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DETOUR DOMINATION NUMBER OF CORONA PRODUCT OF GRAPHS

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Abstract. Let (G, D) be a graph. For any two vertices$u$ and v the detour distance is a longest $u - v$ path. A subset $D \subseteq V$ is called a detour set of G if every vertex in $V - D$ lie in a detour joining the vertices of D. A subset $D \subseteq V$ which is both a detour set and dominating set is called a detour dominating set of G and the cardinality of a minimum detour dominating set is called the detour domination number of G. This paper evaluates the detour domination number of Corona product of some standard graphs.

Keywords: Domination, Detour Domination, Corona Product.

AMS Subject Classification: 05C78.

1. Introduction

We consider finite graphs without loops and multiple edges. For any graph$\boldsymbol{G}$, the set of vertices is denoted by $\boldsymbol{V}(\boldsymbol{G})$ and the edge set by $\boldsymbol{E}(\boldsymbol{G})$. The order and size of $\boldsymbol{G}$ are denoted by $\boldsymbol{p}$ and $\boldsymbol{q}$ respectively. We consider connected graphs with atleast two vertices. For basic definitions and terminologies, we refer [1,7]. For vertices$\boldsymbol{u}$ and $\boldsymbol{v}$ in a connected graph $\boldsymbol{G}$, the detour distance $\boldsymbol{D}(\boldsymbol{u}, \boldsymbol{v})$ is the length of a longest $\boldsymbol{u} - \boldsymbol{v}$ path in G. A $\boldsymbol{u} - \boldsymbol{v}$ path of length $\boldsymbol{D}(\boldsymbol{u}, \boldsymbol{v})$ is called a $\boldsymbol{u} - \boldsymbol{v}$ detour. These concepts were studied by Chartrand et al. [2,3]. A vertex $\boldsymbol{x}$ is said to lie on a $\boldsymbol{u} - \boldsymbol{v}$ detour $\boldsymbol{P}$ if $\boldsymbol{x}$ is a vertex of a $\boldsymbol{u} - \boldsymbol{v}$ detour path $\boldsymbol{P}$ including the verticles $\boldsymbol{u}$ and $\boldsymbol{v}$. A set $\boldsymbol{S} \subseteq \boldsymbol{V}$ is called a detour set if every vertex $\boldsymbol{v}$ in $\boldsymbol{G}$ lies on a detour joining a pair of vertices of $\boldsymbol{S}$. The detour number $\boldsymbol{dn}(\boldsymbol{G})$ is called a minimum order of a detour set and any detour set of order $\boldsymbol{dn}(\boldsymbol{G})$ is called a minimum detour set of $\boldsymbol{G}$. These concepts were studied by Chartrand [4].A set $\boldsymbol{S} \subseteq \boldsymbol{V}\ (\boldsymbol{G})$is called a dominating set of $\boldsymbol{G}$ if every vertex in $\boldsymbol{V}(\boldsymbol{G}) - \boldsymbol{S}$ is adjacent to some vertex in S. The domination number $\boldsymbol{\gamma}(\boldsymbol{G})$ of $\boldsymbol{G}$ is the minimum order of its dominating sets and any dominating set of order $\boldsymbol{\gamma}(\boldsymbol{G})$ is called a$\boldsymbol{\gamma}$-set of $\boldsymbol{G}$. A detour domianating set is a subset $\boldsymbol{S}$ of $\boldsymbol{V}(\boldsymbol{G})$ which is both a dominating and a detour set of $\boldsymbol{G}$. A detour dominating set is said to be minimal detour dominating set of $\boldsymbol{G}$ if no proper subset of $\boldsymbol{S}$ is a detour dominating set of $\boldsymbol{G}$. A detour dominating set $\boldsymbol{S}$ is said to be minimum detour dominating set of $\boldsymbol{G}$ if there exists no detour dominating set $\boldsymbol{S}'$ such that $|\boldsymbol{S}'| < |\boldsymbol{S}|$. The smallest cardinality of a detour dominating set of $\boldsymbol{G}$ is called the detour domination number of $\boldsymbol{G}$. It is denoted by $\boldsymbol{\gamma_d}(\boldsymbol{G})$. Any detour dominating set S of G of cardinality $\gamma_d(G)$ is called a (γ, d)-set of G.If G_1 and G_2 are graphs and G_1 has n vertices then the corona of G_1 and G_2 denoted by$G_1 \circ G_2$, is the graph obtained by taking one copy of G_1 with an edge to every vertex in the i^{th}copy of $G_2$and $G_1 \circ G_2$has$n_1(1 + n_2)$ vertices and $m_1 + n_1m_2 + n_1n_2$ edges.These concepts were studied by R. Frucht and F. Harary [5]. In this paper, we investigate the detour domination number of Corona product of some standard graphs.

