Advances in Mathematics: Scientific Journal 8 (2019), no.3, 208–213
(Special issue on ICRAPAM)
ISSN 1857-8365 printed version
ISSN 1857-8438 electronic version

STRONG BI-IDEALS OF Γ- NEAR SUBTRACTION SEMIGROUPS

P. Meenakshi [1], N. Meenakumari [2]

[(1,2)]PG & Research Department of Mathematics

A. P. C. Mahalaxmi College for Women, Thoothukudi,Tamil Nadu, India.

Email:[1] meena.bushy@gmail.com[2] meenakumarin123@gmail.com

Abstract: Using the concept of strong bi-ideals in near- subtraction semigroups, we introduce the notion of strong bi-ideals in Γ-near- subtraction semigroups. We show that the arbitrary intersection of strong bi-ideals of a Γ-near subtraction semigroup is again a strong bi-ideals. Also we proved that the intersection of strong bi-ideals of Γ-near subtraction semigroup and sub-Γ-near subtraction semigroup is again a strong bi-ideal of X. Also, we study certain properties of strong bi-ideals in a left permutable Γ-near subtraction semigroups. Throughout this paper, by a Γ- near subtraction semigroup X, we mean a zero symmetric Γ-near subtraction semigroup.

Keywords: Left permutable, quasi-ideal, strong bi-ideal, weak bi-ideal, left self distributive S-Γ-near substraction semigroup.

1.Introduction

By X, we mean that zero- symmetric Γ-near subtraction semigroup. Γ-near subtraction semigroup was introduced by Dr. S. J. Alandkar [2]. For basic terminology in near subtraction semigroup, we refer to Dheena [3] and for Γ-near subtraction semigroup, we refer to Dr. S. J. Alandkar [2]. Tamizh Chelvam and Ganesan [14] introduced the notion of bi-ideals in near-rings and it was extended to Γ-near-ring by T. Tamizh Chelvam and N. Meenakumari [13]. V.Mahalakshmi and S. Jayalakshmi introduced the strong bi-ideals and weak bi-ideals in near-subtraction semigroup [7] &[8]. In this paper we introduce the notion of strong bi-ideal and weak bi-ideal in Γ-near subtraction semigroups by extending the concepts of strong bi-ideals and weak bi-ideal in near-subtraction semigroup.

2.Preliminaries

(X, −), where X is a non-empty set is known as subtraction algebra if it satisfies the following identities: for any u, v, w ∈ X, i) u – (v – u) = u; ii) u – (u – v) = v – (v – u); iii) (u – v) – w = (u – w) – v. A triple (X, −, .)is known as subtraction semigroup if it satisfies the following axioms: for any u, v, w∈ X, i)(X, **.**) is a semigroup ii) (X, –) is a subtraction algebra; iii)u(v – w) = uv – uw and (u – v)w = uw – vw. A triple (X, −, .) is known as near-subtraction semigroup(right) if it satisfies the following axioms: for any u, v, w∈ X, i)(X, **.**) is a semigroup ii) (X, –) is a subtraction algebra; iii) (u –

