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APPLICATION OF INTUITIONISTIC MULTI-FUZZY SET

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Abstract: In this paper, the efficient approach for solving multi-criteria decision making problem using intuitionistic multi-fuzzy set (IMFS) is proposed. The main advantage of the intuitionistic fuzzy sets (IFS) is their property to cope with the hesitancy that may exist due to the lack of information.

Keywords: Intutionistic multi-fuzzy set, new normalized hamming distance, similarity measure based on new normalized hamming distance.

1. Introduction

In this paper,the efficient approach for solving multi-criteria decision making problem using intuitionistic multi-fuzzy set (IMFS) is proposed.The main advantage of the intuitionisticfuzzy sets (IFS)is their property to cope with the hesitancy that may exist due to the lack of information. This is achieved by incorporating a second function, along with the membership function of the conventional fuzzy sets, called non-membership function. In this way, apart from the degree of the belongingness, the intuitionistic fuzzy sets also combine the notation of the non-belongingness in order to better describe the real status of the information.The main advantage of multi-fuzzy set (MFS) is their property to assign multi-membership function to a point in the given set. Multi-fuzzy set is used to handle real life problems with multi-characteristic properties. The combination of multi-fuzzy set (MFS) and intuitionistic fuzzy set (IFS) is intuitionistic multi-fuzzy set (IMFS). Here membership function and non membership function are multi-dimensional. The aim of this chapter is to apply the concept of IMFS to multi-criteria decision making problems.

In this paper, an exclusive model to determine the performance of the students using new normalized Hamming distance measure, score function and accuracy function are made.

2. Basic concepts

In this section, the concept of intuitionistic multi-fuzzy set is reviewed.The New Normalized Hamming distance between two IMFS and the similarity measure between two IMFS based on new normalized Hamming measure is introduced.

Definition 2.1 Let G = {⟨x, A(x), B(x)⟩ : x∈ X} where A(x) = (A_1(x), A_2(x),….., A_k(x)) and B(x) = (B_1(x), B_2(x),….., B_k(x)) such that $0 \le A_i(x) + B_i(x) \le 1, \forall\ x \in X$, A_i: X→[0,1] and B_i(x): X→[0,1] for all i = 1,2,3,……,k. Here, $A_1(x) \ge A_2(x) \ge \ldots \ge A_k(x)$, for all x∈ X.
That is, A_i(x)'s are decreasingly ordered sequence. Then the set G is said to be an intuitionistic multi-fuzzy set (IMFS) with dimension k of the set X.

Furthermore, we have $\pi G_i(x) = 1 - A_i(x) - B_i(x)$ for all i and $\pi G_i(x)$ is called the index of an IMFS or hesitation margin of x in G. $\pi G_i(x)$ is the degree of indeterminacy of x∈ X to the IMFS G. That is, $\pi G_i(x)$: X →[0,1] and $0 \le \pi G_i(x) \le 1$, for all i = 1,2,3,……,k.$\pi G_i(x)$ expresses the lack of knowledge of whether x belongs to IMFS A or not.

Remark: Note that since the membership sequence is in decreasing order, the corresponding non-membership sequence may not be in decreasing or increasing order.

Example 2.2 Let G be an IMFS of dimension two with A(x) = (0.6, 0.7) and B(x) = (0.3, 0.1) which implies that,

$$\pi G(x) = (1 - A_1(x) - B_1(x),\ 1 - A_2(x) - B_2(x)) = (1-0.6-0.3,\ 1-0.7-0.1) = (0.1,\ 0.2).$$

It can be interpreted as "the degree that the object x belongs to IMFS is (0.6, 0.7), the degree that the object x does not belongs to IMFS A is (0.3, 0.1) and the degree of hesitancy of x is (0.1, 0.2)".

Definition 2.3. The cardinality of membership and the non-membership function is the length of an element x in an IMFS G and is denoted by η(G). The cardinality of an IMFS G is defined as η(G) = |A(x)| = |B(x)|. If G, H and I are the IMFSs defined on X, then their cardinality is given by η = max{η(G), η(H), η(I)}.

