Advances in Mathematics: Scientific Journal 8 (2019), no.3, 233–238
(Special issue on ICRAPAM)
ISSN 1857-8365 printed version
ISSN 1857-8438 electronic version

ANTI INDEPENDENCE NUMBER ON ANTI FUZZY GRAPH

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Abstract: In this paper, we introduce the concept of anti-independent set and anti-covering on anti-fuzzy graph. We obtain the bounds on anti-independence number on anti-fuzzy graph and applied these parameters on various types of anti-fuzzy graph. The relations between anti-independence number and domination numbers on anti-fuzzy graph are discussed and obtained bounds on them.

Keywords: Anti fuzzy graph, Domination, Anti independent set, v-nodal anti fuzzy graph, uninodal anti fuzz ygraph.

Mathematical Classification: 05C69, 05C70,05C72, 05E99, 05C07.

1. Introduction

Mohaamad Akram [1] introduced the concept of anti fuzzy structure on graph from the fuzzy relation introduced by Zedah. He derived the notion of connectedness on anti fuzzy graph. R.Seethalakshmi and R.B.Gnanajothi [8] introduced the definition of anti fuzzy graph. R. Muthuraj and A. Sasireka [3 - 6] illustrated the concepts of some types of anti fuzzy graph and derived the domination parameters on anti fuzzy graphs. In this paper, we introduce the concept of anti independent domination number, anti covering,anti matching and anti independent domination number on anti fuzzy graphs. The results are examined and some theorems are derived from them.

2. Preliminaries

In this section, basic concepts of anti fuzzy graph are discussed. Notations and more formal definitions which are followed as in [1,3-6, 8].

Definition 2.1 [8]

A fuzzy graph G= (σ, μ) is said to be an anti fuzzy graph with a pair of functions $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$, where for all u, v ∈ V, we have $\mu(u,v) \geq \sigma(u) \vee \sigma(v)$ and it is denoted by $G_A(\sigma, \mu)$.

Remark: μ is considered as reflexive and symmetric. In all examples σ is chosen suitably. i.e., undirected anti fuzzy graphs are only considered.

Notation: Without loss of generality let us simply use the letter G_A to denote an anti fuzzy graph.

Definition 2.2 *[1]*

The order p and size q of an anti fuzzy graph $G_A = (V, \sigma, \mu)$ are defined to be $p = \sum_{x \in V} \sigma(x)$ and $q = \sum_{xy \in V} \mu(xy)$. It is denoted by O(G) and S(G).

Definition 2.3 *[5]*

Two vertices u and v in G_A are called adjacent if $(\frac{1}{2})[\sigma(u) \vee \sigma(v)] \leq \mu(u,v)$.

Definition 2.4 *[8]*

An anti fuzzy graph $G_A = (\sigma,\mu)$ is a strong anti fuzzy graph of $\mu(u,v) = \sigma(u) \vee \sigma(v)$ for all $(u,v) \in \mu^*$ and G_A is a complete anti fuzzy graph if $\mu(u,v) = \sigma(u) \vee \sigma(v)$ for all $(u,v) \in \sigma^*$. Two vertices u and v are said to be neighbors if $\mu(u,v) > 0$.

Definition 2.5 *[4]*

u is a vertex in an anti fuzzy graph G_A then N(u) = {v: (u,v) is a strong edge} is called the neighborhood of u and N[u] = N(u) ∪{u} is called closed neighborhood of u.

Definition 2.6 *[4]*

The strong neighborhood of an edge e_i in an anti fuzzy graph G_A is $N_s(e_i) = \{e_j \in E(G) \,/\, e_j$ is a strong edge in G_A and adjacent to $e_i\}$.

Definition 2.7 *[3,4]*

A path P_A in an anti fuzzy graph is a sequence of distinct vertices $u_0, u_1, u_2, \ldots, u_n$ such that $\mu(u_{i-1},u_i) > 0$, $1 \leq i \leq n$. Here $n \geq 0$ is called the length of the path P_A. The consecutive pairs (u_{i-1}, u_i) are called the edges of the path.

