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WEAK PRIME BI-IDEALS AND WEAK PRIME FUZZY BI-IDEALS IN NEAR-RINGS

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Abstract: The analysis of weak prime bi-ideals in near-rings is the primaryfocus of our research. The concept of weak prime bi-ideals in near-rings N have been dilated upon by our research team. Now we are taking up the significant concept of fuzzification in weak prime bi-ideals of near-rings and exploring its behaviour &operations. Prime bi-ideals, weak prime bi-ideals in near-rings have been attempted to be defined systematically. This concept motivates the study of different kinds of new bi-ideals in algebraic theory, especially bi-ideals in near-rings and fuzzy algebra.

Keywords:
Prime bi-ideal, Weak prime bi-ideal, Prime fuzzy bi-ideal, Weak prime fuzzy bi-ideal.

1.Introduction

The subject of our research is to understand and to analyse the prominent characteristic of the concept of “fuzzification of prime bi-ideals and weak prime bi-ideals in near-rings”. Fuzzy set was first introduced by L.A.Zadeh in 1965 as a general abstraction of “set theory”.Since then, a lot of concepts and applications developed over “fuzzy set”. Near-ring is any set which needs no additions as abelian and suffices with only one distributive law (i.e) either right distributive law or left distributive law. S. AbouZaid, in 1991, developed upon the idea of fuzzification of subnear-rings, subsequently he evaluated fuzzy left (right) ideals of near-rings and discovered some prominent characteristics of fuzzy prime ideals of a near-rings. Similarly, concepts such as quasi-ideals and bi-ideals in integrative near-rings were systematically explored by researchers Yakabe and TamizhChelvam et al. respectively. To further this discourse, we have thoroughly investigated the concept of weak prime bi-ideals and weak prime fuzzy bi-ideals in near-rings. And, we have further mediated and research fuzzification of weak prime bi-ideals and weak prime fuzzy bi-ideals in near-rings.

2. Preliminaries

In this section, we collect some basic concepts in near-rings, which are used in this paper.

Definition 2.1:

A non-empty set N with two binary operations "+" (addition) and "." (multiplication) is called a near-ring if

(i). $(N,+)$is a group (not necessarily abelian).

(ii). $(N,.)$is a semi group.

(iii). For all x,y,z∈N,

- $x.(y+z)=x.y+x.z$ (left distributive law)
- $(x+y).z=x.z+y.z$(right distributive law)

If N satisfies (i) (ii) & left distributive law is called a left near-ring. If N satisfies (i) (ii) & right distributive law is called a right near-ring.

Remark: In this paper, by a near-ring, we mean only a right near-ring. The symbol N stands for a right near-ring $(N,+,.)$ with atleast two elements. 0 denotes the identity element of the group $(N,+)$.

Definition 2.2:

A near-ring N is called a zero-symmetric near-ring, if $0.x=x.0=0$, for all $x\in N$.

Definition 2.3:

Let $(N,+,.)$ be a near-ring. A non-empty set I of N is called an ideal if

(i). $x+y\in I$, for all $x\in I$&$y\in N$.

(ii). $y+x-y\in I$,for all $x\in I$& y∈ N.

(iii). $x(i+y)-xy\in I$, for all$i\in I$&$x,y\in N$. Incase of zero-symmetric,$IN\subseteq I$.

(iv). $NI\subseteq I$.

If I satisfies (i) (ii) & (iii) then I is called left ideal where as I satisfies (i) (ii) &(iv) then I is called right ideal of N.

Definition 2.4:

Let $(N,+,.)$ be a near-ring. A non-empty set I of N is called a weak ideal if

(i). $x+y\in I$, for all $x\in I$&$y\in I$.

(ii). $y+x-y\in I$,for all $x\in I$& y∈ N.

(iii). $x(i+y)-xy\in I$, for all$i\in I$&$x,y\in N$. Incase of zero-symmetric,$IN\subseteq I$.

(iv). $NI\subseteq I$.

If I satisfies (i) & (iii) then I is called left weak ideal where as I satisfies (i) & (ii) then I is called right weak ideal of N.

