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FUZZY PSEUDO INTRINSIC EDGE-MAGIC GRAPHS

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Abstract: A fuzzy graph G is said to be pseudo intrinsic edge-magic graph if it contains mock constant λ_m which is different from intrinsic super constant. This research article aims at introducing some kinds of pseudo intrinsic edge magic graph like evenly divisible, disjoint, double & poly pseudo intrinsic edge-magic graph and further determining the strength of the above mentioned graphs.

Keywords: Intrinsic constant; mock constant; pseudo intrinsic edge-magic; evenly divisible- double; disjoint & poly pseudo intrinsic edge-magic; strength.

Mathematical Classification Subject: 05C72,05C78.

1. Introduction:

Fuzzy set was initially introduced by Zadeh L.A. [1]. Later various researches added productive concepts to develop fuzzy sets theory like "Fuzzy graphs: In Fuzzy Sets and Their Applications" and "Order and Size in Fuzzy graph", [3] and [8]. In 1987, Bhattacharya succeeded in developing the connectivity notions between fuzzy bridge and fuzzy cut nodes [5]. A fuzzy graph contains many properties similar to crisp graph due to generalization of crisp graphs but it diverges at many places.

A crisp graph G is an order pair of vertex-set V and edge set E such that $E \subseteq V \times V$. In addition $v = |V|$ is called order and $e = |E|$, size of the graph G respectively. In a crisp graph, a bijective function $\rho : V \cup E \to N$ that produces a unique positive integer (To each vertex and/or edge) is termed labelling in "Some results on magic graphs," [4]. Having introduced the notion of magic graph where the labels vertices and edges are natural numbers from 1 to $|V| + |E|$ and the sum of the same must be constant in entire graph, "Super edge-magic graphs." [6]. Extending the concept of magic graph adding a property that vertices always get smaller labels than edges which is named super edge magic labelling. Numerous other authors have explored diverse types of different magic graphs [7], [11] & [12]. The subject of edge-magic labelling of graphs had its origin in Kotzig and Rosa's work on magic valuations of graphs [2]. These labelling are currently referred to as either edge-magic labelling or edge-magic total labelling.

Fuzzy graphs are generalization of graphs. In graphs two vertices are either related or unrelated to each other. Mathematically, the degree of relationship is either 0 or 1. In fuzzy graphs, the degree of relationship takes values from [0, 1]. A fuzzy graph has ability to solve uncertain problems in a wide range of fields. The first definition of a fuzzy graph was introduced by Kaufmann in 1973. Azriel Rosenfield in 1975 [3] developed the structure of fuzzy graphs and obtained analogs of several graph theoretical concepts. In [9] &[11], Nagoor Gani et. al. introduced the concepts of order and size of fuzzy graphs & fuzzy labelling graphs, fuzzy magic graphs. A fuzzy graph contains many properties similar to crisp graph due to generalization of crisp graphs but it diverges at many places. [14], [15] & [16]. This paper highlights on developing some kinds of fuzzy pseudo perfect intrinsic edge-magic graphs like evenly divisible, double, disjoint and poly graph. Further the article evaluates the strength of the above mentioned graphs.

2. Preliminaries

***Definition 2.1**: [1]*

A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ where for all $u, v \in V$, we have $\mu(u,v) \leq \sigma(u) \Lambda \sigma(v)$.

Definition 2.2:

A path P in a fuzzy graph is a sequence of distinct nodes $v_1, v_2, v_3 ..... v_n$ such that $\mu(v_i, v_{i+1}) > 0; 1 \leq i \leq n;$ where $n \geq 1$ is called the length of the path P. The consecutive pairs (v_i, v_{i+1}) are called the edge of the path.