Definition 2.8 *[1]*

A cycle in G_A is said to be an anti fuzzy cycle if it contains more than one weakest edge.

Definition 2.9 *[5]*

Every vertex in G_A has unique fuzzy values then G_A is said to be v-nodal anti fuzzy graph. i.e.,σ(u) = c for all $u \in V(G_A)$.

Definition 2.10 *[5]*

Every edge in G_A has unique fuzzy values then G_A is said to be e-nodal anti fuzzy graph. i.e.,μ(u) = c for all $uv \in E(G_A)$

Definition 2.11 *[5]*

Every vertices and edges in an anti fuzzy graph have the unique fuzzy values then G_A is called as uninodal anti fuzzy graph.

Definition 2.12 *[5]*

An edge e = {u,v} of an anti fuzzy graph G_A is called an effective edge if $\mu(u,v) = \sigma(u) \vee \sigma(v)$.

Definition 2.13 *[6]*

An edge e = {u,v} of an anti fuzzy graph G_A is called an weak edge if $\mu(u,v) \neq \sigma(u) \vee \sigma(v)$.

3.AntiIndependent set on Anti Fuzzy Graph

In this section we introduce the concept of anti independent set on anti fuzzy graph. We define the definition of anti independent set and anti independence number on anti fuzzy graph and characterize the limits on them.

Definition 3.1

Two nodes of an anti fuzzy graph are said to be anti independent if there is no strong edge between them.

Definition 3.2

A subset I_A of $V(G_A)$ is said to be an anti independent set of an anti fuzzy graph G_A if any two vertices of I_A are anti independent.

Definition 3.3

An anti independent set I_A is a maximal anti independent set of G_A, if there is no anti independent set I_A' of G_A with $|I_A'|>|I_A|$.

The maximum fuzzy cardinality taken over all maximum anti independent set of G_A is called the anti independence number of G_A and it is denoted by $\alpha(G_A)$ or α_A.

Definition 3.4

An anti covering of an anti fuzzy graph G_A is a subset K_A of $V(G_A)$ such that every edge of G_A has at least one end in K_A.

The maximum fuzzy cardinality taken over all minimum anti covering of anti fuzzy graph G_A is called the anti covering number of G_A and is denoted by β_A (G_A)

Definition 3.5

An edge anti independent set or anti matching of anti fuzzy graph G_A is a subset M of $E(G_A)$ such that no two edges of M are adjacent.

The maximum fuzzy cardinality of maximum edge anti independent set of G_A is called edge anti independence number of anti fuzzy graph G_A and is denoted by α_A' (G_A).

Definition 3.6

An edge anti covering of an anti fuzzy graph G_A is a subset L_A of $E(G_A)$ such that each vertex of G_A is an end of some edge in L_A. The maximum fuzzy cardinality of minimum edge covering of G_A is called edge covering number of anti fuzzy graph G_A and is denoted by β_A' (G_A).

Theorem 3.7

i) An anti fuzzy graph G_A with two adjacent subgraphs has atleast one vertex in anti independent set of G_A.

ii) If I_{A_1} is an anti independent set of a subgraph of anti fuzzy graph G_A then $\alpha_{A_1} \leq \alpha_A$.

Theorem 3.8

A set $I_A \subseteq V(G_A)$ is an anti independent set of anti fuzzy graph G_A if and only if $V(G_A) \backslash I_A$ is an anti covering of anti fuzzy graph G_A.

Proof: Let us consider I_A is an anti independent set of anti fuzzy graph G_A. By the definition of anti independent set, no two vertices adjacent in I_A.

$\Rightarrow$no edges of G_A has both ends in I_A

$\Rightarrow$each edge has at least one end in $V(G_A) \setminus I_A$

$\Rightarrow V(G_A) \setminus I_A$ is an anti covering of anti fuzzy graph G_A.

Theorem 3.9

$\alpha_A + \beta_A = p$, for G_A is uninodal anti fuzzy graph.

Proof: Let us consider G_A is uninodal anti fuzzy graph. Let I_A be a maximum anti independent set and K_A be a minimum anti covering of unimodal anti fuzzy graph G_A. By theorem3.8, we get $V(G_A)\backslash K_A$ is an anti independent set of uninodal anti fuzzy graph G_A.