Definition 2.5:

A subgroup B of (N,+) is called a bi-ideal of N if $BNB\cap(BN)*B\subseteq B$. In case of zero-symmetric,$BNB\subseteq B$.

Definition 2.6:

A subgroup B of $(N,+)$ is said to be weak bi-ideal of N if $BBB\subseteq B$.

Definition 2.7:

An ideal P of $(N,+,.)$ is called a prime ideal if $AB\subseteq P$ implies $A\subseteq P$or $B\subseteq P$,for any two ideals A & B of N.

Definition 2.8:

An ideal P of $(N,+,.)$ is called a weak prime ideal if $\{0)\neq AB\subseteq P$ implies $A=P$or $B=P$, for any two ideals A & B of N.

Definition 2.9:

A function μ is a mapping from the set N to the unit interval [0,1] is called a fuzzy subset.

Definition 2.10:

Let μ be a fuzzy subset of N. Then the level set is defined by, $\mu_t = \{x \in N/\mu(x) \geq t\}$, where $t \in [0,1)$.

Definition 2.11:

A fuzzy subset μ of N is said to be a fuzzy subgroup if $\mu(x+y) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in N$.

Definition 2.12:

A fuzzy subset μ is called a fuzzy ideal of N if for every $x, y, z \in N$,

(i) $\mu(x+y) \geq \min\{\mu(x), \mu(y)\}$.
(ii) $\mu(y + x - y) \geq \mu(x)$.
(iii) $\mu(xy) \geq \mu(y)$.
(iv) $\mu((x+z)y - xy) \geq \mu(z)$.Incase of zero-symmetric,$\mu(xy) \geq \mu(x)$.

A fuzzy subset with (i), (ii) and (iii) is called a fuzzy left ideal of N, where as a fuzzy subset with (i), (ii) and (iv) is called a fuzzy right ideal of N.

Definition 2.13:

Let μ&λ be two fuzzy subsets of N. Then $\mu \cap \lambda, \mu \cup \lambda, \mu - \lambda, \mu\lambda, \mu * \lambda$ are all fuzzy subsets of Nand it is defined by,

$$(\mu \cap \lambda)(x) = min\{\mu(x), \lambda(x)\}$$
$$(\mu \cup \lambda)(x) = max\{\mu(x), \lambda(x)\}$$
$$(\mu - \lambda)(x) = \begin{cases} \sup_{x=y-z} \min\{\mu(y), \lambda(z)\} & if\ x\ can\ be\ expressed\ as\ x = y - z \\ 0 & otherwise \end{cases}$$
$$. \quad \mu\lambda(x) = \begin{cases} \sup_{x=yz} min\{\mu(y), \lambda(z)\} & if\ x\ can\ be\ expressed\ as\ x = yz \\ 0 & otherwise \end{cases}$$
$$(\mu * \lambda)(x) = \begin{cases} \sup_{x=ac-a(b-c)} min\{\mu(a), \lambda(c)\} & if\ x\ can\ be\ expressed\ as\ x = ac \\ 0 & otherwise \end{cases}$$

Definition 2.14:

Let $\{\mu_i / i \in \Omega\}$ be a family of subsets of a near-ring N, then the intersection of $\{\mu_i/i\epsilon\Omega\}$ is defined by, $\bigcap_{i\epsilon\Omega} \mu_i(x) = \inf\{\mu_i(x)/i\epsilon\Omega\}$.

Definition 2.15:

A fuzzy subgroupμ of N is called a fuzzy bi-ideal of N, if$\mu N \mu \cap (\mu N) * \mu \subseteq \mu$. In case of zero-symmetric, $\mu N \mu \subseteq \mu$.

Definition 2.16:

Let μ be a fuzzy subgroup of N, then μis a fuzzy weak bi-ideal of N if $\mu\mu\mu \subseteq \mu$.

3. Weak prime bi-ideals

Definition 3.1:

A bi-ideal B of $(N, +, .)$ is called a prime bi-ideal if $B_1 B_2 \subseteq B$ implies $B_1 \subseteq B$ or $B_2 \subseteq B$, for any two bi-ideals B_1&B_2 of N.

