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INTUITIONISTIC FUZZY STRONG IMPLICATIVE FILTERS OF LATTICE WAJSBERG ALGEBRAS

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Abstract: In this paper, we introduce the notion of an intuitionistic fuzzy strong implicative filter of lattice Wajsberg algebra. Also, we investigate some properties with illustrations. Further, we obtain the relation between an intuitionistic fuzzy implicative filter and anti intuitionistic fuzzy strong implicative filter in lattice Wajsberg algebra. Finally, we establish the equivalent condition of an intuitionistic fuzzy strong implicative filter.

Keywords: Wajsberg algebra; Lattice Wajsberg algebra; Implicative filter; Strong Implicative filter; Fuzzy implicative filter; Fuzzy Strong implicative filter; Intuitionistic fuzzy implicative filter; Intuitionistic fuzzy strong implicative filter.

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1. Introduction

The concept of the fuzzy set was introduced by Zadeh [10] in 1965. After the introduction of the concept of fuzzy sets by Zadeh, several researches were conducted on the generalizations of the notion of fuzzy sets. The concept of intuitionistic fuzzy sets was first introduced by Atanassov [1, 2] in 1986 which is a generalization of the fuzzy sets. Mordchaj Wajsbreg [9] introduced the concept of Wajsberg algebras in 1935 and studied by Font, Rodriguez and Torrens in [5]. They [5] defined lattice structure of Wajsberg algebras and also, they introduced the notion of an implicative filter of lattice Wajsberg algebras and discussed some of their properties. Basheer Ahamed and Ibrahim [3, 4] introduced the definitions of fuzzy implicative filter and an anti fuzzy implicative filter of lattice Wajsberg algebras and obtained some properties with illustrations. The authors [6, 7, 8] introduced the notions of strong implicative, fuzzy strong implicative, an anti fuzzy strong implicative, an intuitionistic fuzzy implicative and an intuitionistic anti fuzzy implicative filters of lattice Wajsberg algebra, and investigated some properties of them.

In this paper, we introduce the notion of an intuitionistic fuzzy strong implicative filter of lattice Wajsberg algebra. We establish the intuitionistic fuzzification of the concept of strong implicative filters in Wajsberg algebras, and investigate some of their properties. We obtain the relation between an intuitionistic fuzzy strong implicative filter and an intuitionistic fuzzy implicative filter.

2. Preliminaries

In this section, we recall some basic definitions and properties which are useful to develop the main results.

Definition 2.1. *[5]* Let $(A, \rightarrow, *, 1)$ be an algebra with a binary operation "$\rightarrow$" and a quasi complement "$*$" is called aWajsberg algebra if and only if it satisfies the following axioms for all $x, y, z \in A$,

(i) $1 \rightarrow x = x$

(ii) $(x \rightarrow y) \rightarrow ((y \rightarrow z) \rightarrow (x \rightarrow z)) = 1$

(iii) $(x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$

(iv) $(x^* \rightarrow y^*) \rightarrow (y \rightarrow x) = 1$.

Proposition 2.2. *[5]* The Wajsberg algebra $(A, \rightarrow, *, 1)$ satisfies the following axioms for all $x, y, z \in A$,

(i) $x \rightarrow x = 1$

(ii) If $x \rightarrow y = y \rightarrow x = 1$ then $x = y$

(iii) $x \rightarrow 1 = 1$

(iv) $x \rightarrow (y \rightarrow x) = 1$

(v) If $x \rightarrow y = y \rightarrow z = 1$ then $x \rightarrow z = 1$

(vi) $(x \rightarrow y) \rightarrow ((z \rightarrow x) \rightarrow (z \rightarrow y)) = 1$

(vii) $x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$

(viii) $x \rightarrow 0 = x \rightarrow 1^* = x^*$

(ix) $(x^*)^* = x$

(x) $x^* \rightarrow y^* = y \rightarrow x$.

Definition 2.3. *[5]* The Wajsberg algebra $(A, \rightarrow, *, 1)$ is called a lattice Wajsberg algebra if it satisfies the following axioms for all $x, y \in A$,

(i) A partial ordering" $\leq$ "on a lattice Wajsberg algebra A,such that $x \leq y$ if and only if $x \rightarrow y = 1$.

(ii) $(x \vee y) = (x \rightarrow y) \rightarrow y$

(iii) $(x \wedge y) = ((x^* \rightarrow y^*) \rightarrow y^*)^*$. Thus, we have $(A, \vee, \wedge, *, 0, 1)$ is a lattice Wajsberg algebra with lower bound 0 and upper bound 1.

Proposition 2.4.*[5]*The Wajsberg algebra $(A, \rightarrow, *, 1)$ satisfies the following axioms for all $x, y, z \in A$,

(i) If $x \leq y$ then $x \rightarrow z \geq y \rightarrow z$

(ii) If $x \leq y$ then $z \rightarrow x \leq z \rightarrow y$

(iii) $x \leq y \rightarrow z$ if and only if $y \leq x \rightarrow z$

(iv) $(x \vee y)^* = (x^* \wedge y^*)$

(v) $(x \wedge y)^* = (x^* \vee y^*)$

(vi) $(x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z)$

(vii) $x \rightarrow (y \wedge z) = (x \rightarrow y) \wedge (x \rightarrow z)$

(viii) $(x \rightarrow y) \vee (y \rightarrow x) = 1$

(ix) $x \rightarrow (y \vee z) = (x \rightarrow y) \vee (x \rightarrow z)$

(x) $(x \wedge y) \rightarrow z = (x \rightarrow y) \vee (x \rightarrow z)$

(xi) $(x \wedge y) \vee z = (x \vee z) \wedge (y \vee z)$

(xii) $(x \wedge y) \rightarrow z = (x \rightarrow y) \rightarrow (x \rightarrow z)$.

