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VARIOUS TYPES OF *PWI*-IDEALS OF LATTICE PSEUDO-WAJSBERG ALGEBRAS

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Abstract: In this paper, we introduce the notion of lattice pseudo positive implicative-Wajsberg algebra with illustration. Also, we introduce the definitions of a completely closed *PWI*-ideal, a strong *PWI*-ideal, a P-*PWI*-ideal, and a Q-*PWI*-ideal. Further, we discuss some of their properties and relationship between them.

Keywords: Lattice pseudo-Wajsberg algebras; *PWI*-ideal;Latticepseudo positive implicative-Wajsberg algebras; completely closed *PWI*-ideal; strong *PWI*-ideal;P-*PWI*-ideal;Q-*PWI*-ideal.

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1. Introduction

In 1935, Mordchaj Wajsberg [10] introduced the concept of Wajsberg algebras and studied by Font, Rodriguez and Torrenns [3]. Also, they [3] defined lattice structure of Wajsberg algebras. Further, they [3] introduced the notion of an implicative filter of lattice Wajsberg algebras and discussed some properties. Pseudo-Wajsberg algebras are generalizations of Wajsberg algebras. In 2001, Ceterchi Rodica [1] introduced the concept of pseudo-Wajsberg algebras.Ceterchi Rodica [2] introduced the lattice structure of pseudo-Wajsberg algebras and discussed some results in generalized pseudo-Wajsberg algebras. The authors [4] introduced the notions of *PWI*-ideal, pseudo lattice ideal. Also, the authors [5],[7] introduced the definitions of fuzzy *PWI*-ideal, fuzzy pseudo lattice ideal and an intuitionistic fuzzy *PWI*-ideal and investigated some of their properties.

In the present paper, we introduce the notion of lattice pseudo positive implicative-Wajsberg algebras with illustration and also, we introduce the notions of a completely closed *PWI*-ideal, a strong *PWI*-

ideal, a P-*PWI*-ideal and a Q-*PWI*-ideal and we discuss some of their properties and relationship between them.

2. Preliminaries

In this section, we recall some basic definitions and its properties that are needed for developing the main results.

Definition: 2.1 *[3]*. An algebra $(A,\longrightarrow\ ,-\ ,1)$ with a binary operation " $\longrightarrow$ " and a quasi-complement " $-$ " is called a Wajsberg algebra if it satisfies the following axioms for all $x,y,z\in A$,

(i) $1\longrightarrow x=x$

(ii) $(x\longrightarrow y)\longrightarrow y=(y\longrightarrow x)\longrightarrow x$

(iii) $(x\longrightarrow y)\longrightarrow\big((y\longrightarrow z)\longrightarrow(x\longrightarrow z)\big)=1$

(iv) $(x^{-}\longrightarrow y^{-})\longrightarrow(y\longrightarrow x)=1$.

Definition: 2.2 *[1]*. An algebra $(A,\longrightarrow\ ,\rightsquigarrow,\ -\ ,\sim,1)$ with the binary operations " $\longrightarrow$ " , " $\rightsquigarrow$ " and the quasi complements" $-$ " , "$\sim$" is called a pseudo-Wajsberg algebra if it satisfies the following axioms for all $x,y,z\in A$,

(i) (a) $1\longrightarrow x=x$

(b) $1\rightsquigarrow x=x$

(ii) $(x\rightsquigarrow y)\longrightarrow y=(y\rightsquigarrow x)\longrightarrow x=(y\longrightarrow x)\rightsquigarrow x=(x\longrightarrow y)\rightsquigarrow y$

(iii) (a) $(x\longrightarrow y)\longrightarrow\big((y\longrightarrow z)\rightsquigarrow(x\longrightarrow z)\big)=1$

(b) $(x\rightsquigarrow y)\rightsquigarrow\big((y\rightsquigarrow z)\longrightarrow(x\rightsquigarrow z)\big)=1$

(iv) $1^{-}=1^{\sim}=0$

(v) (a) $(x^{-}\rightsquigarrow y^{-})\longrightarrow(y\longrightarrow x)=1$

(b) $(x^{\sim}\longrightarrow y^{\sim})\rightsquigarrow(y\rightsquigarrow x)=1$

(vi) $(x\longrightarrow y^{-})^{\sim}=(y\rightsquigarrow x^{\sim})^{-}$.

