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ON CONTRA PRE-ρ-CONTINUITY AND CONTRA SEMI-ρ-CONTINUITY WHERE $\rho\in\{L,M,R,S\}$

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Abstract: In1996, Dontchev introduced contra-continuous functions. In 2013, R.Selvi and and P.Thangavelu are introduced contra-ρ-continuity where $\rho\in\{L,M,R,S\}$. In 2015, R.Selvi and M.Priyadarshini are introduced pre-ρ-continuity and semi-ρ-continuity betweena topological space and a non-empty set where $\rho \in\{L,M,R,S\}$. The purpose of this paper is to introduce the concepts of contra pre-ρ-continuity and contra semi-ρ-continuity between a topological space and a non-empty set.

Keywords: Contra pre-ρ-continuity, Contra semi-ρ-continuity, pre-open, pre-closed, semi-open, semi-closed, multi-functions, saturated set, continuity.

AMSSubjectClassification:32A12,54C05,54C60,26E25

1.Introduction

Any function f:X→Y induces a multi function $f^{-1}\circ f$: X→℘(X). It is also induces another multi function $f^{-1}\circ f$: Y→℘(Y) provided f is surjective. The purpose of this paper is to introduce the notions of Contra pre-ρ continuity and Contra semi-ρ-continuity where ρ∈{L,M,R,S} of a function f:(X,τ)→(Y,σ) between a topological space and a non-empty set. Navpreetsingh Noorie and Rajni Bala [8] introduced the concept of $f^{\#}$ function to characterize the closed, open and continuous functions. The authors [12] characterized continuity by using $f^{\#}$ functions. In analog way Contra pre-ρ-continuity and Contra semi-ρ-continuity are characterized in this paper.

2.Preliminaries

In this chapter we list some basic definitions and results that are needed to develop the thesis.

Definition 2.1

Let A be a subset of a topological space X. Then A is called

(i) Semi-open [5] if A ⊆ *cl*(*int*(A)) and semi-closed if *int*(*cl*(A))⊆A;

(ii) Pre-open[6] if A⊆ *int*(*cl*(A)) and pre-closed if *cl*(*int*(A))⊆A.

Definition 2.2

Let f:(x,τ)→Y be a function.Then f is

(i) L-continuous if f^{-1}(f(A)) is open in X for every open set A in X. [11]

(ii) M-continuous if f^{-1}(f(A)) is closed in X for every closed set A in X.[11]

Definition 2.3

Let f:X→(Y,σ) be a function. Then f is

(i) R-continuous if f(f^{-1}(B)) is open in Y for every openset B inY. [11]

(ii) S-continuous if f(f^{-1}(B)) is closed inY for every closed set B inY. [11]

Definition 2.4

Let f:X→Y be any map and E be any subset of X. Then the following hold.

(i)$f^{\#}$ (E)={y∈Y:f^{-1}(y)⊆E};(ii)$E^{\#}$ =f^{-1}($f^{\#}$ (E)). [8]

Lemma 2.5

Let E be a subset of X and f:X→Y be a function. Then the following hold.

(i)$f^{\#}$(E)=Y\f(X\E);(ii)f(E)=Y\$f^{\#}$(X\E). [8]

Lemma 2.6

Let E be a subset of X and f:X→Y be a function. Then the following hold.

(i) f^{-1}($f^{\#}$ (E))=X\ f^{-1}(f(X\E));(ii) f^{-1}(f(E))=X\f^{-1}($f^{\#}$(X\E)). [12]

Lemma 2.7

Let E be a subset of X and f:X→Y be a function.Then the following hold.

(i) $f^{\#}$(f^{-1}(E))=Y\f(f^{-1}(Y\E));(ii) f(f^{-1}(E))=Y\$f^{\#}$ (f^{-1}(Y\E)).[12]

Definition 2.8

Letf:X→Y, A⊆X and B⊆Y. Then we say that A is f-saturated, if f^{-1}(f(A))⊆A and B is f^{-1}-saturated if f(f^{-1}(B))⊇B. Equivalently A is f-saturated if and only if f^{-1}(f(A))=A, and B is f^{-1}-saturated if and only if f(f^{-1}(B))=B. [11]

Definition 2.9

Let f:(X,τ)→(Y,σ) be a function.Then f is pre-continuous if f^{-1}(B) is pre-open in X for every openset B inY. [6]

Definition 2.10

(i) Let f:(X,τ)→(Y,σ) be a function. Then f is pre-open if f(A) is pre-open in Y for every pre-open set A in X. [6]

(ii) Let f:(X,τ)→(Y,σ) be a function.Then f is pre-closed if f(A)is pre closed in Y for every pre-closed set A in X. [6]

Definition 2.11

Let$f:(X,\tau)\to(Y,\sigma)$ be a function. Then f is semi-continuous if f^{-1}(B)is semi-open in X for every open set B in Y. [5].