v)w = uw – vw. A Γ-near subtraction semigroup is a triple (X, –, γ),for all $\gamma\epsilon$ Γ, where Γ is a non-empty set of binary operators on X, such that (X, –, γ) is a near-subtraction semigroup for all $\gamma\epsilon$ Γ. In practice, we called simply Γ-near- subtraction semigroup instead of right Γ-near- subtraction semigroup. Similarly, we can define a Γ-near- subtraction semigroup(left). It is clear that $0\gamma a = 0$ for all a∈ X and for all $\gamma\epsilon$ Γ.(S, −) where S is a nonempty subset of a subtraction algebra X is said to be subalgebra of X, if x – y∈ S whenever x, y ∈ S. A subalgebra M of (X, –) with M ΓM ⊆ M is called a sub Γ-near subtraction semigroup of X. A nonempty subset A of X is called i) a left Γ-subalgebra of X if A is a subalgebra of (X, –) and X ΓA⊆A. ie.) XγA⊆A for all $\gamma\epsilon$ Γ. ii) a right Γ-subalgebra of X if A is a subalgebra of (X, –) and AΓX⊆A. ie., AγX⊆A for all $\gamma\epsilon$ Γ. X_0= {x∈ X/ xγ0 = 0 for all $\gamma\epsilon$ Γ} is called the zero-symmetric part of X. X is called Zero-Symmetric, if X = X_0. An element a∈ X is called idempotent if aγa = a for all $\gamma\epsilon$ Γ.X_d = {n∈ X/nγ(x –y) = nγx – nγy for all x, y∈X and for all $\gamma\epsilon$ Γ} is the set of all distributive elements of X. X is called distributive if X = X_d.If a∈ Xγa for all $\gamma\epsilon$ Γ and for all a∈X, then X is called S-Γ-near subtraction semigroup.X is said to be left bipotent if Xγa = Xγaγa for all a∈X and $\gamma\epsilon$ Γ. Let X and X′ be two Γ-near subtraction semigroups. A map f: X→ X′ is said to be Γ-near subtraction semigroup homomorphism if i) f(a–b) = f(a) – f(b) ii) f(aγb) = f(a)γf(b) for all a, b ∈X and $\gamma\epsilon$ Γ. Let X be a zero-symmetric Γ-near subtraction semigroup and if X is strongly regular, then X is regular. For A⊆X, by $\sqrt{A}$ we mean that {a∈ X/a^k ∈ A, for some k>0}.

3. Strong bi-ideal of Γ-near subtraction semigroup

In this section we study the properties involving strong bi-ideal in a left permutable Γ-near subtraction semigroups and obtain the characterization theorem on strong bi-ideals.

Definition 3.1

A subalgebra Q of (X, −) is said to be a quasi-ideal of X if (QΓX)∩(XΓQ)⊆Q.

Definition 3.2

A subalgebra B of (X, −) is said to be a strong bi-ideal of X if XΓBΓB⊆B.

Proposition 3.3

Any homomorphic image of a strong bi-ideal is also a strong bi-ideal.

Proof: Let f: X→ X′ be an homomorphism.Let B be a strong bi-ideal of X.Now, for f(b) ∈f(B) and n′ ∈ X′.n′γf(b) γf(b) = f(n) γf(b) γf(b) = f(nγbγb)⊆ f(XΓBΓB)⊆ f(B) for all γ ∈ Γ. Hence f(B) is a strong bi-ideal of X.

Definition 3.4

A family ζ of subsets of a set A is called a Moore system, if i)A ∈ ζ and ii) $\zeta$is closed under arbitrary intersection.

Proposition 3.5

The arbitrary intersection of strong bi-ideals of a Γ-near subtraction semigroup is again a strong bi-ideals.

Proof: Let $\{B_i\}_{i\in I}$ be a set of strong bi-ideals in X. Let B = $\cap_{i\in I} B_i$. Then XΓBΓB ⊆XΓB_iΓB_i ⊆ B_i, ∀ i. Hence XΓBΓB ⊆ B. Hence the proposition.

Definition 3.6

X is said to be strongly regular if for each a ∈ X, there exists b∈X, such that a = bγaγa for all $\gamma\epsilon$ Γ.

Proposition 3.7

Let X be a Γ-near subtraction semigroup and B a strong bi-ideal of X. If elements of B are strongly regular, then B is a quasi-ideal of X.

