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GALLAI-TYPE THEOREMS ON INDEPENDENCE AND IRREDUNDANCE IN GALLAI FUZZY GRAPHS

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Abstract: The Gallai fuzzy graph Γ(G) of a fuzzy graph G has the fuzzy edges of G as its fuzzy vertices and two distinct fuzzy edges of G are fuzzy incident in G, but do not span a fuzzy triangle in G. The Gallai fuzzy graphs are fuzzy spanning Gallai sub graphs of the well-known Class of fuzzy line graphs. Let $\Gamma(\Gamma(G))$ and $\beta(\Gamma(G))$ denote the maximum fuzzy cardinality of a fuzzy dominating set of a Gallai fuzzy graph $\Gamma(G)= (\sigma, \mu)$ with n fuzzy vertices and minimum fuzzy degree $\delta(\Gamma(G))$, $\Gamma(\Gamma(G))\leq n- \delta(\Gamma(G))$, $\beta(\Gamma(G)) \leq n- \delta(\Gamma(G))$.In this paper, we show for the upper fuzzy irredundance number, $IR(\Gamma(G))$: For a Gallai fuzzy graph Γ(G) with n fuzzy vertices and minimum fuzzy degree $\delta(\Gamma(G))$, $IR (\Gamma(G)) \leq n- \delta(\Gamma(G))$.Characterizations are given for classes of Gallai fuzzy graphs which achieve this upper bound for the upper fuzzy irredundance, upper fuzzy domination and fuzzy independence numbers of a Gallai fuzzy graph.

Key words: Gallai fuzzy graph, Gallai-type theorems, On Domination,independent and irredundance

AMS Classification: 03E72, 05C99**.**

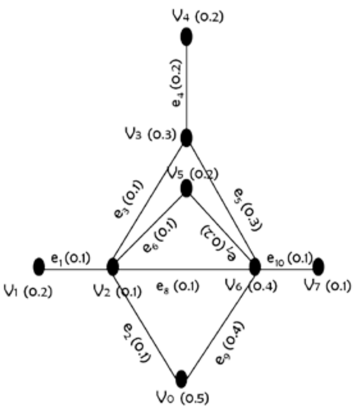
1. Introduction

The study of dominating sets in graphs was begun by Ore and Berge, the domination number, independent domination number are introduced by Cockayne and Hedetniemi. In 1965, L.A. Zadeh [1] introduced a mathematical frame work to explain the concepts of uncertainty in real life through are publication of a seminal paper. In 1975, A. Rosenfeld [2] introduced the notation of fuzzy graph theoretic concept such as paths, cycles and connectedness. In 1996, Van Bang Le [3] was discussed the paper of Gallai graphs and anti-Gallai graphs. Also S. Aparna Lakshmanan and S.B. Rao [4] were discussed in this paper. A. Somasundram and S. Somasundram [5] discussed domination in fuzzy graphs. A. Nagoorgani and

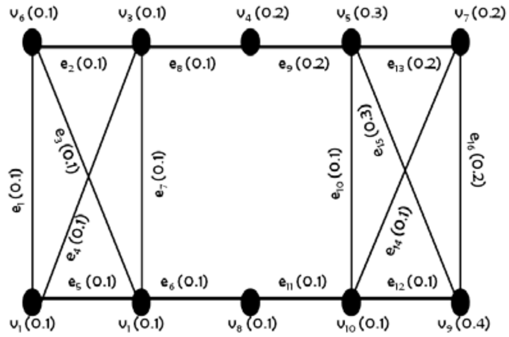
P.Vadivel [6] dealt with domination, independence and irredundance numbers. In this paper we discussed about Gallai- type theorems on domination, independence and irredundance in Gallai fuzzy graphs based on the concept of Gallai type theorems and domination parameters by GaylaS.Domkeet.al. [7]. In [8], our earlier work have discussed the concept of Gallai Fuzzy Graphs on Domination parameters. In this manuscript, we investigate the Gallai-type theorems in Gallai fuzzy graph involving upper fuzzy domination parameters combined with minimum fuzzy degree and establish the relationship with other parameters which is also investigated.