Definition 2.4. Let X be non-empty set such that intuitionistic multi-fuzzy sets G, H, I ∈ X. Then distance measured is a mapping d: X × X →[0, 1] if d(G,H) satisfies the following axioms:

i. $0 \le d(G, H) \le 1$
ii. $d(G, H) = 0 \Leftrightarrow G = H$
iii. $d(G, H) = d(H, G)$
iv. $d(G, H) + d(H, I) \ge d(G, I)$
v. If $G \subseteq H \subseteq I$, then $d(G, I) \ge d(G, H)$ and $d(G,I) \ge d(H,I)$.

Then d(G,H) is a distance measure between IMFSs G and H.

Definition 2.5. Let G = {⟨x, A(x), B(x)⟩ : x∈ X} and H = {⟨x, C(x), D(x)⟩ : x∈ X} be two intuitionistic multi-fuzzy sets where A(x) = (A_1(x), A_2(x),….., A_k(x)) and B(x) = (B_1(x), B_2(x),….., B_k(x)), C(x) = (C_1(x), C_2(x),….., C_k(x)) and D(x) = (D_1(x), D_2(x),….., D_k(x)) such that $0 \le A_i(x) + B_i(x) \le 1$, $0 \le C_i(x) + D_i(x) \le 1$ for all x∈X, X denotes the set of all multi-criteria and η is the cardinality of the IMFSs G and H.

The new normalized Hamming distance measure formula for G and H is given by:

$$d^{new}_{H}(G,H) = \frac{1}{2\eta k}\sum_{i=1}^{\eta}\sum_{j=1}^{k}\left[\begin{array}{l}|A_i(x_j) - C_i(x_j)| + \left\| A_i(x_j) - B_i(x_j)| - |C_i(x_j) - D_i(x_j)\right\| \\ \qquad + \left\| A_i(x_j) - \pi G_i(x_j)| - |C_i(x_j) - \pi H_i(x_j)\right\|\end{array}\right]$$

Definition 2.6. Let S: X × X → [0, 1] be a map. Then S(A, B) is said to be the similarity measure between A and B where A, B ∈ X and X is an IMFS if S(A, B) satisfies the following properties:

i. S(A, B) $\in$[0,1].
ii. S(A, B) = 1 if and only if A = B.
iii. S(A, B) = S(B, A).
iv. If A,B,C$\in$X such that A $\subseteq$ B $\subseteq$ C, then S(A,C) ≤ S(A,B);
v. S(A,C) ≤ S(B,C).
vi. S(A, B) = 0 if and only if A = ϕ and B = A^C or A = B^C and B = ϕ.

Definition 2.7. The Similarity measure S(G, H) between two IMFS's G and H based on the new normalized Hamming distance is defined as $S(G,H) = 1 - d_H^{new}(G,H)$.

Definition 2.8. For a fixed x$\in$X, an object {A_i(x), B_i(x)} where i =1,2,…,k and k > 0, is usually called an intuitionistic multi-fuzzy value (IMFV) or intuitionistic multi-fuzzy number (IMFN).

Definition 2.9. If x = {A_i(x), B_i(x)} is an IMFV where A(x) = (A_1(x), A_2(x),..….., A_k(x)) and B(x) = (B_1(x), B_2(x),….., B_k(x)), then the score function (or net multi-membership) for the IMFS G of dimension k is defined as $S(x) = \sum_{i=1}^{k} \left(\mu_{A_i}(x) - \nu_{A_i}(x) \right)$.

In addition to the score function, the accuracy function for the IMFS G of dimension k is defined as $H(x) = \sum_{i=1}^{k} \left(\mu_{A_i}(x) + \nu_{A_i}(x) \right)$.

Definition 2.10. Let G be an IMFS of dimension k. Ifx and yareIMFV's where x = {A_i(x), B_i(x)} and y = {A_i(y), B_i(y)}, then x and y can be compared as follows:

1. If S(x) > S(y), then x > y;
2. If S(x) = S(y), then
   (i) if H(x) = H(y), then x = y;
   (ii) if H(x) < H(y), then x < y.
   (iii)

Definition 2.11. The intuitionistic multi-fuzzy choice values $\left(A_i^c, B_i^c\right)$ are computed as: $A_i^c = \sum_{j=1}^{n} A_i(x_j)$ and $B_i^c = \sum_{j=1}^{n} B_i(x_j)$ for i=1,2,…,k. Here, n is the number of criteria in X and k is the dimension of IMFS. As per comparisons of IMFV's, the maximum score value gives the optimal alternatives.