Definition: 2.3*[2]*.An algebra$(A,\ \wedge,\ \vee,\longrightarrow\ ,\rightsquigarrow,\ -\ ,\sim,1)$ is called a lattice pseudo-Wajsberg algebras if it satisfies the following axioms for all $x,y\in A$,

(i) A partial ordering "$\leq$" on a lattice pseudo-Wajsberg algebra A such that $x\leq y$ if and only if $x\longrightarrow y=1\Leftrightarrow x\rightsquigarrow y=1$

(ii) $x\vee y\ =(x\longrightarrow y)\rightsquigarrow y=(y\longrightarrow x)\rightsquigarrow x=(x\rightsquigarrow y)\longrightarrow y=(y\longrightarrow x)\longrightarrow x$

(iii)$x\wedge y=(x\rightsquigarrow(x\longrightarrow y)^{\sim})^{-}=\big((x\longrightarrow y)\longrightarrow x^{-}\big)^{\sim}$

$$=\ (y\longrightarrow(y\rightsquigarrow x)^{-})^{\sim}=\big((y\rightsquigarrow x)\rightsquigarrow y^{\sim}\big)^{-}$$

$$=(y\rightsquigarrow(y\longrightarrow x)^{\sim})^{-}\ =\big((y\longrightarrow x)\longrightarrow y^{-}\big)^{\sim}$$

$$=(x\longrightarrow(x\rightsquigarrow y)^{-})^{\sim}\ =\ \big((x\rightsquigarrow y)\rightsquigarrow x^{\sim}\big)^{-}.$$

Proposition: 2.4*[2]***.**In a lattice pseudo-Wajsberg algebra $(A,\longrightarrow\ ,\rightsquigarrow,\ -\ ,\sim,1)$ which satisfies the following axioms for all $x,y,z\in A$,

(i) (a) $x\longrightarrow x=1$

(b) $x\rightsquigarrow x=1$

(ii) (a) If $x\longrightarrow y=1$ and $y\longrightarrow x=1$, then $x=y$

(b) If $x\rightsquigarrow y=1$ and $y\rightsquigarrow x=1$, then $x=y$

(c) If $x\longrightarrow y=1$ and $y\rightsquigarrow x=1$, then $x=y$

(iii) (a) $(x\longrightarrow 1)\rightsquigarrow 1=1$

(b) $(x\rightsquigarrow 1)\longrightarrow 1=1$

(iv) (a) $x\longrightarrow 1=1$

(b) $x\rightsquigarrow 1=1$

(v) (a) If $x\longrightarrow y=1$ and $y\longrightarrow z=1$, then $x\longrightarrow z=1$

(b) If $x\rightsquigarrow y=1$ and $y\rightsquigarrow z=1$, then $x\rightsquigarrow z=1$

(vi) (a) $x\longrightarrow(y\rightsquigarrow x)=1$

(b) $x\rightsquigarrow(y\longrightarrow x)=1$

(vii) $x\longrightarrow(y\rightsquigarrow z)=1\ \Leftrightarrow y\rightsquigarrow(x\longrightarrow z)=1$

(viii) (a) $(x \longrightarrow y) \rightsquigarrow \big((z \longrightarrow x) \longrightarrow (z \longrightarrow y)\big) = 1$

(b) $(x \rightsquigarrow y) \longrightarrow \big((z \rightsquigarrow x) \rightsquigarrow (z \rightsquigarrow y)\big) = 1$

(ix) $x \longrightarrow (y \rightsquigarrow z) = y \rightsquigarrow (x \longrightarrow z)$.