2. Preliminaries.

Definition 2.1 [4] A fuzzy graph G and its Gallai fuzzy graph Γ(G) as shown below:



Γ(G)



G

A fuzzy graph with G as the underlying set is a finite non-empty unordered pair of G= (σ, μ), where $\sigma : V \rightarrow [0,1]$ is a fuzzy subset, $\mu: E \rightarrow [0,1]$ is a fuzzy relation on the fuzzy subset σ such that $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ for all u, v Є V where ∧ and ∨ stands for minimum and maximum. The underlying crisp fuzzy graph of G=(σ, μ) is denoted by $G^*=(V, E)$, where $V =\{ u \epsilon V : \sigma(u) > 0\}andE = \{(u, v) \epsilon VxV : \mu(u, v) > 0 \}$,the fuzzy order P and fuzzy size of the fuzzy graph G=(σ, μ) are defined by $p = \Sigma_{v \epsilon V} \sigma(u)$ and $q =\Sigma_{u, v \epsilon E} \mu(u, v)$. Each pair μ = u, v of fuzzy vertices in σ is a fuzzy edge of G and μ is said to join u and v are fuzzy adjacent vertices, fuzzy vertex u and fuzzy edge μ are fuzzy incident with each other as are σ and μ if two distinct fuzzy edges are incident with a common fuzzy vertex, then they are called fuzzy adjacent edges. A fuzzy edge e = uv of a fuzzy graph is an fuzzy edge if $\mu(u, v) = \sigma(u) \wedge (v)$. $N (u) =\{v \in V / \mu (u, v) = \sigma(u) \wedge \sigma(v) \}$ is called the open fuzzy neighborhood of u and $N[u]= N(u)\cup\{u\}$is the closed fuzzy neighborhood of u.

Definition 2.2 *Let* G be a fuzzy graph and u be a fuzzy vertex in G then there exists a fuzzy vertex v such that (u,v) is a fuzzy edge then we say that u dominates v.

Definition 2.3 Let G=(σ, μ) be a fuzzy graph. A subset D of V is said to be fuzzy dominating set of G if for every v ∈ V-D, there exists u∈ D such that u dominates v.

Definition 2.4 A fuzzy dominating set D of a fuzzy graph G is called minimal fuzzy dominating set of G, if for every fuzzy vertex v ∈ D,D-{v} is not a fuzzy dominating set.

Definition 2.5 Two uzzy vertices in a fuzzy graph G are said to be fuzzy independent if there is no fuzzy edge between them. A subset S of V is said to be fuzzy independent set of G if every two fuzzy vertices of S are fuzzy independent.

Definition 2.6 A fuzzy independent set S of G is said to be maximal fuzzy independent, if for every fuzzy vertex v∈ V-S, the set S∪{v} is not fuzzy independent.

3. Fuzzy independent and irredundant in Gallai sets.

Definition 3.1 Let Γ(G) be Gallai fuzzy graph and S be a set of fuzzy vertices. A fuzzy vertex v is said to be fuzzy private neighbour or private neighbour of u ∈S with respect to S if N[v]∩S={u}. Furthermore, we define fuzzy private neighborhood of u ∈ S with respect to S to be PN[u,S]={v:N[v]∩S={u}}.Stated in other words PN[u,S]=N[u]-N[S-{u}].If u ∈ PN[u,S], then u is an isolate fuzzy vertex in <S>.If is also stated that u is its own fuzzy private neighbor.

Definition 3.2 A Gallai fuzzy set of fuzzy vertices S is said to be Gallai fuzzy irredundant set if PN [u,S]$\neq \varphi$ for everyfuzzy vertex in S.

Definition 3.3 A Gallai fuzzy irredundant set S is a maximal fuzzy irredundant if for every fuzzy vertex u ∈V-S, the set S∪{u} is not Gallai fuzzy irredundant set, which means that there exists at least one fuzzy vertex w ∈ S∪{u} which does not have fuzzy private neighbor.

Definition 3.4 Minimum fuzzy cardinality among all minimal fuzzy dominating sets in Γ(G) is called fuzzy domination number of Γ(G) and is denoted by γ(Γ(G)).

Definition 3.5 Maximum fuzzy cardinality among all minimal fuzzy dominating sets in Γ(G) is called upper fuzzy domination number of Γ(G) and is denoted by Γ(Γ(G)).
A fuzzy dominating set D of a Gallai fuzzy graph Γ(G) is a minimum fuzzy dominating set if $|D|$= γ(Γ(G)).

Definition 3.6 The maximum fuzzy cardinality among all maximal fuzzy independent set is called fuzzy independent number of Γ(G) and is denoted by β(Γ(G)).