Example 3.2: Let $N = \{0,1,2\}$ with "+" and "." are defined as,

+	0	1	2
0	0	1	2
1	1	0	1
2	2	1	0

.	0	1	2
0	0	0	0
1	0	1	1
2	0	1	2

Clearly, N is a commutative near-ring and {0},{0,1} and {0,1,2} are prime bi-ideals of N.

Definition 3.3:

A bi-ideal B of $(N, +, .)$ is called a weak prime bi-ideal if $\{0\} \neq B_1B_2 \subseteq B$ implies$B_1 = B$, or $B_2 = B$, for any two bi-ideals B_1&B_2 of N.

Example 3.4: Let $N = \{0, a, b, c\}$ with "+" and "." are defined as,

+	0	a	b	c
0	0	a	b	c
a	a	0	c	c
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	0	0	0
c	0	a	b	c

Clearly $(N, +, .)$is a near-ring. Note that {0,a} is a weak prime bi-ideal.

Remark: Every prime bi-ideal B is a weak prime bi-ideal but the converse is not true.

Example 3.5: Let $N = \{0, a, b, c\}$ with "+" and "."are defined as,

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

.	0	1	2
0	0	0	0
1	1	0	1
2	2	2	0

Clearly$(N, +, .)$ is a near-ring. Note that {0,2} is a weak prime bi-ideal but not prime bi-ideal. Since {0,1}{0,1} ⊆ (0,2}. But {0,1}⊈ {0,2}.

Theorem 3.6:

Intersection of any family of weak prime bi-ideals of a near-ring N is also a weak prime bi-ideal of N.

Proof: Let $\{B_i, i \in I\}$ be any family of weak prime bi-ideals of a near-ring N. To Prove: $B = \bigcap_{i \in I} B_i$ is a weak prime bi-ideal of N. By [9], Intersection of all bi-ideals of a near-ring N is also a bi-ideal of N., (i.e) $B = \bigcap_{i \in I} B_i$ is a bi-ideal of N.

Let P & Q be any two bi-ideals of N such that $\{0\} \neq PQ \subseteq B = \bigcap_{i \in I} B_i$.

$$\Longrightarrow \{0\} \neq PQ \subseteq B_i, \forall i \in I.$$

Since each$B_i$is a weak prime bi-ideal of N. Therefore, $P = B_i$ or$Q = B_i, \forall i \in I$.

(i.e.,) $P = \bigcap_{i \in I} B_i$ or$Q = \bigcap_{i \in I} B_i$.

Therefore, $B = \bigcap_{i \in I} B_i$ is a weak prime bi-ideal of N.

Theorem 3.7:

Every bi-ideal of a near-ring N is weak prime bi-ideal iff for any bi-ideals B_1, B_2 in N, we have $B_1B_2 = B_1$or$B_1B_2 = B_2$or $B_1B_2 = 0$.

Proof: Assume that every bi-ideal of N is weak prime bi-ideal. Let B_1&B_2 be two bi-ideals of N. Suppose $B_1B_2 \neq N$. Then, B_1B_2 is weak prime. If $\{0\} \neq B_1B_2 \subseteq B_1B_2$, then we have $B_1 = B_1B_2$ or $B_2 = B_1B_2$. Since B_1B_2 is a weak prime bi-ideal of N (By [10], product of two bi-ideals of N is also abi-ideal). If $B_1B_2 = N$, then we have $B_1 = B_2 = N$ whence $N^2 = N$.

Conversely, let I be any proper bi-ideal of N and suppose that $\{0\} \neq B_1B_2 \subseteq I$, for any bi-ideals B_1&$B_2$of N. Then we have either $B_1 = B_1B_2 \subseteq I$or $B_2 = B_1B_2 \subseteq I$.

Corollary 3.8:

Let N bea near-ring in which every bi-ideal of N is weak prime bi-ideal. Then, for any bi-ideal B of N, we have either $B^2 = B$or $B^2 = 0$.

Theorem 3.9:

Let N be a near-ring and B be a weak prime bi-ideal of N. If B is not a prime bi-ideal, then $B^2 = 0$.