Proposition: 2.5*[2]***.**In a lattice pseudo-Wajsberg algebra $(A, \longrightarrow, \rightsquigarrow, -, \sim, 1)$ which satisfies the following axioms for all $x, y \in A$,

(i) (a) $(x^{-} \rightsquigarrow 0) \longrightarrow x = 1$ (b) $(x^{\sim} \longrightarrow 0) \rightsquigarrow x = 1$

(ii) $0 \longrightarrow x = 1 = 0 \rightsquigarrow x$

(iii) (a) $x \longrightarrow 0 = x^{-}$(b) $x \rightsquigarrow 0 = x^{\sim}$

(iv) $(x^{-})^{\sim} = x = (x^{\sim})^{-}$

(v) (a) $x^{\sim} \longrightarrow y^{\sim} = y \rightsquigarrow x$ (b) $x^{-} \rightsquigarrow y^{-} = y \longrightarrow x$

(vi) $x^{\sim} \longrightarrow y = y^{-} \rightsquigarrow x$

(vii) $x \le y \Leftrightarrow y^{-} \le x^{-} \Leftrightarrow y^{\sim} \le x^{\sim}$

(viii) (a) $(x \longrightarrow y)^{\sim} = (y^{\sim} \rightsquigarrow x^{\sim})^{-}$ (b) $(x \rightsquigarrow y)^{-} = (y^{-} \longrightarrow x^{-})^{\sim}$.

Definition 2.6*[4]***.** Let A be lattice pseudo-Wajsberg algebra. Let F be non-empty subset of $A$is called a *PWI*- ideal of A if it satisfies the following axioms for all$x, y \in A$,

(i) $0 \in F$

(ii) $y \in F$ and$(x \longrightarrow y)^{-} \in F$ imply $x \in F$

(iii) $y \in F$ and $(x \rightsquigarrow y)^{\sim} \in F$ imply $x \in F$.

3. Main results

3.1 Various typesof PWI-Ideals of a Lattice pseudo-Wajsberg Algebras

In this section, we define a lattice pseudo positive implicative-Wajsberg algebras. Also, we define various types of *PWI*-Ideals and discuss some properties of a lattice pseudo-Wajsberg algebras.

Definition 3.1.1Alattice pseudo-Wajsberg algebra of A is called a lattice pseudo positive implicative if it satisfies the following axioms for all$x, y, z \in A$,

(i) $[(x \longrightarrow z)^{\sim} \rightsquigarrow (y \longrightarrow z)^{\sim}]^{-} = [(x \longrightarrow y)^{\sim} \rightsquigarrow z]^{-}$

(ii) $[(x \rightsquigarrow z)^{-} \longrightarrow (y \rightsquigarrow z)^{-}]^{\sim} = [(x \rightsquigarrow y)^{-} \longrightarrow z]^{\sim}$.

Example 3.1.2.Consider a set $A = \{0, a, b, 1\}$. Define a partial ordering "$\le$"on A, such that $0 \le a \le 1$; $0 \le b \le 1$ and the binary operations "$\longrightarrow$" , "$\rightsquigarrow$" and the quasi complements "$-$" ,"$\sim$" given by the following tables (3.1), (3.2), (3.3) and (3.4).

x	x^{-}
0	1
a	b
b	a
1	0

Table: 3.1 Complement

$\longrightarrow$	0	a	b	1
0	1	1	1	1
a	b	1	1	1
b	a	b	1	1
1	0	a	b	1

Table: 3.2 Implication

x	$x^{\sim}$
0	1
a	b
b	a
1	0

Table: 3.3 Complement

$\rightsquigarrow$	0	a	b	1
0	1	1	1	1
a	b	1	1	1
b	a	a	1	1
1	0	a	b	1

Table: 3.4 Implication

Then A is a lattice pseudo positive implicative-Wajsberg algebra.

Definition 3.1.3.Let A bea lattice pseudo-Wajsberg algebra. Let I be a non-empty subset of A is called a completely closed *PWI*-ideal, if it satisfies the following axioms for all$x, y \in I$,

(i) $0 \in I$

(ii) $(x \longrightarrow y)^{\sim}, (x \rightsquigarrow y)^{-} \in I, y \in I$ implies $x \in I$.