$\therefore |V\backslash K_A| \leq |I_A|$

$\Rightarrow p- \beta_A \leq \alpha_A$

$\Rightarrow p \leq \alpha_A + \beta_A \rightarrow (1)$

By theorem3.8, I_A is an anti independent set and $V(G_A)\backslash I_A$ is an anti covering of uninodal anti fuzzy graph G_A. Since K_A is a minimum anti covering. We have,

$|K_A| \leq |V(G_A)\backslash I_A|$

$\Rightarrow \beta_A \leq p - \alpha_A$

$\Rightarrow \alpha_A + \beta_A \leq p \rightarrow (2)$

From (1) and (2) $\Rightarrow \alpha_A + \beta_A = p$

Theorem 3.10

For any anti fuzzy graph G_A, with $\delta>0$, $\alpha_A' + \beta_A' \neq p$.

Results

1. $\alpha' + \beta' \neq p$ where $p=\sum_{u\in V} \sigma(u)$.
2. $\alpha_A' + \beta_A' \leq p$ where G_A is anuninodal anti fuzzy graph.
3. $\alpha_A' + \beta_A' \geq p$ where G_A is an e-nodal anti fuzzy graph.
4. For any anti fuzzy graph G_A, $\alpha_A \geq \delta_A$.
5. If G_A is a complete bipartite anti fuzzy graph then $q \leq \alpha_A \times \beta_A$.

Theorem 3.11

Anti independent set of an anti fuzzy graph is a maximal anti independent set if and only if it is minimal dominating set.

Proof: Let us consider G_A be an anti fuzzy graph and I_A be an anti independent set. Suppose, I_A is a maximal anti independent set. Let $u \in V(G_A)\backslash I_A$ then $I_A \cup \{u\}$ is not an anti independent set of anti fuzzy graph G_A. There is a strong edge between u and some vertex in I_A(say v). Therefore, (u,v) is an effective edge.

That is, every vertex in $V(G_A)\backslash I_A$ is adjacent to at least one vertex in I_A. Therefore, I_A is a dominating set.

To prove that I_A is a minimal dominating set.

Let us consider that $I_A' = I_A\backslash\{v\}$ is a dominating set then at least one vertex in $V(G_A)\backslash I_A$ dominates $V(G_A)$ and have strong neighbor to $V(G_A)$ which is contradict to I_A is anti independent set. Hence I_A is a minimal dominating set.

Conversely I_A is a minimal dominating set. To prove that, I_A is a maximal anti independent set of G_A. Suppose I_A is not a maximal anti independent set of anti fuzzy graph G_A then $u \in V(G_A)\backslash I_A$. and $I_A \cup \{u\}$ is a anti independent set of anti fuzzy graph , there is no vertex in I_A are strongly adjacent to u which is contradict to our assumption that I_A is a dominating set. Hence I_A is maximal anti independent set.

Theorem 3.12

For an anti fuzzy graph G_A, $\gamma(G_A) \leq \beta(G_A)$

Theorem 3.13

For an anti fuzzy graph G_A without isolated vertices,γ $(G_A) \leq$ p- α_A where α_A is the anti independence number of anti fuzzy graph G_A.

Proof: Let D be a minimal dominating set with n vertices and γ_A domination number of anti fuzzy graph G_A. i.e., $D=\{v_1,v_2,\ldots, v_\gamma\}$.

To prove that D is maximal anti independent set of anti fuzzy graph G_A. Let $u \in D$ and $N(u)=\{v,w,x\}$. Therefore no one vertex in N(u) is a member of D. Suppose there exists $x \in D$ which is contradict to D as an anti independent set. Then D is a maximal anti independent set. Since G_A has no isolated vertices then $V(G_A) \backslash D$ is a minimal dominating set with $p-\alpha_A$ vertices. Hence $\gamma (G_A) \leq p- \alpha_A$.

Theorem 3.14

Every r-regular bipartite anti fuzzy graph G_A ($r \geq 1$) has a perfect matching.