Definition 3.7 The minimum fuzzy cardinality among all maximal fuzzy independent set is called independent fuzzy domination number of Γ(G) and is denoted by i (Γ(G)).

Definition 3.8 Minimum fuzzy cardinality among all maximal fuzzy irredundant set is called fuzzy irredundance number and is denoted by ir(Γ(G)).

Definition 3.9 Maximum fuzzy cardinality among all maximal Gallai fuzzy irredundant set is called upper fuzzy irredundance number and is denoted by IR(Γ(G)).

Definition 3.10 A property P of a Gallai fuzzy graph Γ(G) is hereditary if every Gallai fuzzy sub graph of Γ(G) also has this property. A fuzzyset S of fuzzy vertices of Γ (G) is called a fuzzy P-set if the induced Gallai fuzzy sub graph [S] has property P. A fuzzy set is called fuzzy $\bar{P}$- set, if the fuzzy set does not satisfy the property P. A property P is super hereditary if whenever a fuzzy set S has property P, so does every fuzzy super set $S_1^{'}$ ⊃ S.

A fuzzy P-set S is a maximal fuzzy P-set if every proper fuzzy super set $S_1^{'}$ ⊃ S is a fuzzy $\bar{P}$ – set. A fuzzy P-set S is 1- maximal fuzzy P-set if for every fuzzy vertex u ∈ V-S, S∪ $\{u\}$ is a fuzzy $\bar{P}$ – set.A fuzzy P-set S is a minimal fuzzy P-set if every proper fuzzy subset $S_1^{'}$ ⊂ S is a fuzzy $\bar{P}$ – set.A fuzzy P-set S is 1- minimal fuzzy P-set if for every fuzzy vertex
v∈ S, S$-\{v\}$ is a fuzzy$\bar{P}$ – set.

Proposition 3.11 Let Γ(G) =(σ,μ) be a Gallai fuzzy graph. Let P be hereditary property. Then a set D is a maximal fuzzy P-set if and only if D is 1-maximal fuzzy P-set.

Proposition 3.12 Let Γ(G) =(σ,μ) be a Gallai fuzzy graph. Let P be super hereditary property. Then a set D is a minimal fuzzy P-set if and only if D is 1-minimal fuzzy P-set.

Proposition 3.13 If D is independent fuzzy dominating set, then D is both a minimal fuzzy independent set and a maximal fuzzy independent set. Conversely, if D is a maximal fuzzy independent set then D is an independent fuzzy dominating set.

Definition3.14T he Gallai fuzzy graph Γ(G) of a fuzzy graph G has the fuzzy edges of G as its fuzzy vertices and two distinct fuzzy edges of G are fuzzy incident in G, but do not span a fuzzy triangle in G. The line fuzzy graph L(G) of a fuzzy graph G has the fuzzy edges of G as its fuzzy vertices and two distinct fuzzy edges of G are adjacent in L(G) if they are fuzzy incident in G.

Example 3.15 A fuzzy graph G and its Gallai fuzzy graph Γ(G) as shown below

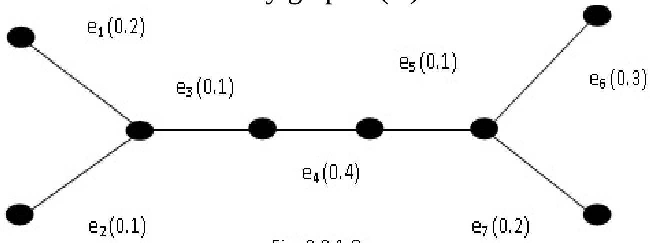


Fig: 2.2.1 G

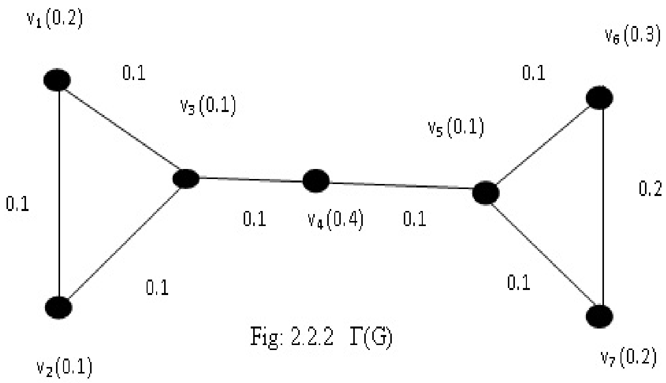


Fig: 2.2.2 Γ(G)

Theorem: 3.16 *(cockayne)* For any Gallai fuzzy graph Γ(G), we have ir(Γ(G)) ≤ γ(Γ(G)) ≤ i(Γ(G)) ≤ β(Γ(G)) ≤ Γ(Γ(G)) ≤ IR(Γ(G)).

