

INTERVAL VALUED FUZZY IDEALS IN NEAR-SUBTRACTION SEMIGROUPS

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Abstract: The analysis of Interval valued Fuzzy Ideals in Near-Subtraction semigroup is the primary focus of our research. In this paper, we introduce the concept of interval valued fuzzy Ideals in Near Subtraction Semigroups which is a generalized concept of Fuzzy Ideals in Near Subtraction Semigroup. We also characterize some properties of Interval valued Fuzzy Ideals in Near- Subtraction Semigroups.

Keywords: Fuzzy Ideals, Interval valued Fuzzy Ideals, Subtraction Algebra, Near- Subtraction Semigroup.

1. Introduction

The concepts of fuzzy sets and fuzzy subsets, fuzzy logic finds roots in seminal work of L. A. Zadeh [11] in 1965. In this work, he introduced “Interval valued fuzzy subsets” which had membership functions of closed intervals numbers rather than single members. Fuzzy Logic and fuzzification is a transformative development in set theory, having bearing in many latest scientific applications. Ideal of subtraction semigroup is thoroughly examined by K. H. Kim et al [7]. Interval valued fuzzy ideals of near-ring researched and characterized by V. Thillaigovidan and Chinnadurai [10].

Our present study is inspired by the above study and we have examined the concept of Interval valued fuzzy ideals of near- subtraction semigroups and its characteristics.

2. Preliminaries

Definition 2.1:

Let X be a non-empty set together with a binary operation $' - '$ is said to be subtraction algebra if it satisfies the following conditions

- (i) $x - (y - x) = x$.
- (ii) $x - (x - y) = y - (y - x)$
- (iii) $(x - y) - z = (x - z) - y$ for all x, y and $z \in X$.

Definition 2.2:

Let X be a non-empty set together with the binary operation $' - '$ and $' \cdot '$ is said to be a right near-subtraction semigroup if it satisfies the following conditions

- (i) $(X, -)$ is a subtraction algebra
- (ii) $(X, \cdot)$ is a semigroup
- (iv) $(x - y)z = xz - yz$ for all $x, y, z \in X$.

It is clear that $0 \cdot x = 0x = 0$ for all $x \in X$. Similarly we also define for left near-subtraction semigroup. Throughout this paper, we define a near- subtraction semi group refers to right near-subtraction semigroup, unless we mentioned otherwise.

Definition 2.3:

A near- subtraction semigroup X is said to be zero-symmetric if $x0 = 0$ for all $x \in X$.

Definition 2.4:

Let S be a non-empty subset of a subtraction algebra X is said to be a sub algebra of X , if $x - y \in S$.

Definition 2.5:

Let X be a near- subtraction semigroup. A non-empty subset I of X is called

- (1) A left ideal if I is a subalgebra of $(X, -)$ and $xi - x(y - i) \in I$ for all $x, y \in X$ and $i \in I$.
- (2) A right ideal if I is a subalgebra of $(X, -)$ and $iX \subseteq I$.
- (3) An ideal if I is both a left and right ideal.

Remark 2.6:

- (i) Suppose, Let X be a subtraction semigroup and I is a left ideal of X , then for $i \in I$ and $x, y \in X$, We have $xi - x(y - i) = xi - (xy - xi) = xi \in I$ by property (1) of subtraction algebra. Thus we have, $XI \subseteq I$.
- (ii) Now, if X is a zero-symmetric near-subtraction semigroup, Then we have $xi - x(0 - i) = xi - 0 = xi \in X$ for all $i \in I$ and $x \in X$.

Definition 2.7:

Define a mapping $\mu: X \rightarrow [0, 1]$ is called fuzzy set of X and its complement is denoted by μ' given by $\mu'(x) = 1 - \mu(x)$ for all $x \in X$.