Definition 2.5.*[6]* A lattice Wajsberg algebra$(A, \rightarrow, *, 1)$ is called a lattice *H*-Wajsberg algebra, if it satisfies $x \vee y \vee ((x \wedge y) \rightarrow z) = 1$ for all $x, y, z \in A$. In a lattice *H*-Wajsberg algebra A, the following hold

(i) $x \rightarrow (x \rightarrow y) = (x \rightarrow y)$

(ii) $x \rightarrow (y \rightarrow z) = (x \rightarrow y) \rightarrow (x \rightarrow z)$ for all $x, y, z \in A$.

Definition 2.6.*[6]* Let $(A_1, \rightarrow, *, 1)$ and $(A_2, \rightarrow, *, 1)$be lattice Wajsberg algebras, $f : A_1 \rightarrow A_2$be a mapping from A_1 to A_2, if for any $x, y \in A_1$, $f(x \rightarrow y) = f(x) \rightarrow f(y)$ holds, then f is called an implication homomorphism from A_1 to A_2.

Definition 2.7.*[6]* Let $(A_1, \rightarrow, *, 1)$ and $(A_2, \rightarrow, *, 1)$ be lattice Wajsberg algebras, $f : A_1 \rightarrow A_2$ be an implication homomorphism from A_1 to A_2, is called a lattice implication homomorphism from A_1 to A_2 if it satisfies the following axioms for all $x, y \in A_1$,

(i) $f(x \wedge y) = f(x) \wedge f(y)$

(ii) $f(x \vee y) = f(x) \vee f(y)$

(iii) $f(x^*) = [f(x)]^*$.

Definition 2.8. *[5]* Let $(A, \rightarrow, *, 1)$ be a lattice Wajsberg algebra. A subset F of A is called an implicative filter of A if it satisfies the following axioms for all $x, y \in A$.

(i) $1 \in F$

(ii) $x \in F$ and $x \rightarrow y \in F$ imply $y \in F$.

Definition 2.9.*[10]* Let X be a set. A function $\mu: X \longrightarrow [0, 1]$ is called a fuzzy subset on X, for all $x \in X$ the value of $\mu(x)$ describes a degree of membership of x in μ.

Definition 2.10. *[10]* Let μ be a fuzzy subset of X then the complement of μ is denoted by $\mu^c$and defined as $\mu^c(x) = 1 - \mu(x)$ for all $x \in X$.

Definition 2.11. *[10]* Let μ be a fuzzy set in a set A. Then for $t \in [0,1]$, the set $\mu_t = \{x \in A / \mu(x) \geq t\}$ is called a level subset of μ.

Definition 2.12. *[10]* Let μ be a fuzzy set in a set A. Then for $t \in [0,1]$, the set $\mu^t = \{x \in A / \mu(x) \leq t\}$ is called a lower t-level cut of μ.

Definition 2.13. *[3]* Let $(A, \rightarrow, *, 1)$ be a lattice Wajsberg algebra. A fuzzy subset μ of A is called a fuzzy implicative filter of A if it satisfies the following axioms for all $x, y, z \in A$.

(i) $\mu(1) \geq \mu(x)$

(ii) $\mu(z) \geq \min\{\mu(y), \mu(y \rightarrow z)\}$.

Proposition 2.14. *[3]* Let μ be a fuzzy implicative filter of a lattice Wajsberg algebra A, then $x \leq y$ implies $\mu(x) \leq \mu(y)$ for all $x, y \in A$.

Definition 2.15. *[6]* Let $(A, \rightarrow, *, 1)$ be a lattice Wajsberg algebra. A subset F of A is called a strong implicative filter of A if it satisfies the following axioms for all $x, y, z \in A$

(i) $1 \in F$

(ii) $x \rightarrow (y \rightarrow z) \in F$ and $x \rightarrow y \in F$ imply $x \rightarrow z \in F$.

Definition 2.16. *[7]* Let $(A, \rightarrow, *, 1)$ be a lattice Wajsberg algebra. A fuzzy subset μ of A is called a fuzzy strong implicative filter of A if it satisfies the following for all $x, y, z \in A$,

(i) $\mu(1) \geq \mu(x)$

(ii) $\mu(x \rightarrow z) \geq \min\{\mu(x \rightarrow y), \mu(x \rightarrow (y \rightarrow z))\}$.

Definition 2.17.*[1]* An intuitionistic fuzzy set S of a non-empty set A is an object having the form $S = \{(x, \mu_S(x), \gamma_S(x)) / x \in A\} = (\mu_S, \gamma_S)$ where the functions $\mu_S: A \rightarrow [0, 1]$ and $\gamma_S: A \rightarrow [0, 1]$ denotes the degree of membership and denotes the non-membership respectively, and 0≤ $\mu_S(x) + \gamma_S(x)$≤ 1 for any $x \in A$.