Proof: Let x∈ (BΓX)∩(XΓB). Then x = $b_1\gamma n_1 = n_2\gamma b_2$ for some b_1, b_2 ∈ B and n_1, n_2 ∈ X for allγ ∈ Γ.Since elements of B are strongly regular,b_1 = c$\gamma b_1\gamma b_1$ and b_2 = d$\gamma b_2\gamma b_2$ for some c, d ∈ X and for all$\gamma \in \Gamma$. Hence x = $b_1\gamma n_1$= $(c\gamma b_1\gamma b_1)\gamma n_1$= $c\gamma b_1\gamma(b_1\gamma n_1)$= $c\gamma b_1\gamma(n_2\gamma b_2)$= $c\gamma b_1\gamma(n_2\gamma(d\gamma b_2\gamma b_2))$= $(c\gamma b_1\gamma n_2\gamma d)\gamma(b_2\gamma b_2) \in X\Gamma B\Gamma B \subseteq B$ for all $\gamma \in \Gamma$. Hence $(B\Gamma X)\cap(X\Gamma B) \subseteq B$. Therefore B is a quasi-ideal of X.

Definition 3.8

X is known as left permutable if $a\gamma b\gamma c = b\gamma a\gamma c$ for all a, b, c ∈ X and for all $\gamma \in \Gamma$.

Proposition 3.9

Let X be a left permutable and B is a bi-ideal of X. If the elements of B are strongly regular, then B is strong bi-ideal of X if and only if B is a quasi-ideal of X.

Proof: If part follows from the proposition 3.7. Conversely assume that B is a quasi-ideal. From the definition of quasi-ideal, is a bi-ideal. Further x ∈ XΓBΓB, then, x = $n\gamma b\gamma b \in X\Gamma B$ for all $\gamma \in \Gamma$. Again since X is left permutable,x = $b\gamma n\gamma b \in B\Gamma X$. for all $\gamma \in \Gamma$.i.e.) $x \in (X\Gamma B)\cap(B\Gamma X) \subseteq B$. Hence $X\Gamma B\Gamma B \subseteq B$. Therefore, B is a strong bi-ideal.

Proposition 3.10

Let X be a left permutable. If B is a strong bi-ideal of X, then $n\gamma B$ and $B\gamma n'$ $\forall\gamma \in \Gamma$ are strong bi-ideals of X, where n, n' ∈X and n is distributive element in X.

Proof: Clearly $B\gamma n'$ is a subalgebra of (X, −) for all$\gamma \in \Gamma$.Also, $X\Gamma(B\gamma n')\Gamma(B\gamma n') \subseteq X\Gamma B\Gamma(B\gamma n') \subseteq B\gamma n'$for all$\gamma \in \Gamma$.And so we get that $B\gamma n'$ is a strong bi-ideal of X. Since n is distributive, $n\gamma B$ is a subalgebra of (X, −) for all $\gamma \in \Gamma$.Further let $x \in X\Gamma(n\gamma B)\Gamma(n\gamma B)$. Then:

$$x = n_1\gamma n\gamma b\gamma n\gamma b' = n\gamma n_1\gamma b\gamma n\gamma b' = n\gamma n\gamma n\gamma b\gamma b' \in n\gamma X\Gamma B\Gamma B \subseteq n\gamma B \text{ for all } \gamma \in \Gamma.$$

Hence $n\gamma B$ is a strong bi − ideal.

Proposition 3.11

Let B be a strong bi-ideal of a Γ-near subtraction semigroup X and S be a sub Γ-near subtraction semigroup of X, then intersection of B with S is again a strong bi-ideal of S.

Proof: Let C = B∩S. Now,

$S\Gamma C\Gamma C = S\Gamma(B\cap S)\Gamma(B\cap S)= S\Gamma((B\cap S)\Gamma B\cap(B\cap S)\Gamma S \subseteq S\Gamma(B\Gamma B\cap S\Gamma S) \subseteq (S\Gamma B\Gamma B)\cap S\Gamma S \subseteq B\cap S = C$.

Hence $S\Gamma C\Gamma C \subseteq C$. Hence the proof.

Proposition 3.12

Let X be a left permutable Γ-near subtraction semigroup. Then B is a strong bi-ideal if and only if B is a bi-ideal.