3.Application of IMFS to determine the performance of the students

In this section, an exclusive model to determine the performance of the students using new normalized Hamming distance measure, score function, and accuracy function are made. The performance is measured by calculating the smallest distance between each student and each subject performance.

3.1 Illustrative Example

Student performance evaluation means examination of a student's performance in studies, sports and behaviour in general by a teacher or student counsellor. It is very important to conduct evaluation of the student's performance because it helps him to get a perspective in life and he can make a plan on how to prioritize his studies, find out where he needs to put in more effort and how to make a schedule. The comments of the evaluator are supposed to help him to do better and that way he can be a better student. It also makes communication between the student and teacher clearer.

Let S = {S_1, S_2, S_3,S_4, S_5} be the set of students, and Subjects C = {English(ENG), Environmental Science (ES), Engineering Mathematics(EM), Engineering chemistry (EC),Engineering Drawing (ED), Engineering Mechanics (E.Mech)}be the set of multi-criteria. For each subject, three internal test marks mark is considered. For each internal test mark, the membership function (the marks of the internal examination based on the number of correct answers), non-membership function (the marks of the internal examinations based on the number of incorrect answers) and hesitation function (the marks allocated to the questions which the student did not attempt) is considered. The bench mark for each subject and each internal test is given by the institution.

Table 1 Bench mark for each subject and each internal test

Criteria					
ENG	ES	EM	EC	ED	E.MECH
(0.8,0.8,0.7) (0.2,0.1,0.2) (0.0,0.1,0.1)	(0.7,0.7,0.8) (0.2,0.1,0.1) (0.1,02,0.1)	(0.7,0.6,0.9) (0.1,0.2,0.1) (0.2,0.2,0.0)	(0.8,0.7,0.6) (0.0,0.1,0.2) (0.2,0.2,0.2)	(0.6.0.6,0.7) (0.3,0.1,0.2) (0.1,0.3,0.1)	(0.7,0.6,0.8) (0.1,0.3,0.1) (0.2,0.1,0.1)

The following table show the students marks in each internal test in each subject**.** The performances of each students on each criteria is described the three dimensional IMFSs. That is, membership degree A_i(marks based on the correct answer), non-membership degree B_i (marks based on incorrect answer) and corresponding hesitation margin $\pi G_i(x)$(marks allotted the question which the student did not attempt) for i = 1, 2, 3 and for each student is given below.