1.1. ***Theorem***[7]For the path $G = P_p$ $(p \geq 2)$, $\gamma(G) = \left\lceil \frac{p}{2} \right\rceil$.

1.2. ***Theorem:[6]****Every end vertex of G belongs to every detour dominating set of G.*

1.3. ***Observation:[6]****If the set of all end vertices forms a detour dominating set of G, then S is the unique minimum detour dominating set of G.*

2. Detour domination number of Corona product of graphs

2.1 Theorem: For $n \geq 2$,$\gamma_d(P_n \circ K_1) = n$.

Proof.Let$G = P_n \circ K_1$

Let $V(P_n) = \{v_1, v_2, \dots, v_n\}$ and let u_i be the vertices of the i^{th}copy ofK_1 attached to v_i.

Then $V(G) = \{v_i, u_i / i = 1 \; to \; n\}$.

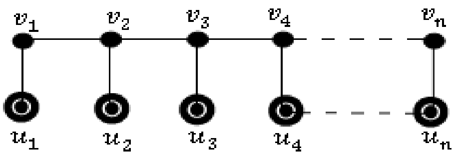

Figure 2.1

$S = \{u_1, u_2, u_3, \dots, u_{n-1}, u_n\}$, being the set of all end verticesis a subset of every detour dominating set of G. Further, S detour dominates all the vertices of$G$. Hence,by 1.3, $S$is theunique minimum detour dominating set of G. Therefore, $\gamma_d(G) = |S| = n$.

2.2 Theorem: For $n \geq 2$,$\gamma_d(P_n \circ K_2) = n$.

Proof.Let $G = P_n \circ K_2$.

Let $V(P_n) = \{v_1, v_2, \dots, v_n\}$ and let $\{u_{i1}, u_{i2}\}$ be the vertex set of the i^{th} copy of K_2 attached to v_i

Then $V(G) = \{v_1, v_2, \dots, v_n, u_{11}, u_{12}, u_{21}, u_{22}, \dots \dots, u_{n1}, u_{n2}\}$

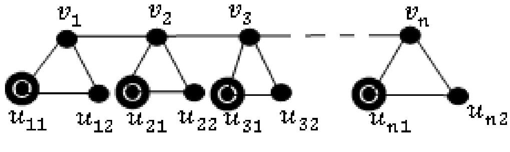

Figure 2.2

$S_1 = \{u_{11}, u_{21}, u_{31}, \dots, u_{(n-1)1}, u_{n1}\}$ and$S_2 = \{u_{12}, u_{22}, u_{32}, \dots, u_{(n-1)2}, u_{n2}\}$ are some detour dominating set of G. Therefore,$\gamma_d(G) = n$.

The above two theorems lead to the generalized result as below.

2.3 Theorem:In general for$n \geq 2$,$\gamma_d(P_n \circ K_m) = n$.

Proof. Let $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ and let $\{u_{i1}, u_{i2}, \ldots \ldots, u_{im}\}$ be the vertex set of i^{th} copy of K_m adjoint to v_i.

$$\therefore V(P_n \circ K_m) = \{v_1, v_2, \ldots, v_n, u_{11}, u_{12}, \ldots, u_{1m}, u_{21}, u_{22}, \ldots, u_{2m}, \ldots \ldots, u_{n1}, u_{n2}, \ldots \ldots, u_{nm}\}$$

Obviously, for $j = 1\ to m$, $S_j = \{u_{1j}, u_{2j}, u_{3j}, \ldots, u_{(n-1)j}, u_{nj}\}$ are some detour sets of $P_n \circ K_m$.

Also, they dominate all the vertices. Further, no set of less than $|S_j| = n$ vertices is a detour dominating set. Hence, each S_j is a minimum detour dominating set of$P_n \circ K_m$.