Proof: If part is trivial**.** Conversely suppose B is a bi-ideal of X. Let x∈ XΓBΓB.Since X is left permutable,x = $n\gamma b\gamma b = b\gamma n\gamma b \in B\Gamma X\Gamma B \subseteq B$, for all $\gamma \in \Gamma$. Hence the proof.

Proposition 3.13

If X is strongly regular, then B = XΓBΓB for every strong bi-ideal B of X.

Proof: Let B be a strong bi-ideal of X. Let b ∈ B. Since X is strongly regular, there exists x ∈ X such that b = $x\gamma b\gamma b \in X\Gamma B\Gamma B$ for some x∈ X and for all $\gamma \in \Gamma$. i.e.)., B ⊆ XΓBΓB. Hence from the definition of a strong bi-ideal, B = XΓBΓB.

Proposition 3.14

Let X be a left permutable S - Γ-near subtraction semigroup. Then X is strongly regular if and only if B = XΓBΓB for every strong bi-ideal B of X.

Proof: If part follows from the proposition 3.13.Conversely, let a ∈ X. By the proposition 3.10, $X\gamma a$ is a strong bi-ideal of X.Now, $a \in X\gamma a = X\Gamma(X\gamma a)\Gamma(X\gamma a) \subseteq (X\gamma a)\Gamma(X\gamma a)$for all $\gamma \in \Gamma$. i.e.).,

$a = n_1\gamma a\gamma n_2\gamma a = n_1\gamma n_2\gamma a\gamma a \in X\gamma a\gamma a$ for all $\gamma \in \Gamma$. i.e), X is strongly regular.

Proposition 3.15

Let X be a left permutable S **-** Γ-near subtraction semigroup. Then X is a left bi-potent if and only if B = XΓBΓB for every strong bi-ideal B of X.

Proof: Every left bi-potent is strongly regular and by proposition 3.14, the result is true.

Proposition 3.16

Let X be a left permutable. Then B = BΓXΓB, for every bi-ideal B of X if and only if B = XΓBΓB.

Proof: Let B = BΓXΓB for a bi-ideal B of X. By proposition 3.12, B is a strong bi-ideal. If x ∈ B = BΓXΓB, then x = bγnγbfor someb∈ B and n∈ X for all $\gamma \in \Gamma$. Since X is left permutable, x = nγbγb ∈XΓBΓB for all $\gamma \in \Gamma$. i.e.)., B⊆ XΓBΓB and so B= XΓBΓB. Conversely let B= XΓBΓB for every bi-ideal B of X. If x∈B = XΓBΓB, then x = nγbγb = bγnγb ∈BΓXΓB for all $\gamma \in \Gamma$. i.e.)., B ⊆BΓXΓB⊆ B. Hence B = BΓXΓB.

Definition 3.17

A S- Γ-near subtraction semigroup X is said to be left self-distributive S-Γ-near subtraction semigroup if aγbγc = aγbγaγc for all a, b, c ∈ X and for all $\gamma \epsilon \Gamma$.

Theorem 3.18

Let X be a left self-distributive S-Γ-near subtraction semigroup. Then B=XΓBΓB, for every strong bi-ideal B of X if and only if X is strongly regular.

Proof: Let B=BΓBΓB, for every strong bi-ideal B of X. Since Xγa is astrong bi-ideal of X and X is a S-Γ-near subtraction semigroup, we have a ∈ Xγa = XΓ(Xγa)Γ(Xγa)⊆ (Xγa)Γ(Xγa) for all $\gamma \in \Gamma$. (ie)., a = $n_1\gamma a\gamma n_2\gamma$afor all $\gamma \in \Gamma$. Since X is a left self-distributive S-Γ-near subtraction semigroup, a = $n_1\gamma a\gamma n_2\gamma$a= a = $n_1\gamma a\gamma n_2\gamma$a γa ∈ Xγa γafor all $\gamma \in \Gamma$. (ie)., X is strongly regular. Conversely, let B be a strong bi-ideal of X. Since X is strongly regular, for b ∈ B, b = nγb γb ∈ XΓBΓBfor all $\gamma \in \Gamma$. (ie)., B⊆ XΓBΓB. Hence the proof.