Definition 2.18.*[8]* Let (A, →, ∗, 1) be a lattice Wajsberg algebra. An intuitionistic fuzzy set $S = (\mu_S, \gamma_S)$ of A is called an intuitionistic fuzzy implicative filter of A if it satisfies the following inequalities for all $x, y \in A$,
(i) $\mu_S(1) \geq \mu_S(x)$ and $\gamma_S(1) \leq \gamma_S(x)$
(ii) $\mu_S(y) \geq \min\{\mu_S(x), \mu_S(x\rightarrow y)\}$
(iii) $\gamma_S(y) \leq \max\{\gamma_S(x), \gamma_S(x\rightarrow y)\}$.

3. Intuitionistic fuzzy strong implicative filters
In this section, we introduce an intuitionistic fuzzy strong implicative filter of lattice Wajsberg algebra with illustrations and investigate some properties.

Definition 3.1. Let (A, →, ∗, 1) be a lattice Wajsberg algebra. An intuitionistic fuzzy set $S = (\mu_S, \gamma_S)$ of A is called an intuitionistic fuzzy strong implicative filter of A if it satisfies the following for all $x, y, z \in A$,
(i) $\mu_S(1) \geq \mu_S(x)$ and $\gamma_S(1) \leq \gamma_S(x)$
(ii) $\mu_S(x\rightarrow z) \geq \min\{\mu_S(x\rightarrow y), \mu_S(x\rightarrow(y\rightarrow z))\}$
(iii) $\gamma_S(x\rightarrow z) \leq \max\{\gamma_S(x\rightarrow y), \gamma_S(x\rightarrow(y\rightarrow z))\}$.

Example 3.2. Consider a set$A = \{0, a, b, c, d, 1\}$with Figure 3.1 as a partial ordering. Define a quasi complement "∗" and a binary operation "→" on A as in Tables 3.1 and 3.2.

x	x^*
0	1
a	C
b	D
c	A
d	B
1	0

Table:3.1
Complement

→	0	a	b	c	d	1
0	1	1	1	1	1	1
a	d	1	a	c	c	1
b	c	1	1	c	c	1
c	b	a	b	1	a	1
d	a	1	a	1	1	1
1	0	a	b	c	d	1

Table:3.2
Implication

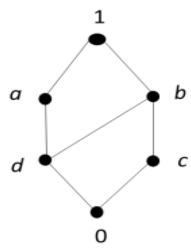

Figure:3.1
Lattice diagram

Define '∨' and '∧' operations on A as follows:
$(x \vee y) = (x \rightarrow y) \rightarrow y$; $(x \wedge y) = ((x^* \rightarrow y^*) \rightarrow y^*)^*$for all$x, y \in A$.
Then, A is a lattice Wajsberg algebra.
Consider an intuitionistic fuzzy set $S = (\mu_S, \gamma_S)$ on A as
$\mu_S(x) = \begin{cases} 1 & \text{if } x \in \{a, 1\} \\ 0.4 & \text{otherwise} \end{cases}$; $\gamma_S(x) = \begin{cases} 0 & \text{if } x \in \{a, 1\} \\ 0.6 & \text{otherwise} \end{cases}$ is an intuitionistic fuzzy strong implicative filter of A.
In the same example 3.2, consider the intuitionistic fuzzy set $S = (\mu_S, \gamma_S)$ on A as,
$\mu_S(x) = \begin{cases} 0.7 & if\ \ x \in \{1, a, c\} \\ 0.3 & if\ \ x \in \{0, b, d\} \end{cases}$; $\gamma_S(x) = \begin{cases} 0.4 & if\ \ x \in \{1, a, c\} \\ 0.6 & if\ \ x \in \{0, b, d\} \end{cases}$.
Then, we have $S = (\mu_S, \gamma_S)$ is not an intuitionistic fuzzy strong implicative filter of lattice Wajsberg algebra A. Since, we have $\mu_S(c \rightarrow b) = 0.3 \ngeq \min\{\mu_S(c \rightarrow d), \ \mu_S(c \rightarrow (d \rightarrow b)\} = \min\{\mu_S(a), \mu_S(1)\} = 0.7$.
$\gamma_S(c \rightarrow b) = 0.6 \nleq \max\{\gamma_S(c \rightarrow d), \ \gamma_S(c \rightarrow (d \rightarrow b)\} = \max\{\gamma_S(a), \gamma_S(1)\} = 0.4$.

Proposition 3.3. Let A be a lattice Wajsberg algebra, and let $S = (\mu_S, \gamma_S)$ be an intuitionistic fuzzy strong implicative filter of A, then $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy implicative filter of A.

Proof. Let $S = (\mu_S, \gamma_S)$ be an intuitionistic fuzzy strong implicative filter of a lattice Wajsberg algebra A.

From the Definition 3.1, we have $\mu_S(x \to z) \geq \min\{\mu_S(x\to y), \mu_S(x \to (y\to z))\}$ for all $x, y, z \in A$.

Take $x = 1$, then from (i) of Definition 2.1, we have

$$\mu_S(1\to z) \geq \min\{\mu_S(1\to y), \mu_S(1\to(y\to z))\}.$$

Hence, $\mu_S(z) \geq \min\{\mu_S(y), \mu_S(y\to z)\}$. (3.1)

From the Definition 3.1, we have $\gamma_S(x \to z) \leq \max\{\gamma_S(x\to y), \gamma_S(x \to (y\to z))\}$ for all $x, y, z \in A$.