Definition 2.8:

An interval valued number $\bar{a}$ on $[0, 1]$ is a closed subinterval of $[0, 1]$, i.e, $\bar{a} = [a^-, a^+]$ such that $0 \leq a^- \leq a^+ \leq 1$, where a^- and a^+ are the lower and upper limits of $\bar{a}$ respectively. In this notation, $\bar{0} = [0, 0]$ and $\bar{1} = [1, 1]$. For, any two interval number $\bar{a} = [a^-, a^+]$ and $\bar{b} = [b^-, b^+]$ on $[0, 1]$. Define

- (1) $\bar{a} \leq \bar{b} \Leftrightarrow a^- \leq b^-$ and $a^+ \leq b^+$

$$(2) \bar{a} = \bar{b} \Leftrightarrow a^- = b^- \text{ and } a^+ = b^+$$

Definition 2.9:

Let X be a non-empty set. Define a mapping $\mu: X \rightarrow D[0,1]$ is called an interval valued fuzzy subset (briefly i-v fuzzy subset) of X , where $D[0,1]$ denotes the family of all closed subintervals of $[0,1]$ and $\bar{\mu}(x) = [\mu^-(x), \mu^+(x)]$, μ^- and μ^+ are fuzzy subsets of X such that $\mu^-(x) \leq \mu^+(x)$ for all $x \in X$. Thus $\bar{\mu}(x)$ is an interval (a closed subset of $[0,1]$) as in the case of a fuzzy set.

Definition 2.10:

A mappings $\min^i: D[0,1] \times D[0,1] \rightarrow D[0,1]$ defined by $\min^i(\bar{a}, \bar{b}) = [\min(a^-, b^-), \min(a^+, b^+)]$ for all $\bar{a}, \bar{b} \in D[0,1]$ is called an interval min-norm. A mapping $\max^i: D[0,1] \times D[0,1] \rightarrow D[0,1]$ defined by $\max^i(\bar{a}, \bar{b}) = [\max(a^-, b^-), \max(a^+, b^+)]$ for all $\bar{a}, \bar{b} \in D[0,1]$ is called an interval max-norm. Let $\min^i$ and $\max^i$ be the interval min-norm and interval max-norm on $D[0,1]$ respectively. Then the following are true.

- (1) $\min^i\{\bar{a}, \bar{a}\} = \bar{a}$ and $\max^i\{\bar{a}, \bar{a}\} = \bar{a}$ for all $\bar{a} \in D[0,1]$.
- (2) $\min^i\{\bar{a}, \bar{b}\} = \min^i\{\bar{b}, \bar{a}\}$ and $\max^i\{\bar{a}, \bar{b}\} = \max^i\{\bar{b}, \bar{a}\}$ for all $\bar{a}, \bar{b} \in D[0,1]$.
- (3) If $\bar{a} \geq \bar{b} \in D[0,1]$, then $\min^i\{\bar{a}, \bar{c}\} \geq \min^i\{\bar{b}, \bar{c}\}$ and $\max^i\{\bar{a}, \bar{c}\} \geq \max^i\{\bar{b}, \bar{c}\}$ for all $\bar{c} \in D[0,1]$.

Definition 2.11:

If $\bar{\mu}$ is an i-v fuzzy subset of a set and $[t_1, t_2] \in D[0,1]$. Then the set $\bar{U}(\bar{\mu}; [t_1, t_2]) = \{x \in X / \bar{\mu}(x) \geq [t_1, t_2]\}$, is called the upper level set of $\bar{\mu}$.

Definition 2.12:

Let $\bar{\mu}, \bar{v}, \bar{\mu}_i (i \in \Omega)$ be i-v fuzzy subsets of X . The following are defined by

- (1) $\bar{\mu} \leq \bar{v} \Leftrightarrow \bar{\mu}(x) \leq \bar{v}(x)$
- (2) $\bar{\mu} = \bar{v} \Leftrightarrow \bar{\mu}(x) = \bar{v}(x)$.
- (3) $(\bar{\mu} \cup \bar{v})(x) = \max^i\{\bar{\mu}(x), \bar{v}(x)\}$.
- (4) $(\bar{\mu} \cap \bar{v})(x) = \min^i\{\bar{\mu}(x), \bar{v}(x)\}$
- (5) $\bigcup_{i \in \Omega} \bar{\mu}_i(x) = \sup^i\{\bar{\mu}_i(x) / i \in \Omega\}$
- (6) $\bigcap_{i \in \Omega} \bar{\mu}_i(x) = \inf^i\{\bar{\mu}_i(x) / i \in \Omega\}$.