Proof: Suppose that $B^2 \neq 0$.To show that B is prime. Let $B_1\&B_2$ be two bi-ideals of N such that $B_1B_2 \subseteq B$. If $B_1B_2 \neq \{0\}$, then $B_1 \subseteq B$or $B_2 \subseteq B$.
If $B_1B_2 = \{0\}$, Since $B^2 \neq 0$, there exists $x, y \in B$ such that $< x >< y > \neq 0$
Then $(B_1 + < x >)(B_2 + < y >) \neq 0$. Suppose $(B_1 + < x >)(B_2 + < y >) \nsubseteq B$.
Then there exists $b_1 \in B_1\&b_2 \in B_2$ and $x' \in < x >\&y' \in < y >$ such that $(b_1 + x')(b_2 + y') \notin B$, which implies $b_1(b_2 + y') \notin B$.
Since $B_1B_2 = 0, b_1(b_2 + y') = b_1(b_2 + y') - b_1b_2 \in B$.Which is a contracdiction. So $\{0\} \neq (B_1 + < x >)(B_2 + < y >) \subseteq B$ which implies $B_1 \subseteq B$or $B_2 \subseteq B$.

Corollary 3.10:

Let N be near-ring & B be a weak bi-ideal of N. If $B^2 \neq 0$, then B is prime bi-ideal iff B is weak prime bi-ideal.

Theorem 3.11:

Let N be a decomposable near-ring with identity. If B is aweak prime bi-ideal of N, then either $B = 0$ or B is prime.

Proof: **S**uppose that $N = N_1 \times N_2$ and let $B = B_1 \times B_2$ be aweak prime bi-ideal of N. We assume that $B \neq 0$. Now, let P be a non-zero bi-ideal of N_1 and Q be a non-zero bi-ideal of N_2such that $\{0\} \neq (P, Q) \subseteq B$. Then$\{0\} \neq (P, N_2)(N_1, Q) \subseteq B$, which implies $(P, N_2) \subseteq B$ or $(N_1, Q) \subseteq B$.
Suppose that $(P, N_2) \subseteq B$, Then $(0, N_2) \subseteq B$ andso $B = B_1 \times N_2$. We show that B_1 is a prime bi-ideal of.N_1. Let $P_1\&Q_1$ bea bi-ideal of N_1 such that $P_1Q_1 \subseteq B_1$. Then $\{0\} \neq (P_1, N_2)(Q_1, N_2) = (P_1Q_1, N_2) \subseteq B$. So $(P_1, N_2) \subseteq B$ or $(Q_1, N_2) \subseteq B$ and hence $P_1 \subseteq B_1$ or$Q_1 \subseteq B_1$. So B is prime bi-ideal of N. The case where $(N_1, B_2) \subseteq B$ is similar.

4. Weak prime fuzzy bi-ideals

Definition 4.1:

A fuzzy bi-ideal f of a near-ring N is called prime fuzzy bi-ideal of N if for any two fuzzy bi-idealsg, h of N such that $g \circ h \leq f$ which implies $g \leq f$ or $h \leq f$.

Example 4.2: Let $N = \{0, a, b, c\}$with "+" and "." are defined as,

+	0	a	b	c
0	0	a	b	c
a	a	0	c	c
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	a	c	b
c	0	a	b	c

Clearly$(N, +, .)$ is a near-ring . Let f, g & h be fuzzy subsets of N such that ,

$$f(0) = 1,\ f(a) = 0.8,\ f(b) = 0.7,\ f(c) = 0.5.$$
$$g(0) = 1,\ g(a) = 0.8,\ g(b) = 0.6,\ g(c) = 0.3.$$
$$h(0) = 1,\ h(a) = 0.7,\ h(b) = 0.5,\ h(c) = 0.2.$$

Then $f$is a prime fuzzy bi-ideal of N.

Definition 4.3:

A fuzzy bi-ideal f of a near-ring N is called weak prime fuzzy bi-ideal of N if for any fuzzy bi-idealsg, h of N containing f such that $g \circ h \leq f$ implies $g = f$ or $h = f$.