Take $x = 1$, then from (i) of Definition 2.1, we have

$$\gamma_S(1\to z) \leq \max\{\gamma_S(1\to y), \gamma_S(1\to(y\to z))\}.$$

Hence, $\gamma_S(z) \leq \max\{\gamma_S(y), \gamma_S(y\to z)\}$. (3.2)

From (3.1) and (3.2), we have $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy implicative filter of A.■

Proposition 3.4. Let A be a lattice Wajsberg algebra. Then A is a lattice H-Wajsberg algebra if and only if each intuitionistic fuzzy implicative filter of A is an intuitionistic fuzzy strong implicative filter.

Proof. Let A be a lattice H-Wajsberg algebra and $S = (\mu_S, \gamma_S)$ be a intuitionistic fuzzy implicative filter of A.

From (i) of Definition 2.18, we have $\mu_S(1) \geq \mu_S(x)$ and $\gamma_S(1) \leq \gamma_S(x)$ for all $x \in A$. (3.3)

From (ii) of Definition 2.5, we have $x\to(y \to z) = (x \to y)\to (x \to z)$.

Therefore, $\min\{\mu_S(x\to y), \mu_S(x \to(y\to z))\} = \min\{\mu_S(x\to y), \mu_S((x \to y)\to (x \to z))\} \leq \mu_S(x\to z)$ and $\max\{\gamma_S(x\to y), \gamma_S(x\to(y \to z))\} = \max\{\gamma_S(x\to y), \gamma_S((x \to y)\to (x \to z))\} \geq \gamma_S(x\to z)$, for all $x, y, z \in A$ [From (ii) and (iii) of Definition 3.1].

Thus, $\mu_S(x \to z) \geq \min\{\mu_S(x\to y), \mu_S(x \to(y\to z))\}$ and

$\gamma_S(x\to z) \leq \max\{\gamma_S(x\to y), \gamma_S(x\to(y \to z))\}$ for all $x, y, z \in A$ (3.4)

From (3.3) and (3.4), we have $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy strong implicative filter.

Conversely, we consider an intuitionistic fuzzy implicative filter $S = (\mu_S, \gamma_S)$ of A is an intuitionistic fuzzy strong implicative filter.

Define a mappings $\mu_S: A \to [0, 1]$ and $\gamma_S: A \to [0, 1]$ as

$\mu_S(x) = \begin{cases} 0.8 & \text{if } x = 1 \\ 0 & \text{if } x \neq 1 \end{cases}$ for all $x \in A$; $\gamma_S(x) = \begin{cases} 0 & \text{if } x = 1 \\ 0.7 & \text{if } x \neq 1 \end{cases}$ for all $x \in A$.

Then, μ is a fuzzy implicative filter of A and hence it is a fuzzy strong implicative filter. It follows that $\{1\} = \mu_{0.8}$ and $\{0\} = \gamma_{0.7}$ are implicative filters of A. This implies that A is a lattice H-Wajsberg algebra.■

Proposition 3.5. Let A be a lattice Wajsberg algebra and $S = (\mu_S, \gamma_S)$ be an intuitionistic fuzzy set of A, if $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy strong implicative filter, then the following are satisfied and equivalent:

(i) $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy implicative filter and for all $x, y \in A$,
$\mu_S(x\to y) \geq \mu_S(x\to(x\to y))$; $\gamma_S(x \to y) \leq \gamma_S(x\to(x\to y))$.

(ii) $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy implicative filter and for all $x, y, z \in A$,
$\mu_S((x \to y) \to(x \to z)) \geq \mu_S(x\to(y \to z))$; $\gamma_S((x \to y) \to(x \to z)) \leq \gamma_S(x\to(y \to z))$.

(iii) $\mu_S(1) \geq \mu_S(x)$ and $\gamma_S(1) \leq \gamma_S(x)$ and for all $x, y, z \in A$,
$\mu_S(x \to y) \geq \min\{\mu_S(z \to(x\to(x\to y))), \mu_S(z)\}$; $\gamma_S(x \to y) \leq \max\{\gamma_S(z \to(x\to(x\to y))), \gamma_S(z)\}$.

Proof. (i) ⇒ (ii).

Let $S = (\mu_S, \gamma_S)$ be an intuitionistic fuzzy strong implicative filter of A.

Then, from the Definition 3.1, we have $\mu_S(1) \geq \mu_S(x)$ and $\gamma_S(1) \leq \gamma_S(x)$,
$\mu_S(x\to z) \geq \min\{\mu_S(x\to y), \mu_S(x \to(y\to z))\}$ and $\gamma_S(x \to z) \leq \max\{\gamma_S(x\to y), \gamma_S(x\to(y \to z))\}$.

Put $x = 1$ and (i) of Proposition 2.1, we get $\mu_S(z) \geq \min\{\mu_S(y), \mu_S(y\to z)\}$ and
$\gamma_S(z) \leq \max\{\gamma_S(y), \gamma_S(y\to z)\}$ for all $x, y, z \in A$.

Therefore, $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy implicative filter of A.