Hence, $\gamma_d(P_n \circ K_m) = |S_j| = n$.

2.4 Illustration:For$n \geq 2$,$\gamma_d(P_n \circ K_6) = n$.

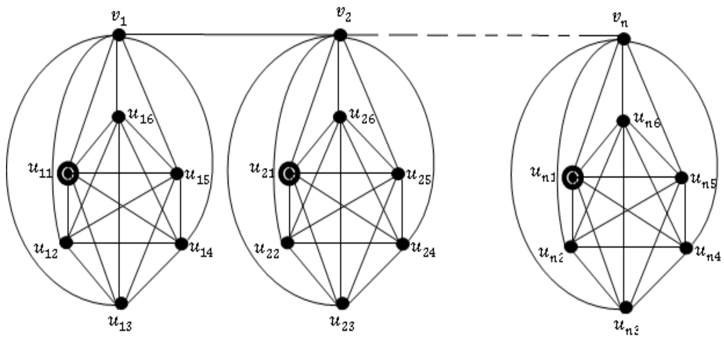

Figure 2.3

Here, $S = \{u_{11}, u_{21}, u_{31}, \ldots, u_{(n-1)1}, u_{n1}\}$ is a minimum detour dominating set.

Hence, $\gamma_d(P_n \circ K_6) = n$.

2.5 Theorem: For $n \geq 3$,$\gamma_d(C_n \circ K_1) = n$.

Proof.Let $G = C_n \circ K_1$.

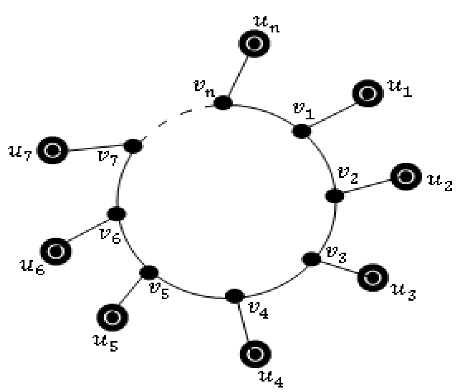

Figure 2.4

Let $\boldsymbol{V(C_n) = \{v_1, v_2, \dots, v_n\}}$ and let $\boldsymbol{u_i}$ be the vertices of the $\boldsymbol{i}^{\text{th}}$ copy of $\boldsymbol{K_1}$ attached to $\boldsymbol{v_i}$.

Then$\boldsymbol{V(G) = \{v_i, u_i / i = 1 \ to \ n\}}$.

$\boldsymbol{S = \{u_1, u_2, u_3, \dots, u_{n-1}, u_n\}}$, being the set of all end vertices is a subset of every detour dominating set of $\boldsymbol{G}$. Further, $\boldsymbol{S}$ detour dominates all the vertices of $\boldsymbol{G}$. Hence, by 1.3, $\boldsymbol{S}$ is the unique minimum detour dominating set of $\boldsymbol{G}$. Therefore, $\boldsymbol{\gamma_d(G) = |S| = n}$.

2.6 Theorem: For $\boldsymbol{n \geq 3, \gamma_d(C_n \circ K_2) = n}$.

Proof.Let $\boldsymbol{G = C_n \circ K_2}$.

Let $\boldsymbol{V(C_n) = \{v_1, v_2, \dots, v_n\}}$ and let $\boldsymbol{\{u_{i1}, u_{i2}\}}$ be the vertex set of the $\boldsymbol{i}^{\text{th}}$ copy of $\boldsymbol{K_2}$ attached to $\boldsymbol{v_i}$

Then $\boldsymbol{V(G) = \{v_1, v_2, \dots, v_n, u_{11}, u_{12}, u_{21}, u_{22}, \dots \dots, u_{n1}, u_{n2}\}}$

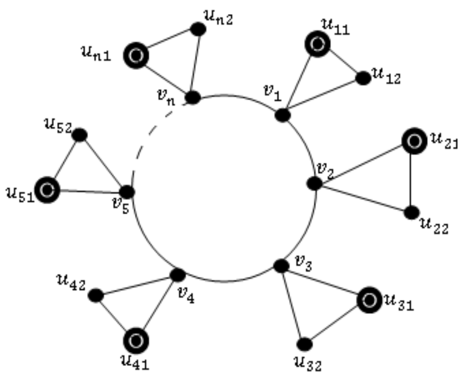

Figure2.5

$\boldsymbol{S_1 = \{u_{11}, u_{21}, u_{31}, \dots, u_{(n-1)1}, u_{n1}\}}$ and$\boldsymbol{S_2 = \{u_{12}, u_{22}, u_{32}, \dots, u_{(n-1)2}, u_{n2}\}}$ are some detour dominating set of $\boldsymbol{G}$. Therefore, $\boldsymbol{\gamma_d(G) = n}$.