Proof:First we have to prove that every minimal fuzzy dominating set in a maximal fuzzy irredundant set. Let D be a γ-set of Γ(Γ(G)), then for every fuzzy vertex v ∈ D, there exist a fuzzy vertex w ∈ V-(D-{v}), which is not fuzzy dominated by D-{v}. This implies that every fuzzy vertex in D has a fuzzy private neighbour. Then D is a fuzzy irredundant set.

Now, we have to prove that D is a maximal fuzzy irredundant set of Γ(G). The property of a set D being a fuzzy irredundant is a hereditary property. By Proposition 3.8, if P is hereditary property of a Gallai fuzzy graph Γ (G), then a set D is a maximal fuzzy P-set if and only if D is a 1-maximal fuzzy P-set. It is enough to prove that D is maximal fuzzy irredundant set. Suppose D is not a maximal fuzzy irredundant set. Then, there exist a fuzzy vertex u ∈ V-D for which D∪{u} is fuzzy irredundant. This means, in particular that PN[u, D∪{u}]≠ φ.That is there exist at least one fuzzy vertex W, which is a fuzzy private neighbor of u with request to D∪{u}.This implies no fuzzy vertex in D is fuzzy adjacent to W. Then D is not a fuzzy dominating set, which is a contradiction to the assumption that D is a fuzzy dominating set. Hence D is a maximal fuzzy irredundant set. Therefore, $(\Gamma(G)) \leq \gamma(\Gamma(G))$.

Now, to prove the second inequality, we have to show that every maximal fuzzy independent set in a Gallai fuzzy graph$\Gamma(G)$ is a minimal fuzzy dominating set. Let D be a maximal fuzzy independent set in $\Gamma(G)$.

By proposition 3.10, a fuzzy independent set is maximal fuzzy independent if and only if it is fuzzy independent and fuzzy dominating. Let D be a maximal fuzzy independent set.
By the definition of maximal fuzzy independent set, for every fuzzy vertex u ∈ V-D, there is a fuzzy vertex v∈ D such that u is fuzzy adjacent to V. This implies that D is a fuzzy dominating set.

Now, we have to prove that D is a minimal fuzzy dominating set. By proposition 3.9, if P is super hereditary property of a Gallai fuzzy graph$\Gamma(G)$, then a set D is a 1-minimal fuzzy P-set if and only if D is minimal fuzzy P-set. A set being a fuzzy dominating set has a super hereditary property.It is enough to prove that, the set D is 1-minimal fuzzy P-set. A fuzzy dominating set D is a minimal fuzzy dominating set if for every fuzzy vertex v ∈ D, the set D-{v}is not a fuzzy dominating set.

Suppose that D is not a minimal fuzzy dominating set. Then there exist at least one fuzzy vertex v ∈ D for which D-{v} is a fuzzy dominating set. But if D-{v} dominates V-(D-{v}), then at least one fuzzy vertex in D-{v} fuzzy adjacent to v. This is a contradiction to the fact that D is fuzzy independent. Then D is a minimal fuzzy dominating set.
Therefore $\gamma(\Gamma(G)) \leq i(\Gamma(G))$.

By definition, $i(\Gamma(G))$ is a minimum fuzzy cardinality of maximal fuzzy independent sets of $\Gamma(G)$and$\beta(\Gamma(G))$ is the maximum fuzzy cardinality of maximal fuzzy independent sets of$\Gamma(G)$. Then $i(\Gamma(G)) \leq \beta(\Gamma(G))$.

Let S be a minimal fuzzy dominating set with maximum fuzzy cardinality. That is,
$|S| = \Gamma(\Gamma(G))$. To prove that $\beta(\Gamma(G)) \leq \Gamma(\Gamma(G))$, we have to prove that S maximal fuzzy independent set with maximal fuzzy cardinality.

Suppose S is not a maximal fuzzy independent set. Then there is a fuzzy vertex W ∈ S-V such that S ∪{w} is a fuzzy independent set. This implies that W is not fuzzy adjacent to any fuzzy vertex in S, then S is not a fuzzy dominating set of $\Gamma(G)$, a contradiction. Therefore, S is a maximal fuzzy independent set. Then $\beta(\Gamma(G)) \leq \Gamma(\Gamma(G))$.