Example 3.15

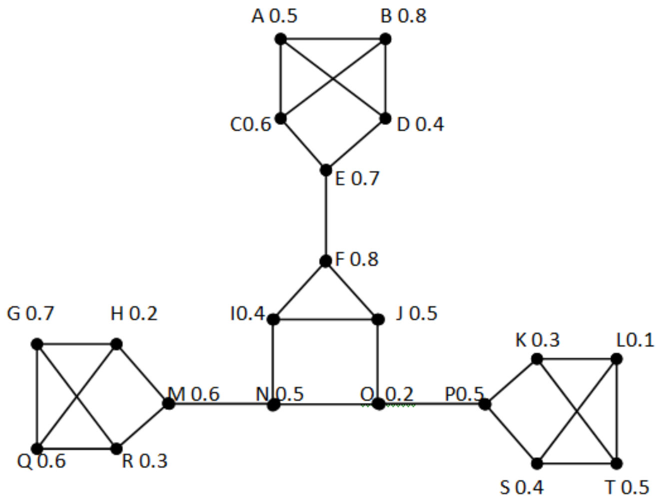

Figure 1. Anti Fuzzy Graph G_A

From Figure 1, consider the anti fuzzy graph as strong anti fuzzy graph. That is, all edges are effective edges. Here p = 9.6

The maximal anti independent set I_A is, $I_A = \{ B,C,F,N,R,H,P,T\}$, $\alpha_A = 4.2$.

The minimum anti covering K_A is, $K_A = \{ A, B, E, F, J, N, Q, G, M, P, S, T \}$, $\beta_A = 7.1$

$\alpha_A + \beta_A = 4.2 + 7.1 = 11.3 \neq p$.

Remark:

1. For any anti fuzzy graph, $\gamma'(G_A) \leq \beta_A' (G_A)$.
2. For any strong anti fuzzy cycle, $\alpha_A(G_A) \leq \beta_A (G_A) \leq \gamma'(G_A)$.
3. For a e-nodal anti fuzzy graph, $\alpha_A'(G_A) \leq \beta_A' (G_A)$.
4.

4. Anti Independent Domination on Anti Fuzzy Graph

Definition 4.1

A dominating set $D \subseteq V(G_A)$ is said to be anti independent dominating set of an anti fuzzy graph G_A if no two vertices are adjacent in D.

An anti independent dominating set D is called a minimal anti independent dominating set if no proper subset of D is an anti independent dominating set.

The maximum fuzzy cardinality taken over all minimal anti independent dominating set in G_A is called its anti independent domination number of G_A and is denoted by $i_A (G_A)$.

Theorem 4.2

Let G_A be an anti fuzzy graph without isolated vertices.

i. If G_A is star then $\gamma(G_A) \leq i(G_A)$
ii. If G_A is cycle then $i(G_A) \leq \gamma(G_A)$

Proof: i) Let G_A be an anti fuzzy star without isolated vertices. Let D be a minimal dominating set and I_A be a maximal anti independent set. By the definition of minimal dominating set, the root vertex is in D. That is $\gamma(G_A) = \sigma(r)$. By the definition of minimal anti independent dominating set, the pendent vertices are in I_A. That is $i(G_A) = \Sigma\sigma(p_i) \geq \gamma(G_A)$.

ii.) Let G_A be an anti fuzzy cycle. Then $\gamma(G_A) \leq 3p/4$. The anti independent dominating set D contains even place vertices or odd place vertices.

$i_A = \max (\sum_{i=1,3} v_i \ or \ \sum_{i=2,4} v_i)$.

Hence $i(G_A) \leq \gamma(G_A)$

Theorem 4.3

For any anti fuzzy graph G_A, $\gamma_A' \leq \alpha_A'$.

Theorem 4.4

If G_A is an anti fuzzy graph of order p without isolated vertices then $i_A(G_A) \leq \frac{p}{2}$.

Remark

1. Dominating set of anti fuzzy path is always an anti independent set.
2. Dominating set of anti fuzzy cycle is always an anti independent set.
3.

5. Conclusion

In this paper, anti independence domination number, anti independent set, anti covering and anti matching is defined on anti fuzzy graph and relation between them is described.

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