leq q - \Delta_N$

$(ii) \ p - \delta_N \leq q - \delta_N$

$(iii) \ p - \Delta_E \leq q - \Delta_E$

$(iv) \ p - \delta_E \leq q - \delta_E$

Example:3.6 In an e-nodal anti fuzzy graph,



$T_A = \{a,b,c,f,h,i\}, V - T_A = \{d,e,g,j\}, \gamma_{T_A} = 3.1, \ p = 5.4, \ q = 7.8, \ \Delta_N = 1.8, \ \delta_N = 0.8, \Delta_E = 2.4, \ \delta_E = 1.2$

Relations: An uni nodal anti fuzzy graph

$(i) \ p - \Delta_N \leq q - \Delta_N$

$(ii) \ p - \delta_N \leq q - \delta_N$

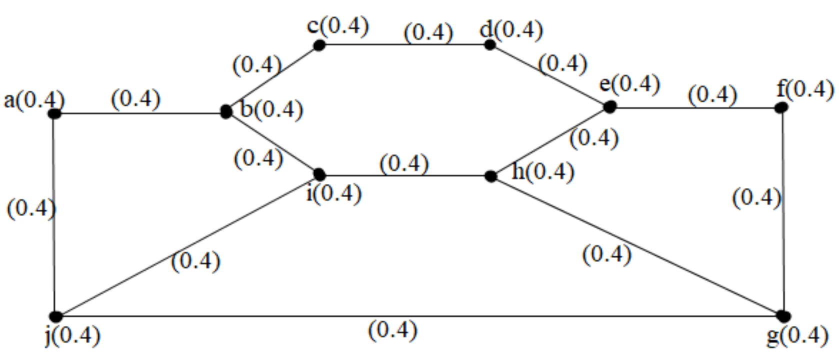
$(iii) \ p - \Delta_E \leq q - \Delta_E$

$(iv) \ p - \delta_E \leq q - \delta_E$

$(v) \ \Delta_N = \delta_N$

$(vi) \ \Delta_E = \delta_E$

Example:3.7 In an uni nodal anti fuzzy graph,



$$T_A = \{a,b,d,e,g,j\}, V - T_A = \{c,f,h,i\}, \gamma_{T_A} = 2.4,\ p = 4,\ q = 5.2,\ \Delta_N = 1.2 = \delta_N, \Delta_E = \delta_E = 0.8$$

4. Conclusion

In this paper, the concept of total strong (weak) domination on anti fuzzy graph is introduced. We have done some theorems with examples and Relations of e-nodal and uninodal anti fuzzy graph were discussed with the suitable examples.

Further, we will introduce all types of graph in an appropriate conditions and some suitable examples.

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