Definition 2.12

(i) Let $f:(X,\tau)\to(Y,\sigma)$be a function.Then f is semi-open if f(A) is semi-open in Y for every semi-open set A in X.[5]

(ii) Let $f:(X,\tau)\to(Y,\sigma)$ be a function.Then f is semi-closed if f(A)is semi-closed inY for every semi-closed set A in X.[5]

Definition 2.13

Let $f:(X,\tau)\to Y$ be a function. Then f is[13]

(i) Contra L-continuous if $f^{-1}(f(A))$ is open in (X,τ) for every closed set A in (X,τ).

(ii) Contra M-continuous if $f^{-1}(f(A))$ is closed in$(X,\tau)$for every open set A in (X,τ).

Definition 2.14

Let $f:X\to(Y,\sigma)$ be a function. Then f is [13]

(i) Contra R-continuous if $f(f^{-1}(B))$ is openin(Y,σ) for every closed set B in (Y,σ).

(ii) Contra S-continuous if $f(f^{-1}(B))$ is closed in (Y,σ) for every open set B in (Y,σ).

Definition 2.15

Let$f:(X,\tau)\to Y$be a function.Then f is [9]

(i) pre-L-continuous if $f^{-1}(f(A))$ is pre-open in $(X,\tau)$for every open set A in (X,τ).

(ii) semi-L-continuous if $f^{-1}(f(A))$ is semi-open in (X,τ) for every opens et A in (X,τ).

(iii) pre-M-continuous if $f^{-1}(f(A))$ is pre-closed in (X,τ) for every closed set A in (X,τ).

(iv) semi-M-continuous if $f^{-1}(f(A))$is semi-closed in (X,τ) for every closed set A in (X,τ).

Definition 2.16

Let $f:X\to(Y,\sigma)$ be a function. Then f is [9]

(i) pre-R-continuous if $f(f^{-1}(B))$ is pre-open in (Y,σ) for every open set B in (Y,σ).

(ii) semi-R-continuous if $f(f^{-1}(B))$ is semi-open in (Y,σ) for every open set B in (Y,σ).

(iii) pre-S-continuous if $f(f^{-1}(B))$ is pre-closed in (Y,σ) for every closed set B in (Y,σ).

(iv) semi-S-continuous if $f(f^{-1}(B))$ is semi-closed in (Y,σ) for every closed set B in (Y,σ).

3. Contra pre-ρ-continuity and contra semi -ρ-continuity where $\rho\in\{L,M,R,S\}$

Definition 3.1

Let$f:(X,\tau)\to Y$beafunction.Thenfis

(i) Contra pre-L-continuous if $f^{-1}(f(A))$ is pre-open in (X,τ) for every closed set A in (X,τ).

(ii) Contra semi-L-continuous if $f^{-1}(f(A))$ is semi-open in (X,τ) for every closed set A in (X,τ).

(iii) Contra pre-M-continuous if $f^{-1}(f(A))$ is pre-closed in (X,τ) for every open set A in (X,τ).

(iv) Contra semi-M-continuous if $f^{-1}(f(A))$ is semi-closed in (X,τ) for every open set A in (X,τ).

Definition 3.2

Let f:X→(Y,σ) be a function. Then f is

(i) Contra pre-R-continuous if $f(f^{-1}(B))$ is pre-open in (Y,σ) for every closed set B in (Y,σ).

(ii) Contra semi-R-continuous if $f(f^{-1}(B))$ is semi-openin(Y,σ) for every closed set B in (Y,σ).

(iii) Contra pre-S-continuous if $f(f^{-1}(B))$ is pre-closed in(Y,σ) for every open set B in (Y,σ).

(iv) Contra semi-S-continuous if $f(f^{-1}(B))$ is semi-closed in (Y,σ) for every open set B in (Y,σ).

Example 3.3

Let X={a,b,c} and Y={1,2,3}. Let τ={Φ,X,{a},{b},{a,b},{a,c}}.Letf:(X,τ)→Y defined by f(a)=1, f(b)=2, f(c)=1.Then f is both Contra pre-ρ-continuous and Contra semi-ρ-continuous where ρ∈{L,M}.

Example 3.4

Let X={a,b,c} and Y={1,2,3,4}. Let τ={Φ,X,{a},{a,b}} and σ={Φ,Y,{1},{2},{1,3,4}}. Let g:X→(Y,σ) defined by g(a)=2, g(b)=2 and g(c)=2.Then g is both Contra pre-ρ-continuous and Contra Semi-ρ-continuous where ρ∈{R,S}.

Definition 3.5

Let f:(X,τ)→(Y,σ) be a function. Then f is

(i) Contra pre-LR-continuous, if it is both Contra pre-L-continuous and Contra pre-R-continuous.