where $\inf^i\{\bar{\mu}_i(x) / i \in \Omega\} = [\inf\{\mu_i^-(x)\}, \inf\{\mu_i^+(x)\}]$ is the i-v infimum norm and $\sup^i\{\bar{\mu}_i(x) / i \in \Omega\} = [\sup\{\mu_i^-(x)\}, \sup\{\mu_i^+(x)\}]$ is the i-v supremum norm

3. Interval valued fuzzy ideals of `Near-Subtraction semigroups

Here, we introduce the notion of interval valued fuzzy ideal of X . In this Session, X denote a near- subtraction semigroup.

Definition 3.1:

An interval valued fuzzy set $\bar{\mu}$ in X is called i-v fuzzy ideal of X if it satisfies the following conditions

- (1) $\bar{\mu}(x - y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\}$ for all $x, y \in X$
- (2) $\bar{\mu}(ax - a(b - x)) \geq \bar{\mu}(x)$ for all $a, b, x \in X$
- (3) $\bar{\mu}(xy) \geq \bar{\mu}(x)$ for all $x, y \in X$

Note, $\bar{\mu}$ is an interval valued fuzzy left ideal of X if it satisfies the condition (1) and (2) and $\bar{\mu}$ is a interval valued fuzzy right ideal of X if it satisfies the conditions (1) and (3).

Example 3.2: Let $X = \{0, a, b\}$ in which $' - '$ and $' \cdot '$ are defined by

-	0	a	b
0	0	0	0
a	a	0	a
b	b	b	0

.	0	a	b
0	0	0	0
a	0	a	0
b	a	0	b

Then $(X, -, \cdot)$ is a near- subtraction semigroup. Let $\bar{\mu}$ be a interval valued fuzzy set on X defined by, $\bar{\mu}(0) = 0.8$, $\bar{\mu}(a) = 0.5$ and $\bar{\mu}(b) = 0.3$. Then by the calculation, it can be easy to prove that, $\bar{\mu}$ is an interval valued fuzzy ideal of X .

Theorem 3.3:

If $\{\bar{\mu}_i / i \in \Omega\}$ is a family of i-v fuzzy ideals of a near-subtraction semigroup of X , then $\bigcap_{i \in \Omega} \bar{\mu}_i$ is also a i-v fuzzy ideal of X , where Ω is any index set.

Proof: Let $x, y, a, b \in X$. Then

$$\begin{aligned} \bigcap_{i \in \Omega} \bar{\mu}_i(x - y) &= \inf^i \{\bar{\mu}_i(x - y) : i \in \Omega\} \\ &\geq \inf^i \{ \min^i \{\bar{\mu}_i(x), \bar{\mu}_i(y)\} : i \in \Omega \} = \min^i \{ \{ \inf^i \bar{\mu}_i(x) : i \in \Omega \}, \{ \inf^i \bar{\mu}_i(y) : i \in \Omega \} \} \\ &= \min^i \{ \bigcap_{i \in \Omega} \bar{\mu}_i(x), \bigcap_{i \in \Omega} \bar{\mu}_i(y) \}. \\ \bigcap_{i \in \Omega} \bar{\mu}_i(xy) &= \inf^i \{\bar{\mu}_i(xy) : i \in \Omega\} \geq \inf^i \{ \min^i \bar{\mu}_i(x) : i \in \Omega \} = \{ \bigcap_{i \in \Omega} \bar{\mu}_i(x) \} \\ \bigcap_{i \in \Omega} \bar{\mu}_i(ax - a(b - x)) &= \inf^i \{\bar{\mu}_i(ax - a(b - x)) : i \in \Omega\} \geq \inf^i \{\bar{\mu}_i(x) : i \in \Omega\} = \{ \bigcap_{i \in \Omega} \bar{\mu}_i(x) \} \end{aligned}$$

Therefore, $\bigcap_{i \in \Omega} \bar{\mu}_i$ is an i-v fuzzy ideal of X .