Example 3.1.4.Consider a set $A = \{0, a, b, c, 1\}$. Define a partial ordering " $\leq$ "on A, such that $0 \leq a \leq b \leq c \leq 1$ and the binary operations "$\longrightarrow$" , " $\rightsquigarrow$ " and quasi complements " $-$ " ,"$\sim$" given by the following tables (3.5), (3.6), (3.7) and (3.8).

x	x^-
0	1
a	b
b	a
c	a
1	0

Table: 3.5
Complement

$\longrightarrow$	0	a	b	c	1
0	1	1	1	1	1
a	b	1	1	1	1
b	a	a	1	1	1
c	a	a	b	1	1
1	0	a	b	c	1

Table: 3.6
Implication

x	$x^\sim$
0	1
a	c
b	a
c	b
1	0

Table: 3.7
Complement

$\rightsquigarrow$	0	a	b	c	1
0	1	1	1	1	1
a	c	1	1	1	1
b	a	a	1	1	1
c	b	a	b	1	1
1	0	a	b	c	1

Table: 3.8
Implication

Then, $A = (A, \wedge, \vee, \longrightarrow, \rightsquigarrow, 0,1)$ is a lattice pseudo-Wajsberg algebra and consider the subset$I = \{0,1\}$, then easily verify that I is a completely closed *PWI*-ideal of A. But, $F = \{0, a, 1\}$ is not a pseudo completely closed *PWI*-ideal of A, Since$(1 \longrightarrow a)^\sim = a^\sim = c \notin F$ andalso $(1 \rightsquigarrow a)^- = a^- = b \notin F$.

Definition 3.1.5. Let Abea lattice pseudo-Wajsberg algebra. Let I be a non-empty subset of A is called a strong *PWI*-ideal of A, if it satisfies the following axioms for all$x, y, z \in A$,

(i) $0 \in I$

(ii) $[(x \longrightarrow y)^\sim \rightsquigarrow z]^-$, $y \in I$ implies $(x \longrightarrow z)^\sim \in I$

(iii) $[(x \rightsquigarrow y)^- \longrightarrow z]^\sim$, $y \in I$ implies$(x \rightsquigarrow z)^- \in I$.

Example 3.1.6. Consider a set $A = \{0, a, b, c, 1\}$. Define a partial ordering " $\leq$ "on A, such that$0 \leq a \leq b \leq c \leq 1$ and the binary operations "$\longrightarrow$" , " $\rightsquigarrow$ " and quasi complements " $-$ " ,"$\sim$" given by the following tables (3.9), (3.10), (3.11) and (3.12).

x	x^-
0	1
a	c
b	c
c	b
1	0

Table: 3.9
Complement

$\longrightarrow$	0	a	b	c	1
0	1	1	1	1	1
a	c	1	1	1	1
b	c	c	1	c	1
c	b	b	b	1	1
1	0	a	b	c	1

Table: 3.10
Implication

x	$x^\sim$
0	1
a	b
b	c
c	b
1	0

Table: 3.11
Complement

$\rightsquigarrow$	0	a	b	c	1
0	1	1	1	1	1
a	b	1	1	1	1
b	c	c	1	c	1
c	b	b	b	1	1
1	0	a	b	c	1

Table: 3.12
Implication

Then, $A = (A, \wedge, \vee, \longrightarrow, \rightsquigarrow, 0, 1)$ is a lattice pseudo-Wajsberg algebra and consider the subset $I = \{0, 1\}$, then easily verify that I is a strong *PWI*-ideal of A. But, $F = \{0, a, 1\}$ is not a strong*PWI*-ideal of A.

Since $[(1 \longrightarrow 0)^\sim \rightsquigarrow a]^- = (1 \longrightarrow a)^\sim = b \notin Fand[(1 \rightsquigarrow 0)^- \longrightarrow a]^\sim = (1 \rightsquigarrow a)^- = c \notin F$.

Proposition 3.1.7. Let A be a lattice pseudo-Wajsberg algebra and I be a strong *PWI*-ideal of A, then I is a completely closed *PWI*-ideal of A.