Let S be a maximal fuzzy irredundant set with maximum fuzzy cardinality. That is $|S| = IR(\Gamma(G))$. To prove that $\Gamma(\Gamma(G)) \leq IR(\Gamma(G))$, we have to prove that S is a minimal fuzzy dominating set with maximum fuzzy cardinality. Suppose S is not a minimal fuzzy dominating
Set. Then S-{v} is a fuzzy dominating set. Then S-{v} is a fuzzy dominating set of $\Gamma(G)$,for some v in S. Then v has no fuzzy private neighbour. This implies that S is not an fuzzy irredundant set. Therefore, S is a minimal fuzzy dominating set. Then $\Gamma(\Gamma(G)) \leq IR(\Gamma(G))$.

Hence $ir(\Gamma(G)) \leq \gamma(\Gamma(G)) \leq i(\Gamma(G)) \leq \beta(\Gamma(G)) \leq \Gamma(\Gamma(G)) \leq IR(\Gamma(G))$.

Example: 3.17 The Gallai fuzzy sets $S_1 = \{v_3,v_5\}$, $S_2 = \{v_1,v_5\}$, $S_3=\{v_3,v_6\}$, $S_4=\{v_2,v_5,\}$, $S_5=\{v_3,v_7\}$, $S_6=\{v_1,v_4,v_6\}$, $S_7=\{v_2,v_4,v_7\}$ are maximal fuzzy irredundant sets in the Gallai fuzzy graph given in fig: 2.2.2$\Gamma(G)$. Here $PN[v_{3,}S_1]=\{v_1,v_2,v_3\}$, $PN[v_5,S_1]=\{v_5,v_6,v_7\}$, $PN[v_{1,}S_6]=\{v_1,v_2\}$, $PN[v_{4,}S_6]=\{v_4\}$, $PN[v_{6,}S_6]=\{v_6,v_7\}$.For this Gallai fuzzy graph with fuzzy domination numbers, independent fuzzy domination numbers and fuzzy irredundant numbers are as follows $\gamma(\Gamma(G)) = 0.2$,$i(\Gamma(G)) = 0.2$, $\Gamma(\Gamma(G)) = 0.9$, $\beta(\Gamma(G)) = 0.9$ $ir(\Gamma(G)) = 0.2$, and $IR(\Gamma(G)) = 0.9$.

4. The upper fuzzy domination parameters and minimum fuzzy degree.

The upper fuzzy domination parameters, $\beta(\Gamma(G))$, $\Gamma(\Gamma(G))$ and $IR(\Gamma(G))$ will be combined with minimum fuzzy degree for Gallai-type results of Gallai fuzzy graph.

Theorem 4.1 For any Gallai fuzzy graph$\Gamma(G)$, $IR(\Gamma(G)) + \delta(\Gamma(G)) \leq n$.
Proof: Let S be a maximal fuzzy irredundant set of size $IR(\Gamma(G))$ and let u∈ S. Since S is fuzzy irredundant, there is a fuzzy vertex u such that u∈ N[u]-N[S-{u}].

We consider two cases,

Case (i) :v= u. Then u is not fuzzy adjacent to any fuzzy vertex in S, and must have at least δ(Γ(G)) fuzzy neighbours in V-S. Thus n- IR(Γ(G)) = $|V - S|$ ≥ δ(Γ(G)) and IR((Γ(G)) + δ(Γ(G)) ≤ n.
Case (ii): v≠ u. By the choice of v, v does not belongs to S and N(v) ∩ S = {u}. Then N[v]-{u} is a subset of V-S, so that n-IR(Γ(G)) = $|V - S|$ ≥ $|N[v] - \{u\}|$≥ δ(Γ(G)).
This implies, - IR(Γ(G)) ≥ δ(Γ(G)) – n.

IR(Γ(G)) ≤ - δ(Γ(G)) + n

IR(Γ(G)) +δ(Γ(G)) ≤ n

Using theorem 4.1 and 3.11, we get the following corollary:

Corollary 4.2 For any Gallai fuzzy graph Γ(G), Γ(Γ(G)) + δ(Γ(G)) ≤ n. and β(Γ(G)) + δ(Γ(G)) ≤ n.
Proof:
By Theorem 3.11, Γ(Γ(G)) ≤ IR(Γ(G))

Γ(Γ(G)) + δ(Γ(G)) ≤ IR(Γ(G)) + δ(Γ(G))

By Theorem 4.1, IR(Γ(G)) + δ(Γ(G)) ≤ n.