The above two theorems lead to the generalized result as below.

2.7 Theorem: In general for $\boldsymbol{n \geq 3, \gamma_d(C_n \circ K_m) = n}$.

Proof. Let $\boldsymbol{V(C_n) = \{v_1, v_2, \dots, v_n\}}$ and let $\boldsymbol{\{u_{i1}, u_{i2}, \dots \dots, u_{im}\}}$ be the vertex set of $\boldsymbol{i}^{\text{th}}$ copy of $\boldsymbol{K_m}$ adjoint to $\boldsymbol{v_i}$.

$\therefore$V$(\boldsymbol{C_n \circ K_m})$ =$\boldsymbol{\{v_1, v_2, \dots, v_n, u_{11}, u_{12}, \dots, u_{1m}, u_{21}, u_{22}, \dots, u_{2m}, \dots \dots, u_{n1}, u_{n2}, \dots \dots, u_{nm}\}}$

Obviously, for $\boldsymbol{j = 1 \ tom, S_j = \{u_{1j}, u_{2j}, u_{3j}, \dots, u_{(n-1)j}, u_{nj}\}}$ are some detour sets of $\boldsymbol{C_n \circ K_m}$.

Also, they dominate all the vertices. Further, no set of less than $\boldsymbol{|S_j| = n}$ vertices is a detour dominating set. Hence, each $\boldsymbol{S_j}$ is a minimum detour dominating set of$\boldsymbol{C_n \circ K_m}$.

Hence, $\boldsymbol{\gamma_d(C_n \circ K_m) = |S_j| = n}$.

2.8 Illustration:For$\boldsymbol{n \geq 3, \gamma_d(C_n \circ K_3) = n}$.

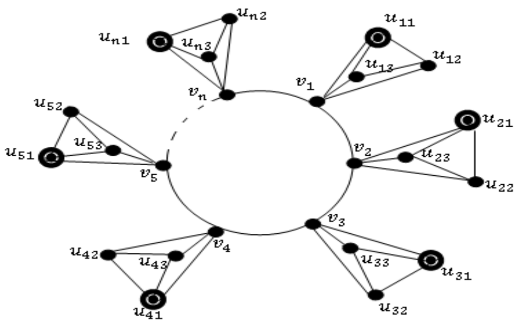

Figure 2.6

Here, $S = \{u_{11}, u_{21}, u_{31}, \ldots, u_{(n-1)1}, u_{n1}\}$ is a minimum detour dominating set.

Hence, $\gamma_d(C_n \circ K_3) = n$.

3.9 Theorem: For $n \geq 4, \gamma_d(W_n \circ K_1) = n$.

Proof.Let $G = W_n \circ K_1$.

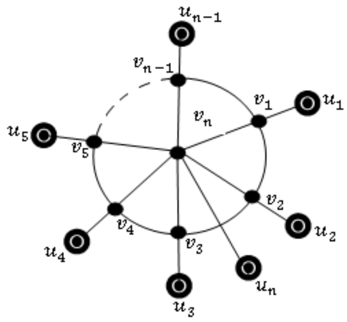

Figure 2.7

SinceW_n contains a central vertex attached to each vertex of a cycle of C_{n-1}.

Let $V(W_n) = \{v_1, v_2, \ldots, v_{n-1}, v_n\}$ with v_n as its central vertex and $v_1, v_2, \ldots, v_{n-1}$ as the verticesof the outer cycle and let u_i be the vertices of the i^{th} copy of K_1 attached to v_i.

Then $V(G) = \{v_i, u_i / i = 1 \text{ to } n\}$.