Proof. Let I be a strong *PWI*-ideal of A. Definition 3.1.4 shows that I satisfies (i) of definition 3.1.2.
Let $x, y \in I$ such that $(x \longrightarrow y)^\sim, (x \rightsquigarrow y)^- \in I$ and $y \in I$.
Then $(x \longrightarrow y)^\sim = [(x \longrightarrow y)^\sim \rightsquigarrow 0]^-$ and [from (iii)(b) of proposition 2.5]
$(x \rightsquigarrow y)^- = [(x \rightsquigarrow y)^- \longrightarrow 0]^\sim$ [from (iii)(a) of proposition 2.5]
This implies that,$[(x \longrightarrow y)^\sim \rightsquigarrow 0]^-, [(x \rightsquigarrow y)^- \longrightarrow 0]^\sim \in I$ and $y \in I$
It follows that $(x \longrightarrow 0)^\sim, (x \rightsquigarrow 0)^- \in I$ [from (ii) & (iii) of definition 3.1.4]
Then, we have $(x^-)^\sim, (x^\sim)^- \in I$ implies $x \in I$ [from (iv) of proposition 2.5]
Hence, I is a completely closed *PWI*-ideal of A.■

Remark 3.1.8. The converse of Proposition 3.1.7 may not be true. In Example 3.1.5, $I = \{0,1\}$ is a completely closed *PWI*-ideal of A. But, it is not a strong *PWI*-ideal of lattice pseudo-Wajsberg algebra. Since$[(1 \longrightarrow 0)^{\sim} \rightsquigarrow a]^{-} = (1 \rightsquigarrow a)^{-} = a^{-} = b \notin I,$

$[(1 \rightsquigarrow 0)^{-} \longrightarrow a]^{\sim} = (1 \longrightarrow a)^{\sim} = a^{\sim} = c \notin I.$

Definition 3.1.9.Let Abe lattice pseudo-Wajsberg algebra. Let I be non-empty subset of A is called a P-*PWI*-ideal of A if it satisfies the following axioms for all$x, y, z \in A$,

(i) $0 \in I$

(ii) $[(x \longrightarrow z)^{\sim} \rightsquigarrow (y \longrightarrow z)^{\sim}]^{-} \in I,$ $y \in I$ implies $x \in I$

(iii) $[(x \rightsquigarrow z)^{-} \longrightarrow (y \rightsquigarrow z)^{-}]^{\sim} \in I,$ $y \in I$ implies $x \in I$.

Example3.1.10. In Example 3.1.4, consider the subset $I = \{0, 1\}$ of A,theneasily verify that I is aP-*PWI*-ideal of$A$.But, $F = \{0, b, 1\}$ is not a P-*PWI*-ideal of A,

Since$[(1 \longrightarrow b)^{\sim} \rightsquigarrow (0 \longrightarrow b)^{\sim}]^{-} = [b^{\sim} \rightsquigarrow 1^{\sim}]^{-} = [a \rightsquigarrow 0]^{-} = [c]^{-} = a \notin F$

Also, $[(1 \rightsquigarrow b)^{-} \longrightarrow (0 \rightsquigarrow b)^{-}]^{\sim} = [b^{-} \longrightarrow 1^{-}]^{\sim} = [a \longrightarrow 0]^{\sim} = [b]^{\sim} = a \notin F$.

Proposition 3.1.11. Let A be a lattice pseudo-Wajsberg algebra and I be a P-*PWI*-ideal of A, then I is a completely closed *PWI*-ideal of A.

Proof. Let I be a P-*PWI*-ideal of A. Definition 3.1.7 shows that I satisfies (i) of definition 3.1.2. Let $x, y \in I$ such that $(x \longrightarrow y)^{\sim}, (x \rightsquigarrow y)^{-} \in I$ and $y \in I$.

Then $(x \rightsquigarrow y)^{-} = (y^{-} \longrightarrow x^{-})^{\sim}$ [from (viii)(b) of proposition 2.5]

$= [(y \longrightarrow 0) \longrightarrow (x \longrightarrow 0)]^{\sim}$ [from (iii)(a) of proposition 2.5]

$= [(x \longrightarrow 0)^{\sim} \rightsquigarrow (y \longrightarrow 0)^{\sim}]^{-}$ [from (viii)(a) of proposition 2.5]

And$(x \longrightarrow y)^{\sim} = (y^{\sim} \rightsquigarrow x^{\sim})^{-}$ [from (viii)(a) of proposition 2.5]

$= [(y \rightsquigarrow 0) \rightsquigarrow (x \rightsquigarrow 0)]^{-}$ [from (iii)(b) of proposition 2.5]

$= [(x \rightsquigarrow 0)^{-} \longrightarrow (y \rightsquigarrow 0)^{-}]^{\sim}$ [from (viii)(b) of proposition 2.5]

This implies that, $[(x \longrightarrow 0)^{\sim} \rightsquigarrow (y \longrightarrow 0)^{\sim}]^{-}, [(x \rightsquigarrow 0)^{-} \longrightarrow (y \rightsquigarrow 0)^{-}]^{\sim} \in I$and $y \in I$ imply $x \in I$.