Definition 2.3:

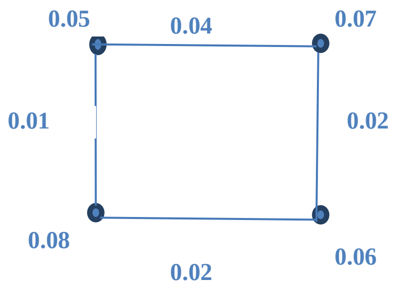
A path P is called a cycle if $v_1 = v_n$ and $n \geq 3$ and a cycle is called a fuzzy cycle if it contains more than one weakest arc.

Definition 2.4:

A bijection ω is a function from the set of all nodes and edges of to [0, 1] which assign each nodes $\sigma^{\omega}(a)$, $\sigma^{\omega}(b)$ and edge $\mu^{\omega}(a,b)$ a membership value such that $\mu^{\omega}(a,b) \leq \sigma^{\omega}(a) \Lambda \sigma^{\omega}(b)$ for all $a, b \in V$ is called fuzzy labelling. A graph is said to be fuzzy labelling graph if it has a fuzzy labelling and it is denoted by G^{ω}.

***Definition 2.5:** [14]*

A fuzzy labelling graph G is said to be fuzzy intrinsic labelling if $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ is bijective such that the membership values of edges and vertices are $z, 2z, 3z, ..., Nz$ where N is the total number of vertices and edges and let $z = 0.1$ *for* $N \leq 6$ & $z = 0.01$ *for* $N > 6$.



Definition 2.6: *[14]*

An edge-magic constant in a fuzzy intrinsic edge-magic graph is said to be mock constant $'\lambda_m'$ if it is equal to $\sigma(v_i)+\mu(v_i v_j)+\sigma(v_j)$ for some $v_i, v_j \in V$ with $\lambda_c \neq \lambda_m$.

Definition 2.7: *[14]*

A fuzzy graph is said to be a pseudo-intrinsic edge-magic graph if it contains mock constant $'\lambda_m'$ which is also denoted by $'G_p'$.

3. Fuzzy evenly divisible pseudo intrinsic edge-magic graph

Definition 3.1:

Let G be a fuzzy pseudo intrinsic edge-magic graph. If the mock constant $'\lambda_m'$ is evenly divisible then G is said to be evenly divisible.

Definition 3.2:

Let G be a fuzzy pseudo intrinsic edge-magic graph. If the mock constant occurs twice that $\lambda_{m_i} = \lambda_{m_j}$ *for all* $i \neq j$ then G is said to be a double pseudo intrinsic edge-magic graph.

Definition 3.3:

Let G be a fuzzy pseudo intrinsic edge-magic graph. If $\lambda_{m_i} \neq \lambda_{m_j}$ *for all* $i \neq j$ then the graph G is said to be a disjoint pseudo intrinsic edge-magic graph.

Definition 3.4:

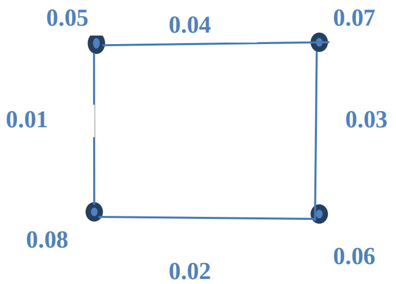
Let G be a fuzzy pseudo intrinsic edge-magic graph. If $\lambda_{m_i} \neq \lambda_{m_j} \neq \lambda_{m_k} \neq \ldots$ and *for all* $i \neq j \neq k \neq \ldots$ then G is said to be a poly pseudo intrinsic edge-magic graph.

Theorem 3.5:

A fuzzy cycle $'C_n'$ is an evenly divisible pseudo intrinsic edge-magic graph for n= 4.

Proof: Let $'C_n'$ be a fuzzy cycle with even number of vertices.

By our assumption, let n = 4.