$S = \{u_1, u_2, u_3, \ldots, u_{n-1}, u_n\}$, being the set of all end vertices is a subset of every detour dominating set of G. Further, S detour dominates all the vertices of G. Hence, by 1.3, S is the unique minimum detour dominating set of G. Therefore, $\gamma_d(G) = |S| = n$.

2.10 Theorem: For $n \geq 4, \gamma_d(W_n \circ K_2) = n$.

Proof. SinceW_n contains a central vertex attached to each vertex of a cycle of C_{n-1}.

Let $G = W_n \circ K_2$.

Let $V(C_n) = \{v_1, v_2, \ldots, v_{n-1}, v_n\}$ with v_n as its central vertex and $v_1, v_2, \ldots, v_{n-1}$ as the vertices of the outer cycle and let $\{u_{i1}, u_{i2}\}$ be the vertex set of the i^{th} copy of K_2 attached to v_i.

Then $V(G) = \{v_1, v_2, \dots, v_n, u_{11}, u_{12}, u_{21}, u_{22}, \dots \dots, u_{n1}, u_{n2}\}$

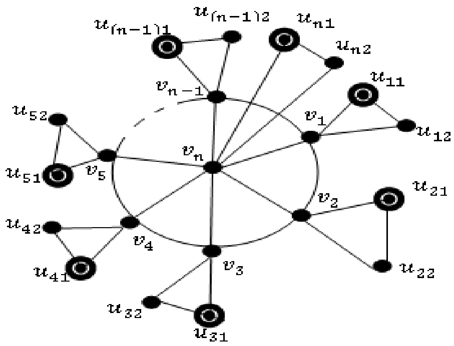

Figure 2.8

$S_1 = \{u_{11}, u_{21}, u_{31}, \dots, u_{(n-1)1}, u_{n1}\}$ and $S_2 = \{u_{12}, u_{22}, u_{32}, \dots, u_{(n-1)2}, u_{n2}\}$ are some detour dominating set of G. Therefore, $\gamma_d(G) = n$.

The above two theorems lead to the generalized result as below.

3.11 Theorem: For $n \geq 4, \gamma_d(W_n \circ K_m) = n$.

Proof: W_n contains a central vertex attached to each vertex of a cycle of C_{n-1}.

Let $V(W_n) = \{v_1, v_2, \dots, v_{n-1}, v_n\}$ with v_n as its central vertex and $v_1, v_2, \dots, v_{n-1}$ as the vertices of the outer cycle and let $\{u_{i1}, u_{i2}, \dots \dots, u_{im}\}$ be the vertex set of i^{th} copy of K_m attached to v_i.

$$\therefore V(W_n \circ K_m) = \{v_1, v_2, \dots, v_n, u_{11}, u_{12}, \dots, u_{1m}, u_{21}, u_{22}, \dots, u_{2m}, \dots \dots, u_{n1}, u_{n2}, \dots \dots, u_{nm}\}$$

The graph $W_n \circ K_m$ looks as in figure 2.9.

From the figure, it is clear that for $j = 1\ to m, S_j = \{u_{1j}, u_{2j}, u_{3j}, \dots, u_{(n-1)j}, u_{nj}\}$ forms a minimum detour dominating set of $W_n \circ K_m$.

Since no set of less than $|S_j| = n$ vertices forms a detour dominating set.

Hence, each S_j is a minimum detour dominating set of $W_n \circ K_m$.

Hence, $\gamma_d(W_n \circ K_m) = |S_j| = n$.

2.12 Illustration: For $n \geq 4, \gamma_d(W_n \circ K_3) = n$.

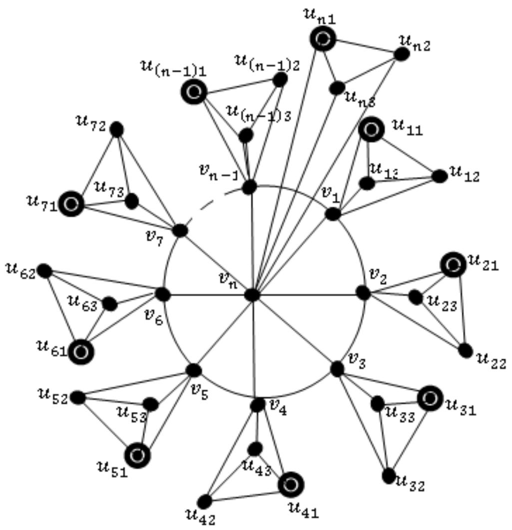

Figure2.9

Here, $\boldsymbol{S} = \{\boldsymbol{u_{11}}, \boldsymbol{u_{21}}, \boldsymbol{u_{31}}, \dots, \boldsymbol{u_{(n-1)1}}, \boldsymbol{u_{n1}}\}$ is a minimum detour dominating set.