From $\mu_S(x \to z) \geq \mu_S(x \to (x \to z), \gamma_S(x \to z) \leq \gamma_S(x \to (x \to z)$,
put $z = y$, we have $\mu_S(x \to y) \geq \mu_S(x\to(x\to y))$; $\gamma_S(x \to y) \leq \gamma_S(x\to(x\to y))$.

If for any $x, y, z \in A$, $\mu_S(x \to y) \geq \mu_S(x\to(x\to y))$; $\gamma_S(x \to y) \leq \gamma_S(x\to(x\to y))$.

Then, we have $\mu_S((x\to y)\to(x\to z)) = \mu_S(x\to((x\to y)\to z)) \geq \mu_S(x\to(x\to((x\to y)\to z)))$,

and $x\to(x\to((x\to y)\to z)) = x\to((x\to y)\to(x\to z)) = x\to((y\to x)\to(y\to z)) \geq x\to(y\to z)$.
From the Proposition 2.14, we have $\mu_S(x\to(x\to((x\to y)\to z))) \geq \mu_S(x\to(y\to z))$.
Therefore, $\mu_S((x \to y) \to(x \to z)) \geq \mu_S(x\to(y \to z))$.
Similarly, $\gamma_S((x\to y)\to(x\to z)) = \gamma_S(x\to((x\to y)\to z))\leq \gamma_S(x\to(x\to((x\to y)\to z)))$ and
$\gamma_S(x\to(x\to((x\to y)\to z))) \leq \gamma_S(x\to(y\to z))$. Thus, $\gamma_S((x \to y) \to(x \to z)) \leq \gamma_S(x\to(y \to z))$.
(ii) ⇒ (iii)**.** Let (ii) be hold. Then, it is clear that $\mu_S(1) \geq \mu_S(x)$ and $\gamma_S(1) \leq \gamma_S(x)$.
If for any $x, y, z \in A$, we have $\mu_S((x \to y) \to (x \to z)) \geq \mu_S(x\to(y \to z))$ and
$\gamma_S((x \to y) \to (x \to z)) \leq \gamma_S(x\to(y \to z))$.
Put $y = x$, then, we get $\mu_S((x \to x) \to(x \to z)) \geq \mu_S(x\to(x \to z))$.
That is, $\mu_S(x \to z) \geq \mu_S(x\to(x\to z))$. [From (i) of Proposition 2.2 and (i) of Definition 2.1]
Thus for any $x, y \in A$, $\mu_S(x \to y) \geq \mu_S(x\to(x\to y))$.
Since μ_S is a fuzzy implicative filter and then, we have
$\mu_S(x\to(x\to y)) \geq \min\{\mu_S(z \to(x\to(x\to y))), \mu_S(z)\}$.
Hence,$\mu_S(x \to y) \geq \min\{\mu_S(z \to(x\to(x\to y))), \mu_S(z)\}$.
Similarly, $\gamma_S(x \to y) \leq \gamma_S(x\to(x\to y))$ and $\gamma_S(x\to(x\to y)) \leq\max\{\gamma_S(z \to(x\to(x\to y))), \gamma_S(z)\}$.
Thus $\gamma_S(x \to y) \leq \max\{\gamma_S(z \to(x\to(x\to y))), \gamma_S(z)\}$.
(iii) ⇒ (i)**.** Let (iii) be hold.
Put $x = 1$ then, we get $\mu_S(y) \geq \min\{\mu_S(z \to y), \mu_S(z)\}$ and $\gamma_S(y) \leq\max\{\gamma_S(z \to y), \gamma_S(z)\}$.
[From (i) of Proposition 2.2 and (i) of Definition 2.1]
Also, $\mu_S(1) \geq \mu_S(x)$ and $\gamma_S(1) \leq \gamma_S(x)$. Hence $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy implicative filter.
Put $z = 1$ then, we get $\mu_S(x \to y) \geq \mu_S(x\to(x\to y))$ and $\gamma_S(x \to y) \leq \gamma_S(x\to(x\to y))$. [From (i) of Definition 2.1 and $\mu_S(1) \geq \mu_S(x)$ and $\gamma_S(1) \leq \gamma_S(x)$].■