Table 2 Each students mark in each subject

	Criteria					
	ENG	ES	EM	EC	ED	E.MECH
S1	(0.3,0.7,0.5) (0.2,0.1,0.4) (0.5,0.2,0.1)	(0.4,0.3,0.5) (0.5,0.4,0.4) (0.1,03,0.1)	(0.7,0.6,0.7) (0.3,0.2,0.1) (0.0,0.2,0.2)	(0.5,0.4,0.5) (0.4,0.4,0.3) (0.1,0.2,0.2)	(0.6.0.2,0.7) (0.3,0.4,0.2) (0.1,0.4,0.1)	(0.6,0.6,0.8) (0.2,0.3,0.1) (0.2,0.1,0.1)
S2	(0.4,0.3,0.4) (0.3,0.6,0.4) (0.3,0.1,0.2)	(0.7,0.6,0.8) (0.2,0.2,0.1) (0.1,0.2,0.1)	(0.3,0.2,0.1) (0.6,0.0,0.7) (0.1,0.8,0.2)	(0.4,0.3,0.4) (0.5,0.3,0.5) (0.1,0.4,0.1)	(0.8,0.3,0.4) (0.1,0.5,0.4) (0.1,0.2,0.2)	(0.4,0.3,0.3) (0.5,0.1,0.2) (0.1,0.6,0.5)
S3	(0.6,0.3,0.5) (0.3,0.3,0.2) (0.1,0.4,0.3)	(0.4,0.3,0.3) (0.4,0.2,0.4) (0.2,0.5,0.3)	(0.7,0.4,0.3) (0.3,0.2,0.4) (0.0,0.4,0.3)	(0.5,0.4,0.2) (0.1,0.3,0.3) (0.4,0.3,0.5)	(0.4,0.3,0.1) (0.4,0.0,0.6) (0.2,0.7,0.3)	(0.7,0.6,0.5) (0.1,0.4,0.4) (0.2,0.0,0.1)
S4	(0.5,0.4,0.3) (0.4,0.3,0.7) (0.1,0.3,0.0)	(0.6,0.4,0.7) (0.0,0.5,0.3) (0.4,0.1,0.0)	(0.5,0.5,0.4) (0.4,0.3,0.0) (0.1,0.2,0.6)	(0.5,0.6,0.4) (0.4,0.3,0.5) (0.1,0.1,0.1)	(0.7,0.6,0.7) (0.3,0.3,0.0) (0.0,0.,10.3)	(0.6,0.6,07) (0.3,0.3,0.3) (0.1,0.1,0.0)
S5	(0.7,0.6,0.7) (0.2,0.3,0.1) (0.1,0.1,0.2)	(0.6,0.4,0.6) (0.3,0.4,0.0) (0.1,0.2,0.4)	(0.5,0.5,0.7) (0.0,0.4,0.3) (0.5,0.1,0.0)	(0.7,0.6,0.6) (0.2,0.3,0.0) (0.1,0.1,0.4)	(0.7,0.3,0.7) (0.2,0.4,0.3) (0.1,0.3,0.0)	(0.7,0.5,0.4) (0.3,0.4,0.6) (0.0,0.1,0.00

3.2 Solution by new normalized Hamming distance measure for IMFS

In this section, the new normalized Hamming distance measure for IMFS is used to determine the performance of the students.

The following algorithm is constructed to solve the above MCDM problem of determining the performance of the students using new normalized Hamming distance measure.

Step 1. Calculate the distance between the bench mark given by the institution and the marks obtained by the each students using new normalized Hamming distance measure.

Step 2. Find the minimum distance between them which gives the solution to the above MCDM problem.

Table below shows the final calculation of new Hamming distance measure between the bench mark given by the institution and the marks obtained by the each students, by using the definition, the new normalized Hamming distance measure of IMFS.

Table 3 The distance between the bench mark and each students mark

S_1	S_2	S_3	S_4	S_5
0.3806	0.3972	0.3639	0.3000	**0.2556**

From the above table, the minimum distance gives the optimal solution to the given MCDM problem. The new normalized Hamming distance between the bench mark given by the institution and the marks obtained by the student S_5 is minimum. Hence, it is identified that the students S_5 perform well in the examination.

The new normalized Hamming distance between the bench mark given by the institution and the marks obtained by the student S_2 is maximum. Hence, it is identified that for the student S_2 , more attention is needed.

3.2.1 Conclusion of Solution by new normalized Hamming distance measure for IMFS

In 3.2, a method of solving multi-criteria decision making problems in intuitionistic multi-fuzzy environment is used. From the above said discussion, thebench mark given by the institution and the marks obtained by the students, the identification of the students who performed well in the examination and the students who needed more attention is made.

3.3 Solution based on score value and accuracy values of IMFS

In this section, the score function and accuracy values of IMFS is used to determine the student's performance.

The following algorithm is constructed to solve the above MCDM problem of selection of best basketball player using score function and accuracy value.

Step 1. Calculate the individual choice values of each student mark and each subject mark.

Step 2. Calculate the corresponding Score values and Accuracy values for each student mark and each subject mark.

Step 3. Choose the maximum score values each students mark and each subject mark which gives that student performed well and the mark in the subject is in the examination.

Step 4. Calculate the difference between the score values of each students mark and subject mark with sign.

Step 5. Analyze the table of Score difference which will give the optimal solution to the given MCDM problem. In the case of difficulties, check only the hesitation difference between the particular alternatives and then analyze we get the optimal decision for the given MCDM problem.