Therefore Γ(Γ(G)) + δ(Γ(G)) ≤ n.

By Theorem 3.11, β(Γ(G)) ≤ IR(Γ(G))

Thenβ(Γ(G)) + δ(Γ(G)) ≤ IR(Γ(G)) + δ(Γ(G))

By Theorem 4.1, IR(Γ(G)) + δ(Γ(G)) ≤ n.

Thereforeβ(Γ(G)) + δ(Γ(G)) ≤ n.

We will first consider Gallai fuzzy graphs for whichβ(Γ(G)) + δ(Γ(G)) = n.

Theorem 4.3 Let Γ(G) be a connected Gallai fuzzy graph and let I be a maximal fuzzy independent set of Γ(G) such that $|I|$ = β(Γ(G)) .Then β(Γ(G)) + δ(Γ(G)) = n if and only if for each u∈ I, we have fuzzy degree (u) = δ(Γ(G)) and V- N(u) is an fuzzy independent.
Proof: Suppose thatβ(Γ(G)) + δ(Γ(G)) = n.Let u∈ I,
Then I is a subset or equal toV-N[u]. So β(Γ(G) = $|I|$≤ n - $|N(u)|$

≤ n - δ(Γ(G)) = β(Γ(G)).

Since $|N(u)|$ ≥ δ(Γ(G)), implies that - $|N(u)|$ ≤ - δ(Γ(G)).
Thus $|N(u)|$ = δ(Γ(G)), and |V − N(u)| = $|I|$
So V- N(u) is a fuzzy independent set.
Now, suppose that for each u∈ I, fuzzy degree (u) = δ(Γ(G)) and V-N(u) is a fuzzy independent set. Then β(Γ(G))≥ |V − N(u)|.

β(Γ(G)) ≥ n - δ(Γ(G)).

β(Γ(G)) + δ(Γ(G)) ≥ n.

By corollary 4.2, β(Γ(G)) + δ(Γ(G)) ≤ n.

Therefore β(Γ(G)) $= n -$ δ(Γ(G)).

Theorem 4.4 For any Gallai fuzzy graphΓ(G), IR(Γ(G)) +δ(Γ(G)) = n if and only if Γ(Γ(G)) + δ(Γ(G)) = n.
Proof: Let Γ(G) be a Gallai fuzzy graph with Γ(Γ(G)) + δ(Γ(G)) = n.
From the fact that Γ(Γ(G)) ≤ IR(Γ(G)), we have Γ(Γ(G)) + δ(Γ(G)) ≤ IR(Γ(G)) + δ(Γ(G)).
By Theorem 4.1, we have,IR(Γ(G)) + δ(Γ(G)) ≤ n.
Then n = Γ(Γ(G)) + δ(Γ(G)) ≤ IR(Γ(G)) + δ(Γ(G)) ≤ n. Hence IR(Γ(G)) + δ(Γ(G)) = n.
Conversely,

Suppose that Γ(G) is a Gallai fuzzy graph with IR(Γ(G)) + δ(Γ(G)) = n.

Let S be a maximal fuzzy irredundant set for Γ(G) with |S| = IR(Γ(G)) . We will show that S is fuzzy dominating and since S is fuzzy irredundant, it will be a minimal fuzzy dominating set. So suppose that S is not fuzzy dominating. Then there is a w ∈ V- S such that w is not fuzzy adjacent to any fuzzy vertex of S. Then N[w] is a subset or equal to V-S. But |N[w]|≥ δ(Γ(G)) +1,
Which implies δ(Γ(G)) + 1 ≤ |N[w]| ≤ |V − S|.

$\delta(\Gamma(G)) + 1 = n - IR(\Gamma(G))$ and so $IR(\Gamma(G)) + \delta(\Gamma(G)) \leq n - 1$, a contradiction. Thus S is a fuzzy dominating set. Since $\Gamma(\Gamma(G)) \geq |S| = IR(\Gamma(G)) \geq \Gamma(\Gamma(G))$,
we must have $IR(\Gamma(G)) = \Gamma(\Gamma(G))$.Therefore $\Gamma(\Gamma(G)) + \delta(\Gamma(G)) = n$.

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