Proposition 3.6. Let $S = (\mu_S, \gamma_S)$ be an intuitionistic fuzzy strong implicative filter of a lattice Wajsberg algebra A if and only if the fuzzy subsets μ_S, γ_S^c are fuzzy strong implicative filters of A, where $\gamma_S^c(x) = 1-\gamma_S(x)$ for all $x \in A$.
Proof. *Let* $S = (\mu_S, \gamma_S)$ be an intuitionistic fuzzy strong implicative filter of A.
From the Definition 3.1, we have the fuzzy subset μ_S is a fuzzy strong implicative filter of A.
Now $\gamma_S^c(1) = 1 -\gamma_S(1) \geq 1 -\gamma_S(x) = \gamma_S^c(x)$
and $\gamma_S^c(x \to z) = 1 -\gamma_S(x \to z) \geq 1 - \max\{\gamma_S(x \to y), \gamma_S(x \to(y\to z))\}$
$= \min\{1 -\gamma_S(x \to y), 1 -\gamma_S(x \to(y\to z))\}$
$= \min\{\gamma_S^c(x \to y), \gamma_S^c(x \to(y\to z))\}$.
Thus, $\gamma_S^c(1) \geq \gamma_S^c(x)$ and $\gamma_S^c(x \to z) \geq \min\{\gamma_S^c(x \to y), \gamma_S^c(x \to(y\to z))\}$.
Hence, γ_S^c is a fuzzy strong implicative filter of A.
Conversely, if μ_S and γ_S^c are fuzzy strong implicative filters of A.
Then we have $\mu_S(1) \geq \mu_S(x)$ and
$1 -\gamma_S(1) = \gamma_S^c(1) \geq \gamma_S^c(x) = 1 -\gamma_S(x)$ implies that $\gamma_S(x) \geq \gamma_S(1)$.
From (i) and (ii) of Definition 2.16, we have $\mu_S(x \to z) \geq \min\{\mu_S(x\to y), \mu_S(x \to(y\to z))\}$ and
$\gamma_S^c(x\to z) \geq \min\{\gamma_S^c(x \to y), \gamma_S^c(x \to(y\to z))\}$
$1 -\gamma_S(x \to z) \geq \min\{1 -\gamma_S(x \to y), 1 -\gamma_S(x \to(y\to z))\}$
$= 1 -\max\{\gamma_S(x\to y), \gamma_S(x\to(y \to z))\}$
$\gamma_S(x\to z) \leq \max\{\gamma_S(x\to y), \gamma_S(x\to(y \to z))\}$.
Thus, $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy strong implicative filter of A. ■

Proposition 3.7. Let $S = (\mu_S, \gamma_S)$ bean intuitionistic fuzzy strong implicative filter of a lattice Wajsberg algebra A if and only if (μ_S, μ_S^c) and (γ_S^c, γ_S) are intuitionistic fuzzy strong implicative filters of A.
Proof. Let $S = (\mu_S, \gamma_S)$ be an intuitionistic fuzzy strong implicative filter of A.
Then, $0 \leq \mu_S(x) + \mu_S^c(x) = 1$ and $0 \leq \gamma_S(x) + \gamma_S^c(x) = 1$.
Thus,(μ_S, μ_S^c) and (γ_S^c, γ_S) are intuitionistic fuzzy subsets of A.
We have $\mu_S(1) \geq \mu_S(x)$, $\mu_S^c(1) = 1-\mu_S(1) \leq 1-\mu_S(x) = \mu_S^c(x)$. (3.5)
$\mu_S(x\to z) \geq \min\{\mu_S(x\to y), \mu_S(x \to(y\to z))\}$ (3.6)
$\mu_S^c(x\to z) = 1-\mu_S(x \to z)$
$\leq 1-\min\{\mu_S(x\to y), \mu_S(x \to(y\to z))\}$

$= \max\{1 - \mu_S(x \to y), 1 - \mu_S(x \to (y \to z))\}$
$= \max\{\mu_S^c(x \to y), \mu_S^c(x \to (y \to z))\}$ for all $x, y \in A$.

Thus $\mu_S^c(x \to z) \leq \max\{\mu_S^c(x \to y), \mu_S^c(x \to (y \to z))\}$. (3.7)

From (3.5), (3.6) and (3.7), we have (μ_S, μ_S^c) is an intuitionistic fuzzy strong implicative filter of A.
Similarly, we have to prove that (γ_S^c, γ_S) is an intuitionistic fuzzy strong implicative filter of A.
Conversely, if (μ_S, μ_S^c) and (γ_S^c, γ_S) are intuitionistic fuzzy strong implicative filters of A then $\mu_S(1) \geq \mu_S(x)$ and $\gamma_S(1) \leq \gamma_S(x)$, $\mu_S(x \to z) \geq \min\{\mu_S(x \to y), \mu_S(x \to (y \to z))\}$,
and $\gamma_S(x \to z) \leq \max\{\gamma_S(x \to y), \gamma_S(x \to (y \to z))\}$.
Hence, $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy strong implicative filter of A. ■

Proposition 3.8. An intuitionistic fuzzy set $S = (\mu_S, \gamma_S)$ of a lattice Wajsberg algebra A is an intuitionistic fuzzy strong implicative filter of A if and only if for each $m, n \in [0, 1]$, the sets μ_{S_n} and $\gamma_S{}^m$ is either empty or fuzzy strong implicative filters of A.