The following table 4, gives the calculations of choice value, score function (or net membership) and accuracy function of students mark vs subjects. Here, we take into consider only the membership and non-membership values

Table 4 the choice value, score function and accuracy value for each student

	Choice value	Score value(A_i)	Accuracy value
S_1	(3.1,2.8,3.7) (1.9,1.8,1.5)	4.4	14.8
S_2	(3.0,2.0,2.4) (2.2,1.7,2.3)	1.2	13.6
S_3	(3.3,2.3,1.9) (1.6,1.4,2.3)	2.2	12.8

S_4	(3.4,3.1,3.2) (1.8,2.0,1.8)	4.1	15.3
S_5	(3.9,2.9,3.7) (1.2,2.2,1.3)	**5.8**	15.2

From the above table 4, the score value of S_5 is maximum when compared to other score values of the remaining students. It gives that the student S_5 is the student who perform well among the students.

The following table 5, gives the calculations of choice value, score function (or net membership) and accuracy function of each subject mark. Here, we take into consider only the membership and non-membership values.

Table 5 the choice value, score value and accuracy value for each subject

	Choice value	Score value (B_i)	Accuracy value
	(2.5,2.3,2.4) (1.4,1.6,1.8)	2.4	12.0
ES	(2.7,2.0,2.9) (1.4,1.7,1.2)	3.3	11.9
EM	(2.7,2.2,2.2) (1.6,1.1,1.5)	2.9	11.3
EC	(2.6,2.3,2.1) (1.6,1.6,1.6)	2.2	11.8
ED	(3.2,1.7,2.6) (1.3,1.6,1.5)	3.1	11.9
E.MECH	(3.0,2.6,2.7) (1.4,1.5,1.6)	**3.8**	12.8

From the above table, the score function for the subject E.Mech is maximum when compared to other score values of the remaining subjects. It gives that students perform well in the subject E.Mech.

The table below shows the calculation of difference between the score values of each students mark and each subject mark.

Table 6 the score difference between score values of each student and score values of each subject

Score difference ($A_i - B_i$)	Subject					
	ENG	**ES**	**EM**	**EC**	**ED**	**E.MECH**
S_1	4.4 – 2.4 = 2.0	4.4 – 3.3 = 1.1	4.4 – 2.9 = 1.5	4.4 – 2.2 = 2.2	4.4 – 3.1 = 1.3	4.4 – 3.8 = 0.6
S_2	1.2 – 2.4 = –1.2	1.2 – 3.3 = – 2.1	1.2 – 2.9 = –1.7	1.2 – 2.2 = –1.0	1.2 – 3.1 = – 1.9	1.2 – 3.8 = –2.6
S_3	2.2 – 2.4 = – 0.2	2.2 – 3.3 = –1.1	2.2 – 2.9 = –0.7	2.2 – 2.2 = 0.0	2.2 – 3.1 = – 0.9	2.2 –3.8 = – 1.6
S_4	4.1 – 2.4 = 1.7	4.1 – 3.3 = 0.8	4.1– 2.9 = 1.2	4.1 – 2.2 = 1.9	4.1– 3.1 = 1.0	4.1 –3.8 = 0.3
S_5	5.8 – 2.4 = 3.4	5.8 – 3.3 = 2.5	5.8 – 2.9 = 2.9	5.8 – 2.2 = 3.6	5.8 – 3.1 = 2.7	5.8 –3.8 = 2.0

From the above table 6, if the score difference between the students mark and each mark is negative, then score value of each subject mark is higher than the students mark. Then it is identified that the students S_5 perform well in all the subjects because the score difference for the student S_5 is positive for the all the subjects. Also it is identified that for the student S_2, more attention is needed because score difference for the students S_2 is negative for the all the subjects.

3.3.1 Conclusion of Solution by new normalized Hamming distance measure for IMFS

In 3.3, a methods of solving multi-criteria decision making problems in intuitionistic multi-fuzzy environment is used. From the above said discussion, the identification of the students who performed well in the examination and the students who needed more attention subject wise is made.

4. Conclusion

In this paper, the solution of MCDM problem was discussed using new normalized Hamming distance measure,score value and accuracy value.

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