Example 4.4: Let $N = \{0, a, b, c\}$with "+" and "." are defined as,

+	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

.	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	a	c	b
c	0	a	b	c

Clearly$(N, +, .)$is a near-ring. letf, g & h be fuzzy subsets of N such that ,

$$f(0) = 1,\ f(a) = 0.8,\ f(b) = 0.7,\ f(c) = 0.5.$$
$$g(0) = 1,\ g(a) = 0.8,\ g(b) = 0.6,\ g(c) = 0.3.$$
$$h(0) = 1,\ h(a) = 0.7,\ h(b) = 0.5,\ h(c) = 0.2.$$

Then f is a weak prime fuzzy bi-ideal of N.

Remark:Every prime fuzzy bi-ideal of N is a weak prime fuzzy bi-ideal of N but the converse is not true.

Example 4.5: Let $N = \{0, a, b, c\}$with "+" and "." are defined as,

+	0	a	b	c
0	0	0	0	0
a	a	0	c	b
b	b	0	0	b
c	c	0	c	0

.	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	0	0	0
c	0	a	b	c

Clearly $(N, +, .)$ is a near-ring. Let f, g & h be fuzzy subsets of N such that,

$$f(0) = 1,\ f(a) = 0.2,\ f(b) = 0.2,\ f(c) = 1.$$
$$g(0) = 0.8,\ g(a) = 0,\ g(b) = 0.8,\ g(c) = 0.$$
$$h(0) = 0.8,\ h(a) = 0,\ h(b) = 0.8,\ h(c) = 0.$$

Here the fuzzy bi-ideal f is a weak prime fuzzy bi-ideal of N but not prime fuzzy bi-ideal of N. Since $g \circ h \le f$ implies neither $g \neq f$nor $h \neq f$.

Theorem 4.6:

Let $\{f_i / i \in \Omega\}$be family of weak prime fuzzy bi-ideals of a near-ring N, then $\bigcap_{i\in\Omega} f_i$ is also a weak prime fuzzy bi-ideal of N, where Ω is any index set.

Proof: By[6], Intersection of any family of fuzzy bi-ideals of N is also a fuzzy bi-deal of N.
To prove: $\bigcap_{i\in\Omega} f_i = f$ is afuzzy bi-ideal of N. Let σ & δ be two fuzzy bi-ideals of N contain f such that $\sigma \circ \delta \le f = \bigcap_{i\in\Omega} f_i$. $\Longrightarrow \sigma \circ \delta \le f_i$, for all $i \in \Omega$.
Since each f_i is aweak prime fuzzy bi-ideal of N. So we get, $\sigma = f_i$ or $\delta = f_i$, for all $i \in \Omega$.

(i.e.,) $\sigma = \bigcap_{i\in\Omega} f_i$ or $\delta = \bigcap_{i\in\Omega} f_i$

Therefore, $\bigcap_{i\in\Omega} f_i = f$ is a weak prime fuzzy bi-ideal of N.

Lemma 4.9:

If f is a non-constant weak prime fuzzy bi-ideal of N, then $Im\, f = \{1, t\}, t \in [0,1)$.

Proof: Let f be a non-constant weak prime fuzzy bi-ideal of N. If $Im\, f = \{t_1, t_2, t_3\}$, for $1 > t_1 > t_2 > t_3 \ge 0$. Then there exists $a, b, c \in N$ such that $f(a) = t_1, f(b) = t_2 \& f(c) = t_3$. Choose $s_1 \& s_2$ in such a way that $1 > s_1 > t_1 > s_2 > t_2 > s_3 > t_3$.Now, we define fuzzy subsets g & h as follows,

$$g(x) = \begin{cases} s_1 & if\ x \in f_{t_1} \\ t_2 & otherwise \end{cases} \quad h(x) = \begin{cases} t_2 & if\ x \in f_{t_1} \\ s_2 & if\ x \in f_{t_2} - f_{t_1} \\ t_3 & otherwise \end{cases}$$