Apply intrinsic edge-magic labelling, we get

$$\sigma(v_1)+\mu(v_1 v_2)+\sigma(v_2)=0.05+0.04+0.07=0.16=\lambda_c$$
$$\sigma(v_2)+\mu(v_2 v_3)+\sigma(v_3)=0.07+0.03+0.06=0.16=\lambda_c$$
$$\sigma(v_3)+\mu(v_3 v_4)+\sigma(v_4)=0.06+0.02+0.08=0.16=\lambda_c$$
$$\sigma(v_4)+\mu(v_4 v_1)+\sigma(v_1)=0.08+0.01+0.05=0.14=\lambda_m$$

Here the mock constant $\lambda_m = 0.14$ which is evenly divisible.

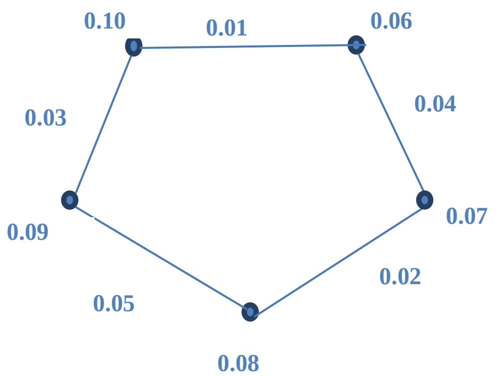
By definition 3.1, the graph $'C_n'$ is an evenly divisible pseudo intrinsic edge-magic graph.

Theorem 3.6:

A fuzzy cycle $'C_n'$ is a double pseudo intrinsic edge-magic graph for n=5.

Proof: Let $'C_n'$ be a fuzzy cycle with odd number of vertices.

By our assumption, we consider n = 5.



Apply intrinsic edge-magic labelling, we get

$$\sigma(v_1)+\mu(v_1 v_2)+\sigma(v_2)=0.10+0.01+0.06=0.17=\lambda_c$$
$$\sigma(v_2)+\mu(v_2 v_3)+\sigma(v_3)=0.06+0.04+0.07=0.17=\lambda_c$$
$$\sigma(v_3)+\mu(v_3 v_4)+\sigma(v_4)=0.07+0.02+0.08=0.17=\lambda_c$$
$$\sigma(v_4)+\mu(v_4 v_5)+\sigma(v_5)=0.08+0.05+0.09=0.22=\lambda_{m_1}$$
$$\sigma(v_5)+\mu(v_5 v_1)+\sigma(v_1)=0.09+0.03+0.10=0.22=\lambda_{m_2}$$

Here, two equal mock constants occur $\lambda_{m_1} = \lambda_{m_2} = 0.22$.

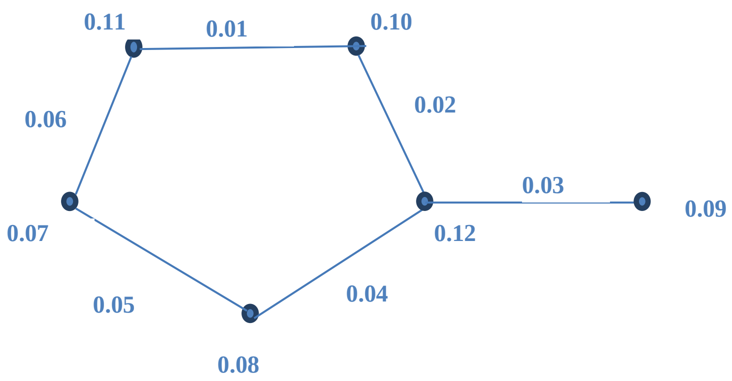
By definition 3.2, the given fuzzy cycle graph with five vertices is a double pseudo intrinsic edge-magic graph.

Theorem 3.7:

A fuzzy n-pan graph is a disjoint pseudo intrinsic edge-magic graph for n=5.

Proof: Let G be a fuzzy n-pan graph with n=5.