[from (ii) & (iii) of definition 3.1.10]

Hence, I is a completely closed *PWI*-ideal of A. ■

Definition 3.1.12. Let A bea lattice pseudo-Wajsberg algebra. Let I be non-empty subset of $A$is called a Q-*PWI*-ideal of A if it satisfies the following axioms for all$x, y, z \in A$,

(i) $0 \in I$

(ii) $[x \longrightarrow (y \rightsquigarrow z)^{-}]^{\sim} \in I$, $y \in I$ implies$(x \longrightarrow z)^{\sim} \in I$

(iii) $[x \rightsquigarrow (y \longrightarrow z)^{\sim}]^{-} \in I$, $y \in I$ implies $(x \rightsquigarrow z)^{-} \in I$.

Example3.1.13. In Example 3.1.6, consider the subset $I = \{0, 1\}$ of$A$, theneasily verify that I is a Q-*PWI*-ideal of A.But, $F = \{0, a, 1\}$ is not a Q-*PWI*-ideal of A.

Since$[1 \longrightarrow (1 \rightsquigarrow a)^{-}]^{\sim} = [1 \longrightarrow a^{-}]^{\sim} = [1 \longrightarrow c]^{\sim} = [c]^{\sim} = b \notin F$ and

$[1 \rightsquigarrow (1 \longrightarrow a)^{\sim}]^{-} = [1 \rightsquigarrow a^{\sim}]^{-} = [1 \rightsquigarrow b]^{-} = [b]^{-} = c \notin F$.

Proposition 3.1.14. Let A be lattice pseudo-Wajsberg algebra and I be a Q-*PWI*-ideal of A, then I is a completely closed *PWI*-idealof A.

Proof. Let$I$ be a Q-*PWI*-ideal of A. Definition 3.1.10 shows that I satisfies (i) of definition 3.1.2. Let $x, y \in I$ such that $(x \longrightarrow y)^{\sim}$, $(x \rightsquigarrow y)^{-} \in I$ and $y \in I$.

Then $(x \longrightarrow y)^{\sim} = [x \longrightarrow (y \rightsquigarrow 0)^{-}]^{\sim}$ [from (iii)(b) of proposition 2.5]

$= (x \longrightarrow 0)^{\sim} \in I$ and $y \in I$ [from (ii) of definition 3.1.12]

$= (x^{-})^{\sim} = x \in I$ [from (iv) of proposition 2.5]

And $(x \rightsquigarrow y)^{-} = [x \rightsquigarrow (y \longrightarrow 0)^{\sim}]^{-}$ [from (iii)(a) of proposition 2.5]

$= (x \rightsquigarrow 0)^{-} \in I$and $y \in I$ [from (iii) of definition 3.1.12]

$= (x^{\sim})^{-} = x \in I$ [from (iv) of proposition 2.5]

Hence, I is a completely closed *PWI*-ideal of A. ■

Remark 3.1.15. The converse of Proposition 3.1.14 may not be true. In Example 3.1.4, $I = \{0,1\}$ is a completely closed *PWI*-ideal of A. But, it is not a Q-*PWI*-ideal of lattice pseudo-Wajsberg algebra.
Since, $[1 \longrightarrow (1 \rightsquigarrow c)^{-}]^{\sim} = [1 \longrightarrow c^{-}]^{\sim} = [1 \longrightarrow a]^{\sim} = [a]^{\sim} = c \notin I$ and
$[1 \rightsquigarrow (1 \longrightarrow c)^{\sim}]^{-} = [1 \rightsquigarrow c^{\sim}]^{-} = [1 \rightsquigarrow b]^{-} = [b]^{-} = a \notin I$.