Hence, $\boldsymbol{\gamma_d(W_n \circ K_3) = n}$.

2.13 Theorem:$\boldsymbol{\gamma_d\left(K_{1,n} \circ K_1\right) = n + 1}$.

Proof:Let$\boldsymbol{G = K_{1,n} \circ K_1}$.

Let $\boldsymbol{V(K_{1,n}) = \{v_1, v_2, \dots, v_{n-1}, v_n, v\}}$ with $\boldsymbol{v}$ as its root vertex and $\boldsymbol{v_1, v_2, \dots, v_{n-1}, v_n}$ be the set of end vertices and let $\boldsymbol{u_i}$ be the vertices of the $\boldsymbol{i}^{\text{th}}$ copy of $\boldsymbol{K_1}$ attached to $\boldsymbol{v_i}$ and $\boldsymbol{x}$ be the vertex of a copy of $\boldsymbol{K_1}$attached to the root vertex $\boldsymbol{v}$.

Then $\boldsymbol{V(G) = \{v_1, v_2, \dots, v_{n-1}, v_n, v\} \cup \{u_1, u_2, \dots, u_{n-1}, u_n, x\}}$

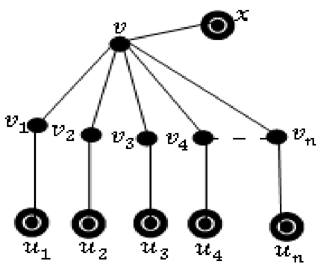

Figure2.10

$\boldsymbol{S = \{u_1, u_2, u_3, \dots, u_{n-1}, u_n, x\}}$, being the set of all end vertices is a subset of every detour dominating set of $\boldsymbol{G}$. Further, $\boldsymbol{S}$ detour dominates all the vertices of $\boldsymbol{G}$. Hence, by 1.3, $\boldsymbol{S}$ is the unique minimum detour dominating set of $\boldsymbol{G}$. Therefore, $\boldsymbol{\gamma_d(G) = |S| = n + 1}$.

2.14 Theorem:$\boldsymbol{\gamma_d\left(K_{1,n} \circ K_2\right) = n + 1}$

Proof.Let $\boldsymbol{G = K_{1,n} \circ K_2}$.

Let $\boldsymbol{V(K_{1,n}) = \{v_1, v_2, \dots, v_{n-1}, v_n, v\}}$ with $\boldsymbol{v}$ as its root vertex and $\boldsymbol{v_1, v_2, \dots, v_{n-1}, v_n}$ be the set of end vertices and let $\boldsymbol{\{u_{i1}, u_{i2}\}}$ be the vertex set of the $\boldsymbol{i}^{\text{th}}$ copy of $\boldsymbol{K_2}$ attached to $\boldsymbol{v_i}$.

and$\{x_1, x_2\}$ be the vertex set of a copy of K_2attached to the root vertex v.

Then $V(G) = \{v_1, v_2, \dots, v_{n-1}, v_n, v\} \cup \{u_{11}, u_{12}, u_{21}, u_{22}, \dots \dots, u_{n1}, u_{n2}, x_1, x_2\}$

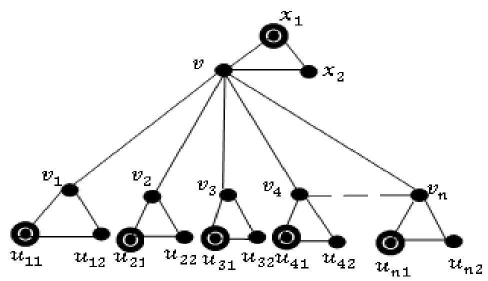

Figure 2.11

$S_1 = \{u_{11}, u_{21}, u_{31}, \dots, u_{(n-1)1}, u_{n1}, x_1\}$ and$S_2 = \{u_{12}, u_{22}, u_{32}, \dots, u_{(n-1)2}, u_{n2}, x_2\}$ are some detour dominating set of G. Therefore,$\gamma_d(G) = n + 1$.