Proof. Let $S = (\mu_S, \gamma_S)$ be an intuitionistic fuzzy strong implicative filter of A and $\mu_{S_n} \neq \phi$, $\gamma_S{}^m \neq \phi$ for all $m, n \in [0, 1]$. From the Definition 3.1, we have $1 \in \mu_{S_n}$ and $1 \in \gamma_S{}^m$.
Let $x \to y \in \mu_{S_n}$ and $x \to (y \to z) \in \mu_{S_n}$ for all $x, y, z \in A$.
Then, we have $\mu_S(x \to y) \geq n$ and $\mu_S(x \to (y \to z)) \geq n$.
Thus, we get $\mu_S(x \to z) \geq \min\{\mu_S(x \to y), \mu_S(x \to (y \to z))\} \geq n$, that is, $x \to z \in \mu_S{}^n$.
Thus, μ_{S_n} is a fuzzy strong implicative filter of A.
Similarly, let $x, y, z \in A$ be such that $x \to y \in \gamma_S{}^m$ and $x \to (y \to z) \in \gamma_S{}^m$.
Then, we have $\gamma_S(x \to y) \leq m$ and $\gamma_S(x \to (y \to z)) \leq m$.
Thus, we get $\gamma_S(x \to z) \leq \max\{\gamma_S(x \to y), \gamma_S(x \to (y \to z))\} \leq m$, that is, $x \to z \in \gamma_S{}^m$.
Thus, $\gamma_S{}^m$ is a fuzzy strong implicative filter of A.
Conversely, for each $m, n \in [0, 1]$, the sets μ_{S_n} and $\gamma_S{}^m$ is either empty or implicative filters of A. For any $x \in A$, let $\mu_S(x) = n$ and $\gamma_S(x) = m$. Then, $x \in \mu_{S_n} \cap \gamma_S{}^m$ and so that $\mu_{S_n} \neq \phi \neq \gamma_S{}^m$.
Since μ_{S_n} and $\gamma_S{}^m$ are fuzzy strong implicative filters of A, therefore $1 \in \mu_{S_n}$ and $1 \in \gamma_S{}^m$.
Hence, $\mu_S(1) \geq n = \mu_S(x)$ and $\gamma_S(1) \leq m = \gamma_S(x)$.
To prove: (i). $\mu_S(x \to z) \geq \min\{\mu_S(x \to y), \mu_S(x \to (y \to z))\}$ and
(ii). $\gamma_S(x \to z) \leq \max\{\gamma_S(x \to y), \gamma_S(x \to (y \to z))\}$.`
(i). If not, then there exists $p, q, r \in A$ such that $\mu_S(p \to r) < \min\{\mu_S(p \to q), \mu_S(p \to (q \to r))\}$.
Taking $n_0 = \frac{1}{2}[\mu_S(p \to r) + \min\{\mu_S(p \to q), \mu_S(p \to (q \to r))\}]$,
we have $\mu_S(p \to r) < n_0 < \min\{\mu_S(p \to q), \mu_S(p \to (q \to r))\}$.
Therefore, $p \to q \in \mu_{S_{n_0}}$ and $p \to (q \to r) \in \mu_{S_{n_0}}$ but $p \to r \notin \mu_{S_{n_0}}$.
Thus, $\mu_{S_{n_0}}$ is not a fuzzy strong implicative filter of A, which is a contradiction.
Therefore, $\mu_S(x \to z) \geq \min\{\mu_S(x \to y), \mu_S(x \to (y \to z))\}$.
(ii). If not, then there exists $u, v, w \in A$ such that $\gamma_S(u \to w) > \max\{\gamma_S(u \to v), \gamma_S(u \to (v \to w))\}$. Taking $m_0 = \frac{1}{2}[\gamma_S(u \to w) + \max\{\gamma_S(u \to v), \gamma_S(u \to (v \to w))\}]$,
we have $\max\{\gamma_S(u \to v), \gamma_S(u \to (v \to w))\} < m_0 < \gamma_S(u \to w)$.
Therefore, $v \in \gamma_S{}^{m_0}$ and $v \to u \in \gamma_S{}^{m_0}$ but $u \notin \gamma_S{}^{m_0}$.
Thus, $\gamma_S{}^{m_0}$ is not a fuzzy strong implicative filter of$A$, which is a contradiction.
Therefore, $\gamma_S(x \to z) \leq \max\{\gamma_S(x \to y), \gamma_S(x \to (y \to z))\}$.
Hence, $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy strong implicative filter of A.■

Proposition 3.9. Let Abe a lattice Wajsberg algebra, V a non-empty subset of [0, 1] and $J_t = \{x \,/\, J(x) \geq t, x \in A\}$ such that $t \in V$ and J is a fuzzy strong implicative filter of A satisfies the following:
(i) $A = \bigcup_{t \in V} J_t$
(ii) $r > t$ if and only if $J_r \subseteq J_t$for all $r, t \in V$
(iii) Let $\mu_P(x) = Sup\{t \in V / x \in J_t\}$ and $\gamma_P(x) = Inf\{t \in V / x \in J_t\}$ for all $x \in A$ then $P = (\mu_P, \gamma_P)$ is an intuitionistic fuzzy strong implicative filter of A.

Proof. Let $P = (\mu_P, \gamma_P)$ be an intuitionistic fuzzy set of A.