Proposition 3.1.16. Let$A$ be lattice pseudo positive implicative Wajsberg algebra such that$x = (x \longrightarrow z)^{\sim}, x = (x \rightsquigarrow z)^{-}$ for all $x, y, z \in A$, then I is a strong *PWI*-ideal if and only if it is a P-*PWI*-ideal of A.
Proof. Let$I$ be a strong *PWI*-ideal of A and $[(x \longrightarrow z)^{\sim} \rightsquigarrow (y \longrightarrow z)^{\sim}]^{-} \in I$ and $y \in I$ for all $x, y, z \in A$, then from (i) of definition 3.1.1, we get $[(x \longrightarrow y)^{\sim} \rightsquigarrow z]^{-}$ and $y \in I$. Since I is a strong *PWI*-ideal of A. Then $(x \longrightarrow z)^{\sim} \in I$. By assumption, we have $x \in I$. Similarly$[(x \rightsquigarrow z)^{-} \longrightarrow (y \rightsquigarrow z)^{-}]^{\sim}$ and $y \in I$ then from (ii) of definition 3.1.1, we get$[(x \rightsquigarrow y)^{-} \longrightarrow z]^{\sim}$and $y \in I$. Since I is a strong *PWI*-ideal of A.Then $(x \rightsquigarrow z)^{-} \in I$. By assumption, we have $x \in I$, then I is a P-*PWI*-ideal of A.
Conversely, let $[(x \longrightarrow y)^{\sim} \rightsquigarrow z]^{-}$and $y \in I$, then from (i) of definition 3.1.1, we get$[(x \longrightarrow z)^{\sim} \rightsquigarrow (y \longrightarrow z)^{\sim}]^{-} \in I$and $y \in I$. Since I is aP-*PWI*-ideal of A, then $x \in I$. By assumption $x = (x \longrightarrow z)^{\sim}$, so $y \in I (x \longrightarrow z)^{\sim} \in I$. Similarly, $[(x \rightsquigarrow y)^{-} \longrightarrow z]^{\sim}$ and $y \in I$implies$(x \rightsquigarrow z)^{-} \in I$. Hence, I is a strong *PWI*-ideal of A. ■

Proposition 3.1.17. Let$A$ be lattice pseudo positive implicative Wajsberg algebra such that $x = (x \longrightarrow z)^{\sim} = (x \rightsquigarrow z)^{-}$for all $x, y, z \in A$,then I is a strong *PWI*-ideal if and only if it is a Q-*PWI*-ideal of A.
Proof. Let$I$ be a strong *PWI*-ideal of A and $[x \longrightarrow (y \rightsquigarrow z)^{-}]^{\sim} \in I, y \in I$ for all $x, y, z \in A$. By assumption $x = (x \rightsquigarrow z)^{-}$ we get $[(x \rightsquigarrow z)^{-} \longrightarrow (y \rightsquigarrow z)^{-}]^{\sim}$ and $y \in I$, then from (ii) of definition 3.1.1, we have $[(x \rightsquigarrow y)^{-} \longrightarrow z]^{\sim} \in I, y \in I$ since I is a strong *PWI*-ideal of $A, (x \rightsquigarrow z)^{-} \in I$ by assumption $[(x \longrightarrow z)^{\sim} = (x \rightsquigarrow z)^{-}]$ implies $(x \longrightarrow z)^{\sim} \in I$. Similarly,$[x \rightsquigarrow (y \longrightarrow z)^{\sim}]^{-} \in I$ and y $\in I$implies $(x \rightsquigarrow z)^{-} \in I$.Hence I is aQ-*PWI*-ideal of A.
Conversely,[$(x \longrightarrow y)^{\sim} \rightsquigarrow$ z$]^{-}$and y $\in I$, then from (i) of definition 3.1.1, we get$[(x \longrightarrow z)^{\sim} \rightsquigarrow (y \longrightarrow z)^{\sim}]^{-} \in I$ and y $\in I$.By assumption $x = (x \longrightarrow z)^{\sim}$implies$[x \longrightarrow (y \rightsquigarrow z)^{-}]^{\sim} \in I$ and y $\in I$, since I is aQ-*PWI*-ideal of A. We get, $(x \rightsquigarrow z)^{-} \in I$. By assumption $[(x \rightsquigarrow z)^{-} = (x \longrightarrow z)^{\sim}]$, we have $(x \longrightarrow z)^{\sim} \in I$. Similarly, $[(x \rightsquigarrow y)^{-} \longrightarrow$ z$]^{\sim}$, y $\in I$ implies $(x \rightsquigarrow z)^{-} \in I$.Hence, I is astrong *PWI*-ideal of$A$. ■