Theorem 3.4:

Suppose $\bar{\mu}$ is an i-v fuzzy subset of X . $\bar{\mu} = [\mu^-, \mu^+]$ is an i-v fuzzy left ideal of X if and only if μ^+, μ^- are fuzzy left ideals of X .

Proof: Now, Let us assume that $\bar{\mu}$ is an i-v fuzzy left ideal of X . For any $x, y \in X$. We have

$$[\mu^-(x - y), \mu^+(x - y)] = \bar{\mu}(x - y) \geq \min^i \{\bar{\mu}(x), \bar{\mu}(y)\} = \min^i \{ [\mu^-(x), \mu^+(x)], [\mu^-(y), \mu^+(y)] \} = [\min \{\mu^-(x), \mu^-(y)\}, \min \{\mu^+(x), \mu^+(y)\}].$$

It follows that $\mu^-(x - y) \geq \min^i \{\mu^-(x), \mu^-(y)\}$ and $\mu^+(x - y) \geq \min \{\mu^+(x), \mu^+(y)\}$. Also, $[\mu^-(ax - a(b - x)), \mu^+(ax - a(b - x))] = \bar{\mu}(ax - a(b - x)) \geq \bar{\mu}(x) = [\mu^-(x), \mu^+(x)]$. It follows that $\mu^-(ax - a(b - x)) \geq \mu^-(x)$ and $\mu^+(ax - a(b - x)) \geq \mu^+(x)$.

Conversely, assume that μ^-, μ^+ are fuzzy left ideal of X .

Let $x, y, a, b \in X$, then:

$$\begin{aligned} \bar{\mu}(x - y) &= [\mu^-(x - y), \mu^+(x - y)] \geq [\min \{\mu^-(x), \mu^-(y)\}, \min \{\mu^+(x), \mu^+(y)\}] = \\ &= \min^i \{ [\mu^-(x), \mu^+(x)], [\mu^-(y), \mu^+(y)] \} = \\ &= \min^i \{ \bar{\mu}(x), \bar{\mu}(y) \} \end{aligned}$$

$$\bar{\mu}(ax - a(b - x)) = [\mu^-(ax - a(b - x)), \mu^+(ax - a(b - x))] = \bar{\mu}(x) \geq \bar{\mu}(x).$$

Hence, $\bar{\mu}$ is an i-v fuzzy left ideal of X .

Theorem 3.5:

$\bar{\mu}$ is an i-v fuzzy subset of X . $\bar{\mu} = [\mu^-, \mu^+]$ is an i-v fuzzy right ideal of X if and only if μ^+, μ^- are fuzzy right ideals of X .

Proof: Let us, assume that $\bar{\mu}$ is an i-v fuzzy right ideal of X . We have: $[\mu^-(x-y), \mu^+(x-y)] = \bar{\mu}(x-y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\} = \min\{[\mu^-(x), \mu^+(x)], [\mu^-(y), \mu^+(y)]\} = [\min\{\mu^-(x), \mu^-(y)\}, \min\{\mu^+(x), \mu^+(y)\}]$ for any $x, y \in X$. From that, it follows that $\mu^-(x-y) \geq \min\{\mu^-(x), \mu^-(y)\}$ and $\mu^+(x-y) \geq \min\{\mu^+(x), \mu^+(y)\}$. It follows that $\mu^-(xy) \geq \min\{\mu^-(x), \mu^-(y)\}$ and $\mu^+(xy) \geq \min\{\mu^+(x), \mu^+(y)\}$.

Also, $[\mu^-(xy), \mu^+(xy)] = \bar{\mu}(xy) \geq \bar{\mu}(x) = [\mu^-(x), \mu^+(x)]$. It follows that $\mu^-(xy) \geq \mu^-(x)$ and $\mu^+(xy) \geq \mu^+(x)$.