Clearly $f \le g$ & $f \le h$

$$g.h(x)=\begin{cases} t_1 & if\ x=yz, \quad y,z\in f_{t_1} \\ s_2 & if\ x=yz, \quad y\in f_{t_2}-f_{t_1}, \quad z\in f_{t_1} \\ t_2 & if\ x=yz, \quad y,z\in f_{t_2}-f_{t_1} \\ t_3 & if\ x=yz, \quad y\in f_{t_3}-f_{t_2} \\ 0 & otherwise \end{cases}$$

Thus $g.h\le f$. But $g(a)=s_1>t_1=f(a)\&h(b)=s_2>t_2=f(b)$.
Then, $g\neq f$ & $h\neq f$. Which is a contradiction.
If $Im\, f=\{t_1,t_2\}$, for $1>t_1>t_2\ge 0$. Then there exists $a_1,b_1\in N$ such that $f(a_1)=t_1$, $f(b_1)=t_2$. Choose $s_1\&s_2$ in such a way that $1>s_1>t_1>s_2>t_2$.

Now, we define fuzzy subsets g & h as follows,

$$g(x)=\begin{cases} s_1 & if\ x\in f_{t_1} \\ t_2 & otherwise \end{cases} \quad h(x)=\begin{cases} t_1 & if\ x\in f_{t_1} \\ s_2 & otherwise \end{cases}$$

Clearly $f\le g$ & $f\le h$

$$g.h(x)=\begin{cases} t_1 & if\ x=yz, \quad y,z\in f_{t_1} \\ s_2 & if\ x=yz, \quad y\in f_{t_2}-f_{t_1}, z\in f_{t_1} \\ t_2 & if\ x=yz, \quad y\notin f_{t_1}, \quad z\notin f_{t_1} \\ 0 & otherwise \end{cases}$$

Thus $g.h\le f$. But $g(a_1)=s_1>t_1=f(a_1)\&h(b_1)=s_2>t_2=f(b_1)$. Then, $g\neq f$ & $h\neq f$.Which is a contradiction.
Hence $Im\, f=\{1,t\}, t\in[0,1)$.

Lemma 4.8:

Let A be a non-empty subset of a near-ring N and μ_A be a fuzzy set in N defined by, $\mu_A(x)=\begin{cases} s & if\ x\in A \\ t & otherwise \end{cases}$ for all $x\in N$ and $s,t\in[0,1]$ with $s>t$. Then μ_A is a fuzzy ideal of N iff A is an ideal of N.

Proof: Let μ_A be a fuzzy ideal of N. To prove: A is an ideal of N.

(i) Let $x,y\in A$. Then $\mu_A(x)=\mu_A(y)=s$
Now, $\mu_A(x+y)\ge\min\{\mu_A(x),\mu_A(y)\}=s$
Which implies $x+y\in A$. (i.e) $x+y\in N$.
Therefore, A is a subgroup of N.

(ii) Let $y\in N$ & $x\in A$. Then $\mu_A(y+x-y)\ge\mu_A(x)=s\Rightarrow y+x-y\in A$.

(iii) Let $x\in A$ & $y\in N$. Then$\mu_A(x)=s$.
Now, $\mu_A(xy)\ge\mu_A(x)=s.\Rightarrow xy\in A$.

(iv) Let $x\in N$ & $y\in A$. Then$\mu_A(y)=s$.
Now,$\mu_A(xy)\ge\mu_A(y)=s.\Rightarrow xy\in A$.

This shows that A is an ideal of N.

Conversely,

Assume that A is an ideal of N.To prove: μ_A is a fuzzy ideal of N.

(i) Let $x,y\in N$.
If $x\notin A$ or $y\notin A$. Then $\mu_A(x+y)\ge t=\min\{\mu_A(x),\mu_A(y)\}\Rightarrow x+y\in\mu_A$.
If $x\in A$ or $y\in A$. Then $x+y\in A$. Now, $\mu_A(x+y)=s\ge\min\{\mu_A(x),\mu_A(y)\}\Rightarrow x+y\in\mu_A$.

(ii) Let $x,y\in N$.
If $x\in A$ or $y\in N$. Then $\mu_A(y+x-y)=s=\mu_A(x)\Rightarrow y+x-y\in\mu_A$.
If $x\notin A$ or $y\in N$. Then $\mu_A(y+x-y)\ge t=\mu_A(x)\Rightarrow y+x-y\in\mu_A$.