Proposition 3.1.18. Let A be lattice pseudo-Wajsberg algebra such that $x = (x \rightsquigarrow z)^{-} = (x \longrightarrow z)^{\sim}$ for all $x, y, z \in A$ and I be a P-*PWI*-ideal of A, then I is a Q-*PWI*-ideal of A.
Proof. Let $[x \longrightarrow (y \rightsquigarrow z)^{-}]^{\sim} \in I$, $y \in I$. By assumption $[x = (x \rightsquigarrow z)^{-}]$, we get$[(x \rightsquigarrow z)^{-} \longrightarrow (y \rightsquigarrow z)^{-}]^{\sim} \in I, y \in I$. Since $I$is a P-*PWI*-ideal of A, so that $x \in I$, by assumption $[x = (x \longrightarrow z)^{\sim}]$. Similarly, $[x \rightsquigarrow (y \longrightarrow z)^{\sim}]^{-} \in I, y \in I$ then, we get$[(x \longrightarrow z)^{\sim} \rightsquigarrow (y \longrightarrow z)^{\sim}]^{-} \in I$and $y \in I$since, $I$is a P-*PWI*-ideal of A, so that $x \in I$. Hence I isa Q-*PWI*-ideal of A. ■

4. Conclusion

In the present paper, we have introduced the notion of lattice pseudo positive implicative-Wajsberg algebras with illustration and also, introduced the notions of a completely closed *PWI*-ideal, a strong *PWI*-ideal, a P-*PWI*-ideal and a Q-*PWI*-ideal and we discussed some of their properties and relationship between them with appropriate illustrations.

References

[1] Ceterchi Rodica., *Pseudo-Wajsberg Algebras*, Multi Valued Logic, Volume 6, (2001), 67-88.
[2] Ceterchi Rodica., *The Lattice Structure of Pseudo-Wajsberg Algebras*, Journal of Universal Computer Science, Volume.6, Number.1, (2000), 22-38.

[3] Font, J. M., Rodriguez, A. J., and Torrens, A., *Wajsberg algebras,* STOCHASTICA, Volume 8, Number 1, (1984), 5-31.

[4] Ibrahim, A., Indhumathi, M., *PWI-Ideals of Lattice Pseudo-Wajsberg algebras,* Advances in Theoretical and Applied Mathematics, Volume 13, Number 1, (2018), 1-14.

[5] Ibrahim, A., Indhumathi, M., *Fuzzy PWI-Ideals of Lattice Pseudo-Wajsberg algebras*, International Journal of Mathematics and its Applications, Volume 6, Number 4, (2018), 21-31.

[6] Ibrahim, A., Indhumathi, M., *Anti-fuzzy PWI-Ideals of Lattice Pseudo-Wajsberg algebras*, Journal of Computer and Mathematical Sciences , Volume 9, Issue 8, (2018), 903-913.

[7] Ibrahim, A., Indhumathi, M., *Intuitionistic fuzzy PWI-Ideals of Lattice Pseudo-Wajsberg algebras*, Journal of Applied Science and Computations , Volume 5, Issue 10, (2018), 1271-1282.

[8] Ibrahim, A., Indhumathi, M., *P-Ideals of Lattice Pseudo-Wajsberg algebras*, MantechPublication, Journal of Engineering Mathematics and Statistics , Volume 3, Issue 1, (2018), 1-10.

[9] Ibrahim, A., Indhumathi, M., Classes of *p*-ideals of Lattice pseudo-Wajsberg Algebras,International Journal of Research in Advent Technology, Volume 7, Issue 5, (2019), 172-179.

[10] Wajsberg, M., *Beiträge zum Metaaussagenkalkül I*, Monat. Mat. Phys. 42, (1935), 221-242.