Conversely, assume that μ^-, μ^+ are fuzzy right ideal of X . Let $x, y, a, b \in X$. Then:

$$\begin{aligned} \bar{\mu}(x-y) &= [\mu^-(x-y), \mu^+(x-y)] \geq \min\{[\mu^-(x), \mu^+(x)], [\mu^-(y), \mu^+(y)]\} = \\ &= \min\{\bar{\mu}(x), \bar{\mu}(y)\} \bar{\mu}(xy) = [\mu^-(xy), \mu^+(xy)] \geq [\min\{\mu^-(x), \mu^-(y)\}, \min\{\mu^+(x), \mu^+(y)\}] = \\ &= [\min\{\mu^-(x), \mu^+(x)\}, \min\{\mu^-(y), \mu^+(y)\}] = \min\{\bar{\mu}(x), \bar{\mu}(y)\}. \end{aligned}$$

Hence, $\bar{\mu}$ is an i-v fuzzy right ideal of X .

Theorem 3.6:

If $\bar{\mu}$ is an i-v fuzzy subset of subtraction semigroup X . $\bar{\mu} = [\mu^-, \mu^+]$ is an i-v fuzzy ideal of X if and only if μ^+, μ^- are fuzzy ideals of X .

Theorem 3.7:

If $\bar{\mu}$ is an i-v fuzzy subset of X . Also, $\bar{\mu}$ is an i-v fuzzy left ideal of X if and only if $\bar{U}(\bar{\mu}; [t_1, t_2])$ is a left ideal of X , for all $[t_1, t_2] \in D[0, 1]$.

Proof: Let us, assume that $\bar{\mu}$ is an i-v fuzzy left ideal of X . Let $[t_1, t_2] \in D[0, 1]$ such that $x, y \in \bar{U}(\bar{\mu}; [t_1, t_2])$. Let $\bar{\mu}(x-y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\} = \min\{[t_1, t_2], [t_1, t_2]\} = [t_1, t_2]$. Thus, $x-y \in \bar{U}(\bar{\mu}; [t_1, t_2])$. Hence $\bar{U}(\bar{\mu}; [t_1, t_2])$ is a subalgebra of X . Let $x \in \bar{U}(\bar{\mu}; [t_1, t_2])$ and $y \in X$. Then, $\bar{\mu}(ax - a(b-x)) \geq \bar{\mu}(x) = [t_1, t_2]$. Therefore, $(ax - a(b-x)) \in \bar{U}(\bar{\mu}; [t_1, t_2])$. Let $a, b \in \bar{U}(\bar{\mu}; [t_1, t_2])$ and $x, y \in X$. Then $\bar{\mu}(ax - a(b-x)) \geq \bar{\mu}(x) = [t_1, t_2]$. Therefore, $(ax - a(b-x)) \in \bar{U}(\bar{\mu}; [t_1, t_2])$. Hence $\bar{U}(\bar{\mu}; [t_1, t_2])$ is a left ideal of X . Conversely, assume that $\bar{U}(\bar{\mu}; [t_1, t_2])$ is a left ideal of X for all $[t_1, t_2] \in D[0, 1]$. Let $x, y \in X$. Suppose $\bar{\mu}(x-y) < \min\{\bar{\mu}(x), \bar{\mu}(y)\}$. Choose an interval $\bar{a} = [a_1, a_2] \in D[0, 1]$ such that $\bar{\mu}(x-y) < [a_1, a_2] < \min\{\bar{\mu}(x), \bar{\mu}(y)\}$. This implies $\bar{\mu}(x) > [a_1, a_2]$ and $\bar{\mu}(y) > [a_1, a_2]$. Then we have $x, y \in \bar{U}(\bar{\mu}; [a_1, a_2])$ and since $\bar{U}(\bar{\mu}; [a_1, a_2])$ is a left ideal of X , $x-y \in \bar{U}(\bar{\mu}; [a_1, a_2])$. Hence $\bar{\mu}(x-y) \geq [a_1, a_2]$, a contradiction. Then, $\bar{\mu}(x-y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\}$. Suppose $\bar{\mu}(ax - a(b-x)) < \bar{\mu}(x)$. Choose an interval $\bar{a} = [a_1, a_2] \in D[0, 1]$ such that $\bar{\mu}(ax - a(b-x)) < [a_1, a_2] < \bar{\mu}(x)$. This implies that $\bar{\mu}(x) > [a_1, a_2]$. Then we have $x \in \bar{U}(\bar{\mu}; [a_1, a_2])$ and $ax - a(b-x) \in \bar{U}(\bar{\mu}; [a_1, a_2])$. Hence, $\bar{\mu}(ax - a(b-x)) \geq [a_1, a_2]$, a contradiction. Thus $\bar{\mu}(ax - a(b-x)) \geq \bar{\mu}(x)$. Therefore $\bar{\mu}$ is an i-v fuzzy left ideal of X .