The above two theorems lead to the generalized result as below.

2.15 Theorem:$\gamma_d(K_{1,n} \circ K_m) = n + 1$

Proof:Let $V(K_{1,n}) = \{v_1, v_2, \dots, v_{n-1}, v_n, v\}$ with v as its root vertex $K_{1,n}$ and let $\{v_1, v_2, \dots, v_{n-1}, v_n\}$ be the set of end vertices.

Assume that $\{u_{i1}, u_{i2}, \dots \dots, u_{im}\}$ be the vertex setof i^{th} copy of K_m attached to $v_i$and $\{x_1, x_2, \dots x_m\}$ be the vertex set of a copy of K_m attached to the root vertex v.

Then $V(K_{1,n} \circ K_m)$

$= \{v_1, v_2, \dots, v_n, v\} \cup \{u_{11}, u_{12}, \dots, u_{1m}, u_{21}, u_{22}, \dots, u_{2m}, \dots \dots, u_{n1}, u_{n2}, \dots \dots, u_{nm}, x_1, x_2, \dots, x_m\}$

The graph $K_{1,n} \circ K_m$ looks as in figure 2.12.

From the figure, it is clear that for $j = 1 \; to \; m$, $S_j = \{u_{1j}, u_{2j}, u_{3j}, \dots, u_{(n-1)j}, u_{nj}, x_j\}$ forms a minimum detour dominating set of $K_{1,n} \circ K_m$. Since no set of less than $|S_j| = n + 1$ vertices forms a detour dominating set. Hence, each S_j is a minimum detour dominating set of$K_{1,n} \circ K_m$.

Hence, $\gamma_d(K_{1,n} \circ K_m) = |S_j| = n + 1$.

2.16 Illustration:$\gamma_d(K_{1,n} \circ K_3) = n + 1$.

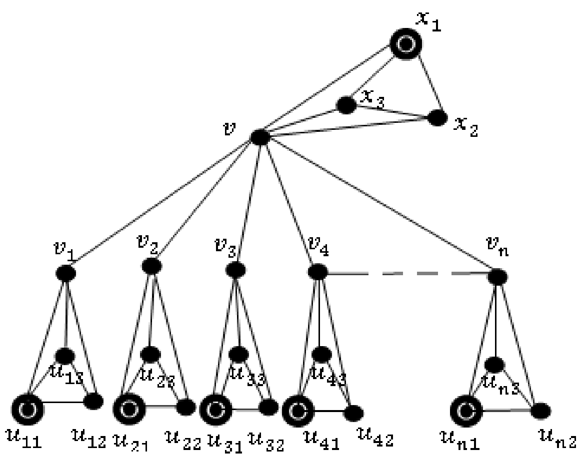

Figure 2.12

Here, $\boldsymbol{S = \{u_{11}, u_{21}, u_{31}, \ldots, u_{(n-1)1}, u_{n1}, x_1\}}$ is a minimum detour dominating set.

$\boldsymbol{\gamma_d(K_{1,n} \circ K_3) = n + 1}$.

References

[1] F.Buckley and F.Harary, Distance in Graphs, Addision-Weseely, Reading MA, (1990).

[2] G.Chartrand, H.Escuadro and P.Zhang, Distance in Graphs, Taking the Long View, AKCE J.Graphs and Combin.,1(1)(2004), 1-13.

[3] G.Chartrand, H.EScuadro and B.Zang, Detour distance in graph, J.Combin.mathcombin, compul 53 (2005) 75-94.

[4] G.Chartrand, L.Johns and P.Zang, Detour Number of graph, Utilitas Mathematics, 64(2003), 97-113.

[5] R. Frucht and F. Harary, On the corona of two graphs, Aequationes Math. 4 (1970), 322–325.

[6] J.John and N. AriyanayagamCommunicatedDiscrete Mathematics, Algorithms and Applications Vol. 9, No. 1 (2017) 1750.

[7] T.W.Haynes, S.T.Hedetniemi and P.J.Slater, Fundamentals of Domination in graphs, Marcel Decker,Inc., New York,(1998).