Apply fuzzy intrinsic edge-magic labelling, we get



$$\sigma(v_1)+\mu(v_1 v_2)+\sigma(v_2)=0.11+0.01+0.10=0.22=\lambda_{m_1}$$
$$\sigma(v_2)+\mu(v_2 v_3)+\sigma(v_3)=0.10+0.02+0.12=0.24=\lambda_c$$
$$\sigma(v_3)+\mu(v_3 v_4)+\sigma(v_4)=0.12+0.03+0.09=0.24=\lambda_c$$
$$\sigma(v_4)+\mu(v_4 v_5)+\sigma(v_5)=0.12+0.04+0.08=0.24=\lambda_c$$
$$\sigma(v_5)+\mu(v_5 v_1)+\sigma(v_1)=0.08+0.05+0.07=0.20=\lambda_{m_2}$$
$$\sigma(v_5)+\mu(v_5 v_1)+\sigma(v_1)=0.07+0.06+0.11=0.24=\lambda_c$$

Here, two mock constants occur $\lambda_{m_1}=0.20$ & $\lambda_{m_2}=0.22$.

By definition 3.3, the given fuzzy n-pan graph with n=5 is a disjoint pseudo intrinsic edge-magic graph.

Theorem 3.8:

Every fuzzy perfect intrinsic edge-magic graph is not always a pseudo intrinsic edge-magic graph and vice-versa.

4. Strength of evenly divisible, double, disjoint & poly pseudo intrinsic edge-magic graph

Definition 4.1:

Let E be a fuzzy evenly divisible pseudo intrinsic edge-magic graph. Then the strength of E is denoted by α_e and is defined as the twice of the mock constant, $\alpha_e=2\lambda_m$.

Assessment 4.1.1:

Evaluate the strength of fuzzy evenly divisible pseudo intrinsic edge-magic for cycle graph with n=4.

By theorem 3.5, we get $\lambda_m=0.14$ and by above definition, strength of the given graph is $\alpha_e=2\lambda_m=2(0.14)=0.28$.

Definition 4.2:

Let D be a fuzzy double pseudo intrinsic edge-magic graph. Then the strength of D is denoted by α_d and is defined as $\alpha_d = \lambda_m, (\lambda_{m_i} = \lambda_{m_j})$ *for all* $i \neq j$.

Assessment 4.2.1:

Evaluate the strength of fuzzy double pseudo intrinsic edge-magic for cycle graph with n=5.

By theorem 3.6, we focussed double pseudo intrinsic edge magic graph with two mock constants which are equal.

$$\lambda_{m_1} = \lambda_{m_2} = 0.22 = \lambda_m$$

By above definition, Strength of the mentioned graph is

$\alpha_d = 0.22, (\lambda_{m_1} = \lambda_{m_2} = 0.22)$ *for all* $i \neq j$.

Definition 4.3:

Let T be a fuzzy disjoint pseudo intrinsic edge-magic graph. Then the strength of T is denoted by α_t and is defined as $\alpha_t = \lambda_{m_i} + \lambda_{m_j}$ *for all* $i \neq j$.

Assessment 4.3.1:

Evaluate the strength of fuzzy disjoint pseudo intrinsic edge-magic for n-pan graph.
By theorem 3.7, we discussed n-pan graph with n=5.
In that place $\lambda_{m_1} = 0.20$ & $\lambda_{m_2} = 0.22$
Through the above definition, strength of the above mentioned graph is

$\alpha_t = \lambda_{m_i} + \lambda_{m_j}$ *for all* $i \neq j$.

$= 0.20 + 0.22$

$\alpha_t = 0.42$

Definition 4.4:

Let P be a fuzzy poly pseudo intrinsic edge-magic graph. Then the strength of T is denoted by α_p and is defined as $\alpha_p = \lambda_{m_i} + \lambda_{m_j} + \lambda_{m_k} + \ldots$ *for all* $i \neq j \neq k \neq \ldots$

5. Conclusion:

The research article has taken the concepts like the evenly divisible, double, disjoint and poly pseudo intrinsic edge-magic graphs. This paper has evaluated the strength of concern graphs like fuzzy cycle with odd and even number of vertices & n-pan graph. In future, the focus would be on poly pseudo intrinsic edge-magic graphs with more examples.

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