Theorem 3.8:

If $\bar{\mu}$ is an i-v fuzzy subset of X . $\bar{\mu}$ is an i-v fuzzy right ideal of X if and only if $\bar{U}(\bar{\mu}; [t_1, t_2])$ is a right ideal of X , for all $[t_1, t_2] \in D[0, 1]$.

Proof: First we, assume that $\bar{\mu}$ is an i-v fuzzy right ideal of X . Let $[t_1, t_2] \in D[0, 1]$ such that $x, y \in \bar{U}(\bar{\mu}; [t_1, t_2])$. Let $\bar{\mu}(x-y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\} = \min\{[t_1, t_2], [t_1, t_2]\} = [t_1, t_2]$. Thus, we have $x-y \in \bar{U}(\bar{\mu}; [t_1, t_2])$. Hence $\bar{U}(\bar{\mu}; [t_1, t_2])$ is a subalgebra of X . Let $y \in \bar{U}(\bar{\mu}; [t_1, t_2])$ and $x \in X$. Then, $\bar{\mu}(xy) \geq \bar{\mu}(x) = [t_1, t_2]$. Therefore, $xy \in \bar{U}(\bar{\mu}; [t_1, t_2])$. Hence $\bar{U}(\bar{\mu}; [t_1, t_2])$ is a right ideal of X . Conversely, we assume that $\bar{U}(\bar{\mu}; [t_1, t_2])$ is a right ideal of X for all $[t_1, t_2] \in D[0, 1]$. Let $x, y \in X$. Suppose that $\bar{\mu}(x-y) < \min\{\bar{\mu}(x), \bar{\mu}(y)\}$. Now, Choose an interval $\bar{a} = [a_1, a_2] \in D[0, 1]$ such that $\bar{\mu}(x-y) < [a_1, a_2] < \min\{\bar{\mu}(x), \bar{\mu}(y)\}$. This implies that $\bar{\mu}(x) > [a_1, a_2]$ and $\bar{\mu}(y) > [a_1, a_2]$.

Then, $x, y \in \bar{U}(\bar{\mu}: [a_1, a_2])$ and since $\bar{U}(\bar{\mu}: [a_1, a_2])$ is a right ideal of X , $x - y \in \bar{U}(\bar{\mu}: [a_1, a_2])$. Hence, We have $\bar{\mu}(x - y) \geq [a_1, a_2]$, a contradiction. Then, $\bar{\mu}(x - y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\}$. Suppose that $\bar{\mu}(xy) < \bar{\mu}(x)$. Choose an interval $\bar{a} = [a_1, a_2] \in D[0, 1]$ such that $\bar{\mu}(xy) < [a_1, a_2] < \bar{\mu}(x)$. This implies that $\bar{\mu}(x) > [a_1, a_2]$. Then $y \in \bar{U}(\bar{\mu}: [a_1, a_2])$ and since $\bar{U}(\bar{\mu}: [a_1, a_2])$ is a right ideal of X , $xy \in \bar{U}(\bar{\mu}: [a_1, a_2])$. Hence $\bar{\mu}(xy) \geq [a_1, a_2]$, a contradiction. Thus, $\bar{\mu}(xy) \geq \bar{\mu}(x)$. Therefore $\bar{\mu}$ is an i-v fuzzy right ideal of X .