(iii) Let $x,y\in N$.
If $x\notin A$. Then clearly, $\mu_A(xy)\ge t=\mu_A(x)\Rightarrow xy\in\mu_A$.
If $x\in A$. Then clearly, $\mu_A(xy)=s=\mu_A(x)\Rightarrow xy\in\mu_A$.

(iv) Let $x,y\in N$.
If $y\notin A$. Then clearly, $\mu_A(xy)\ge t=\mu_A(y)\Rightarrow xy\in\mu_A$.
If $y\in A$. Then clearly, $\mu_A(xy)=s=\mu_A(y)\Rightarrow xy\in\mu_A$.

Therefore, μ_A is a fuzzy ideal of N.

Theorem 4.9:

If f is a non-constant weak prime fuzzy bi-ideal of a near-ring N, thenf_t is a weak prime bi-ideal of N.

Proof: Let f be a non-constant weak prime fuzzy bi-ideal of a near-ring N. Let B_1&B_2 be bi-ideals of N such that $\{0\} \neq B_1.B_2 \subseteq f_t$, Now, we define fuzzy subsets λ & σ as follows:

$$\lambda(x) = \begin{cases} 1 & if\ x \in B_1 \\ t & otherwise \end{cases} \sigma(x) = \begin{cases} 1 & if\ x \in B_2 \\ t & otherwise \end{cases}$$

By 4.9, λ & σ are fuzzy ideals of N, therefore λ & σ are fuzzy bi-ideals of N.

Clearly $f \leq \lambda$& $f \leq \sigma$

$$\lambda.\sigma(x) = \begin{cases} 1 & if x \in B_1.B_2 \\ t & if\ x = yz, y \notin B_1\ or\ z \notin B_2 \\ 0 & otherwise \end{cases}$$

Then $\{0\} \neq\ \lambda.\sigma \leq f$ which implies $\lambda = f\ or\ \sigma = f$. Therefore,$B_1 = f_t\ \ or\ \ B_2 = f_t$.

Hence, f_t is a weak prime bi-ideal of N.

Theorem 4.10:

Let f be a non-constant fuzzy bi-ideal of a near-ring N. Then, f is a weak prime fuzzy bi-ideal of N if and only if

(i) $Im\,f = \{1, t\}, t \in [0,1)$.

(ii) $f_t$is a weak prime bi-ideal of N.

Proof: Let f be a weak prime fuzzy bi-ideal of N. Then, (i)& (ii) follows from Lemma 4.8

Conversely, asume (i) & (ii).To prove: f is a weak prime fuzzy bi-ideal of N.

Suppose g & h be two fuzzy bi-ideals of N both containing f such that $g \circ h \leq f$ with $g \neq f$ & $h \neq f$. Then there exists $x, y \in N$ such that $g(x) = a > t = f(x)$&$h(y) = b > t = f(y)$.

Thus $x \in g_a$ but $x \notin f_t$ and $y \in h_b$but $y \notin f_t$. Clearly, g_a&h_b are bi-ideals of N.

Let $z \in f_t$. Since $f \leq g,\ f(z) = 1 \Longrightarrow g(z) = 1$.

Then, $g(z) \geq a$. Therefore $f_t \subseteq g_a$. Similarly, $f_t \subseteq h_b$.

If $g_a.h_b \subseteq f_t$, then $g_a = f_t\ or\ \ h_b = f_t$, which is a contradiction. Thus $g_a.h_b \nsubseteq f_t$.

Then $x_1 y_1 \notin f_t$, for some $x_1 \in g_a$& for some $y_1 \in h_b$. Thus $f(x_1 y_1) = t$.

$$\begin{aligned} g.h(x_1 y_1) &= \sup \min\{g(x_1), h(x_1)\} \\ &\geq \min\{g(x_1), h(x_1)\} \geq \min\{a, b\} \\ &> t = f(x_1 y_1) \end{aligned}$$

Which is a contradiction to $g \circ h \leq f$.

Hence, f is a weak prime fuzzy bi-ideal of N.

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