4. Weak bi-ideal of Γ-near subtraction semigroup

In this section we establish weak bi-ideal in Γ-near subtraction semigroup and obtain the characterization theorem on weak bi-ideals.

Definition 4.1

A subalgebra B of (X, −) is said to be a weak bi-ideal of X if BΓBΓB⊆B.

Proposition 4.2

Any homomorphic image of a weak bi-ideal is also a weak bi-ideal.

Proof: Let f: X→ X′ be an homomorphism.Let B be a weak bi-ideal of X. Let B′ = f(B). Let b′ ∈f(B). Then b′γb′γb′= f(b)γf(b) γf(b) = f(b) γf(b) γf(b)= f(bγbγb)⊆ f(BΓBΓB)⊆ f(B) for all $\gamma \in \Gamma$. Hence the proof.

Proposition 4.3

The arbitrary intersection of weak bi-ideals of a Γ-near subtraction semigroup is again a weak bi-ideals.

Proof: Let $\{B_i\}_{i \in I}$ be a set of weak bi-ideals in X. Let B = $\cap_{i \in I} B_i$. Then BΓBΓB ⊆$B_i \Gamma B_i \Gamma B_i$ ⊆ B_i, ∀ i. Hence BΓBΓB ⊆ B. Therefore, B is a weak bi-ideal of X.

Proposition 4.4

If B is a weak bi-ideal of a Γ-near subtraction semigroupX and S is a sub Γ-near subtraction semigroup of X, then then intersection of B with S is again a weak bi-ideal of S.

Proof: Let C = B∩S. Now, CΓCΓC = (B∩S)Γ(B∩S)Γ(B∩S)⊆(B∩S)Γ(BΓB∩SΓS)⊆(B∩S)ΓBΓB∩ (B ∩ S)ΓSΓS⊆ BΓBΓB∩ SΓSΓS⊆ B∩S = C. Hence CΓCΓC ⊆ C. Hence the proof.

Proposition 4.5

Let B is a weak bi-ideal of X, then $b'\gamma B$ and $B\gamma b$ for all $\gamma \in \Gamma$are weak bi-ideals of X, where b,b' ∈X and b' is distributive element in X.

Proof: Clearly $B\gamma b$ is a subalgebra of (X, −) for all $\gamma \in \Gamma$. Then:

$$(B\gamma b)\Gamma(B\gamma b)\Gamma(B\gamma b) \subseteq B\Gamma B\Gamma(B\gamma b) \subseteq B\gamma b.$$

We have b' is distributive, $b'\gamma B$ is a subalgebra of (X, −) and $(b'\gamma B)\Gamma(b'\gamma B)\Gamma(b'\gamma B) \subseteq b'\gamma B\Gamma B\Gamma Bn \subseteq b'\gamma B$ for all $\gamma \in \Gamma$. Hence $b'\gamma B$ and $B\gamma b$ for all $\gamma \in \Gamma$ are weak bi-ideals of X.

Corollary 4.6

Let B be a weak bi-ideal of X. For b, c ∈ B, if b is distributive, then $b\gamma B\gamma c$ is a weak bi-ideal of X.

Theorem 4.7

Let X be a left self-distributive S-Γ-near subtraction semigroup. Then BΓBΓB =B, for every weak bi-ideal B of X if and only if X is strongly regular.