We have to prove: $P = (\mu_P, \gamma_P)$ is an intuitionistic fuzzy strong implicative filter of A.
(i) $\mu_P(1) = Sup\{t \in V / 1\in J_t\} \geq \mu_P(x)$, for all $x \in A$. (3.8)
(ii) If $\mu_P(x \to z) < \min\{\mu_P(x\to y), \mu_P(x \to(y\to z))\}$ for all $x, y, z \in A$.
Let $t_1 = \min\{\mu_P(x), \mu_P(x\to y)\}$. So $\mu_P(x\to y) \geq t_1$, $\mu_P(x \to(y\to z)) \geq t_1$.
Then there exists a fuzzy strong implicative filter J_{t_1} such that $x\to y \in J_{t_1}$ and $x \to(y\to z) \in J_{t_1}$. Thus, $x \to z \in J_{t_1}$.
Then, we have $\mu_P(x \to z) = Sup\{t \in V / 1\in J_t\} \geq t_1$, which is a contradiction.
Therefore, $\mu_P(x \to z) \geq\min\{\mu_P(x\to y), \mu_P(x \to(y\to z))\}$ for all $x, y \in A$. (3.9)
(iii) If $\gamma_P(x \to z) > \max\{\gamma_P(x \to y), \gamma_P(x \to(y\to z))\}$ for all $x, y, z \in A$.
Let $\gamma_P(x\to y) \geq t_2$ and $\gamma_P(x \to(y\to z)) \geq t_3$. So $\gamma_P(x \to z) \geq t_2 \wedge t_3$.
Then there exist a fuzzy strong implicative filter $J_{t_2\wedge t_3}$ such that $x\to y \in J_{t_2\wedge t_3}$,
$x\to(y\to z) \in J_{t_2\wedge t_3}$. Hence, $x \to z \in J_{t_2\wedge t_3}$.
We have $\gamma_P(x \to z) = Inf\{t \in V / y \in J_t\} \leq t_2 \wedge t_3$, which is a contradiction.
Therefore, $\gamma_P(x \to z) \leq \max\{\gamma_P(x\to y), \gamma_P(x \to(y\to z))\}$. (3.10)
From (3.8), (3.9) and (3.10), we have $P= (\mu_P, \gamma_P)$ is an intuitionistic fuzzy strong implicative filter of A.■

Proposition 3.10. Let $A_1$and A_2 be any two lattice Wajsberg algebras, f is a lattice implication homomorphism from $A_1$to A_2. Let $S = (\mu_S, \gamma_S)$ and $T = (\mu_T, \gamma_T)$ be the intuitionistic fuzzy sets of A_1 and A_2 respectively. If T is an intuitionistic fuzzy strong implicative filter of A_2, then $f^{-1}(T)$ is an intuitionistic fuzzy strong implicative filter of A_1.
Proof. Let T be an intuitionistic fuzzy strong implicative filter of A_2, since f is a homomorphism, we have for any $x \in A_1$, from $f(1) =1$, and $f^{-1}(\mu_T)(x) = \mu_T(f(x)) \leq \mu_T(1) = \mu_T(f(1))= f^{-1}(\mu_T)(1)$.
That is, $f^{-1}(\mu_T)(1) \geq f^{-1}(\mu_T)(x)$ for all $x \in A_1$.
Similarly, $f^{-1}(\gamma_T)(x) = \gamma_T(f(x)) \geq \gamma_T(1) = \gamma_T(f(1)) = f^{-1}(\gamma_T)(1)$.
That is, $f^{-1}(\gamma_T)(1) \leq f^{-1}(\gamma_T)(x)$ for all$x \in A_1$.
Thus, $f^{-1}(\mu_T)(1) \geq f^{-1}(\mu_T)(x)$ and $f^{-1}(\gamma_T)(1) \leq f^{-1}(\gamma_T)(x)$. (3.11)
For any $x, y, z \in A_1$and $T \subseteq A_2$ is an intuitionistic fuzzy strong implicative filter, we have

$$\begin{aligned} f^{-1}(\mu_T)(x\to z) = \mu_T(f(x \to z)) = \mu_T(f(x)\to f(z)) &\geq \min\{\mu_T(f(x)\to f(y)), \mu_T(f(x)\to(f(y)\to f(z)))\} \\ &= \min\{\mu_T(f(x \to y)), \mu_T(f(x)\to f(y \to z))\} \\ &= \min\{\mu_T(f(x \to y)), \mu_T(f(x \to(y \to z)))\} \\ &= \min\{f^{-1}(\mu_T)(x \to y), f^{-1}(\mu_T)(x \to(y \to z))\}. \end{aligned}$$

Thus, $f^{-1}(\mu_T)(x\to z) \geq \min\{f^{-1}(\mu_T)(x \to y), f^{-1}(\mu_T)(x \to(y \to z))\}$. (3.12)

$$\begin{aligned} f^{-1}(\gamma_T)(x\to z) = \gamma_T(f(x \to z)) = \gamma_T(f(x)\to f(z)) &\leq \max\{\gamma_T(f(x)\to f(y)), \gamma_T(f(x)\to(f(y)\to f(z)))\} \\ &= \max\{\gamma_T(f(x \to y)), \gamma_T(f(x)\to f(y \to z))\} \\ &= \max\{\gamma_T(f(x \to y)), \gamma_T(f(x \to(y \to z)))\} \\ &= \max\{f^{-1}(\gamma_T)(x \to y), f^{-1}(\gamma_T)(x \to(y \to z))\}. \end{aligned}$$

Thus, $f^{-1}(\gamma_T)(x\to z) \leq \max\{f^{-1}(\gamma_T)(x \to y), f^{-1}(\gamma_T)(x \to(y \to z))\}$. (3.13)
From (3.11), (3.12) and (3.13), we have $f^{-1}(T)$ is an intuitionistic fuzzy strong implicative filter of A_1.■

4. Conclusion

We have introduced the notion of an intuitionistic fuzzy strong implicative filter of lattice Wajsberg algebra. Also, we investigated some properties with interesting illustrations. We obtained the relation between an intuitionistic fuzzy implicative filter and strong implicative filter in lattice Wajsberg algebra. Finally, we establish the equivalent condition of an intuitionistic fuzzy strong implicative filter.

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