Theorem 3.9:

Suppose $\bar{\mu}$ is an i-v fuzzy subset of X . Let $\bar{\mu}$ be an i-v fuzzy ideal of X if and only if $\bar{U}(\bar{\mu}: [t_1, t_2])$ is an ideal of X , for all $[t_1, t_2] \in D[0, 1]$.

Theorem 3.10:

If $\bar{\mu}$ is an i-v fuzzy left ideal of X . Then the set $X_{\bar{\mu}} = \{x \in X \mid \bar{\mu}(x) = \bar{\mu}(0)\}$ is a left ideal of X .

Proof: Let $x, y \in X_{\bar{\mu}}$. Suppose that, $\bar{\mu}$ is a i-v fuzzy left ideal of X . Then, $\bar{\mu}(x - y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\} = \bar{\mu}(0)$. Thus, we get $x - y \in X_{\bar{\mu}}$. For all $a, b \in X$ and $x \in X_{\bar{\mu}}$, we have $\bar{\mu}(ax - a(b - x)) \geq \bar{\mu}(x) = \bar{\mu}(0)$. Thus, $ax - a(b - x) \in X_{\bar{\mu}}$. Hence, $X_{\bar{\mu}}$ is a left ideal of X .

Theorem 3.11:

Let $\bar{\mu}$ be an i-v fuzzy right ideal of X . Then the set $X_{\bar{\mu}} = \{x \in X \mid \bar{\mu}(x) = \bar{\mu}(0)\}$ is a right ideal of X .

Theorem 3.12:

Let $\bar{\mu}$ be an i-v fuzzy ideal of X . Then the set $X_{\bar{\mu}} = \{x \in X \mid \bar{\mu}(x) = \bar{\mu}(0)\}$ is an ideal of X .

References

- [1] J. C. Abbott, *Sets, Lattices and Boolean Algebras*, Allyn and Bacon, Boston, 1969.
- [2] P. Dheena and G. Satheeskumar, *On strongly regular near-subtraction semigroups*, Common Korean Math. Soc. 22 (3) (2007) 33-330.
- [3] Y. B. Jun, Kyung Ja Lee and Asghar Khan, *Ideal Theory of Subtraction algebras*, Sci. Math. Jpn 61(2005) 459-464.
- [4] K. J. Lee and C. H. Park, *Some questions on fuzzifications of ideals in subtraction algebras*, Commun. Korean Math. Soc. 22(2007), pp. 35-363.
- [5] G. Pilz, *Near-rings*, North Holland, Amsterdam, 1983.
- [6] D. R. Prince Williams, *Fuzzy ideals in near-subtraction semigroups*, International Scholarly and Scientific Research & Innovation 2 (7) (2008) 625-632.
- [7] E. H. Roh, K. H. Kim, and Jong Geol Lee, *On Prime and semiprime ideals in subtraction semigroups*, Scientiae Mathematicae Japonicae 61(2005), no. 2, 259-266.
- [8] B. M. Schein, *Difference Semigroups*, Communications in algebras 20(1992), 2153-2169.
- [9] N. Thillaigovidan and V. Chinnadurai, *On Interval valued fuzzy quasi-ideals of semigroups*, East Asian Mathematics Journals, 25(2009), no. 4, 49-467.
- [10] N. Thillaigovidan and V. Chinnadurai, *Interval valued fuzzy ideals of near-rings*, J. Fuzzy Math. 23(2)(2015) 471-484.
- [11] L. A. Zadeh, *Fuzzy sets*, Inform and control, 8(1965), 338-353.
- [12] B. Zelinka, *Subtraction semigroups*, Math. Bohemica 120 (1995), 445-447.