Proof: Let B be a weak bi-ideal of X. If X is strongly regular, X is regular. Let b∈B. Since X is regular, $b = b\gamma a\gamma b$, for some a ∈ X for all $\gamma \in \Gamma$. Since X is left distributive, we have $b\gamma a\gamma b = b\gamma a\gamma b\gamma b$for all $\gamma \in \Gamma$. Thus $b = b\gamma a\gamma b = b\gamma a\gamma b\gamma b = b\gamma a\gamma b\gamma b\gamma b = b\gamma b\gamma b \in B\Gamma B\Gamma B$for all $\gamma \in \Gamma$. i.e)., B⊆ BΓBΓB. Hence B = BΓBΓB for every weak bi-ideal B of X. Conversely, let a∈X. Since $X\gamma a$ is a weak bi-ideal B of X and X is S-Γ-near subtraction semigroup, we get $a \in X\gamma a = (X\gamma a)\Gamma(X\gamma a)\Gamma(X\gamma a) \subseteq (X\gamma a)\Gamma(X\gamma a)$.i.e)., $a = n_1\gamma a\gamma n_2\gamma a$for all $\gamma \in \Gamma$. Since X is left self-distributive, $a = n_1\gamma a\gamma n_2\gamma a\gamma a \subseteq X\gamma a\gamma a$ for all $\gamma \in \Gamma$. i.e)., X is strongly regular.

Theorem 4.8

Let X be a left self-distributive S-Γ-near subtraction semigroup. Then BΓBΓB =B, for every weak bi-ideal B of X if and only if B=XΓBΓB, for every strong bi-ideal B of X.

References

[1] Abbott J. C., Sets, Lattices and Boolean Algebras, Allyn and Bacon, Boston 1969.

[2] Alandkar S. J.,A Note on Γ- near subtraction semigroups, Indian Streams Research Journal, Vol.6, Issue-6(2016)

[3] Dheena P. and Satheesh Kumar G., On strong regular near-subtraction semigroups Commun. Korean Math.Soc.22(2007),pp.323-330.

[4] Kaushik J. P. and Moin Khan, On bi-Γ-ideal in Γ-semirings, Int. J. Contemp. Math. Sciences, Vol. 3, no.26(2008),pp.1255-1260

[5] Kim K. H. Roh E. H. and Yon Y. H., A Note on Subtraction Semigroups, Scientiae Mathematicae Japonicae Online, Vol.10,(2004),pp.393-401.

[6] Mahalakshmi V., Maharasi S., Jayalakshmi S.,On bi-ideals of Near-Subtraction Semigroup, Advances in Algebra ISSN 0973-6964, Vol. 6(1)(2013), pp. 35-43.

[7] Mahalakshmi V., Maharasi S., Jayalakshmi S.,On weak bi-ideals in near subtraction semigroup,International Journal of Current Research , Vol. 8, Issue, 08, pp. 35955-35959, August, 2016.

[8] Mahalakshmi V., Jayalakshmi S.,A Study on regularities in near subtraction semigroups, Ph. D thesis, Manonmaniam Sundaranar University, 2016.

[9] Meenakshi, P. and Meenakumari N., On semigamma bi-ideals in Γ-seminear-rings, International Journal of Current Research Vol.9, Issue 10(2017), pp.59172-59175.

[10] Meenakshi, P. and Meenakumari N.,Bi-ideals of Γ- near subtraction semigroups,JETIR February 2019, Volume 6, Issue 2 (ISSN-2349-5162), pp. 448-451.

[11] Pilz Gunter , Near-Rings, North Holland, Amsterdam, 1983.

[12] Schein B. M.,Difference Semigroups, Comm. In Algebra 20(1992),pp. 2153-2169.

[13] Tamizh Chelvam T. And Meenakumari N.,Bi-ideals of Gamma Near-Rings, Southeast Asian Bulletin of Mathematics, 27(2004), pp. 983-988.
[14] Tamil Chelvam and N. GanesanOn Bi-ideals of Near-Rings, Indian J. Pure appl. Math. 18(11)(1987),pp.1002-1005 .
[15] Zelinka B.,Subtraction Semigroups, Math. Bohemica, 120(1995), pp. 445-447.