(ii) Contra pre-LS-continuous, if it is both Contra pre-L-continuous and Contra pre-S-continuous.

(iii) Contra pre-MR-continuous, if it is both Contra pre-M-continuous and Contra pre-R-continuous.

(iv) Contra pre-MS-continuous, if it is both Contra pre-M-continuous and Contra pre-S-continuous.

(v) Contra semi-LR-continuous, if it is both Contra semi-L-continuous and Contra semi-R-continuous.

(vi) Contra semi-LS-continuous, if it is both Contra semi-L-continuous and Contra semi-S-continuous.

(vii) Contra semi-MR-continuous, if it is both Contra semi-M-continuous and Contra semi-R-continuous.

(viii) Contra semi-MS-continuous, if it is both Contra semi-M-continuous and Contra semi-S-continuous.

We now characterize the contra ρ-continuity defined above by using the concept of open, closed and continuous functions.

Theorem 3.6

Let *f*:(X,τ)→(Y,σ) be open and contra continuous. Then *f* is both Contra pre-MR- continuous and Contra semi-MR-continuous.

Proof: Let A⊆X be open in X. Since *f* is open, *f*(A) is open in Y. Again since *f* is contra continuous, $f^{-1}(f(A))$ is closed in X. Since every closed set is both pre-closed and semi-closed, $f^{-1}(f(A))$ is both pre-closed and semi-closed. This proves f is Contra pre-M-continuous and Contra semi-M-continuous.

Now let B be aclosed subset of Y. Since f is contra continuous, f^{-1}(B) is open in X. Since f is open, $f(f^{-1}$(B)) is open in Y. Since every open set is both pre-open and semi-open, $f(f^{-1}$(B)) is both pre-open and semi-open. This proves f is both Contra pre-R-continuous and Contra semi-R-continuous. This shows that f is both Contra pre-MR-continuous and Contra semi-MR-continuous.

Theorem 3.7

Let f:(X,τ)→(Y,σ) be closed and contra continuous. Then f is both Contra pre-LS- continuous and Contra semi-LS-continuous.

Proof: Let A ⊆ X be closed in X. Since f is closed, f(A) is closed in Y. Again since f is contra continuous $f^{-1}(f$(A)) is open in X. Since every open set is both pre-open and semi-open, $f^{-1}(f$(A)) is both pre-open and semi-open.This proves f is Contra pre-L-continuous and Contra semi-L-continuous.

Now let B be an open subset of Y. Since f is contra continuous, f^{-1}(B) is closed in X and since f is closed $f(f^{-1}$(B)) is closed inY. Since every closed set is both pre-closed and semi-closed, $f(f^{-1}$(B)) is both pre-closed and semi-closed. This proves f is Contra pre-S-continuous and Contra semi – S-continuous. This shows that f is both Contra pre-LS-continuous and Contra semi – LS-continuous.

Corollary 3.8

Let f:(X,τ)→(Y,σ) be open, closed and contra continuous. Then f is both Contra pre-ρ continuous and Contra Semi-ρ-continuous where ρ∈{L,M,R,S}.

Proof: Follows from Theorem3.6 and Theorem 3.7.

Theorem 3.9

Let g:Y→Z and f:X→Y be any two functions. Then the following hold.

(i) Let f be closed and continuous. If g is contra L-continuous then g ∘f:X→Z is both Contra pre-L-continuous and Contra semi-L-continuous;

(ii) Let f be open and continuous.If g is contra M-continuous the g ∘f:X→Z is both Contra pre-M-continuous and Contra semi-M-continuous;

(iii) Let f be open, closed and continuous. If g is contra LM-continuous then g ∘f:X→Z is both Contra pre-LM-continuous and Contra semi-LM-continuous;

(iv) Let g be open and continuous. If f is contra R-continuous then g ∘f:X→Z is both Contra pre-R-continuous and Contra semi-R-continuous.

(v) Let g be closed and continuous. If f is contra S-continuous then g ∘f:X→Z is both Contra pre-S-continuous and Contra semi-S-continuous;

(vi) Let g beopen, closed and continuous. If f is contra RS-continuous then g ∘f:X→Z is both Contra pre-RS-continuous and Contra semi-RS-continuous.

Proof: Suppose g is contra L-continuous. Let f be closed and continuous. Let A be closed in X. Then (g∘ $f)^{-1}$(g∘ f)(A)=f^{-1}(g^{-1}(g(f(A)))). Since f is closed, f(A) is closed in Y. Since g is contra L-continuous, g^{-1}(g(f(A))) is open in Y. Since f is continuous, f^{-1}(g^{-1}(g(f(A)))) is open in X. Since every open set is both pre-open and semi-open, f^{-1}(g^{-1}(g(f(A)))) is both pre-open and semi-open. Therefore, g∘f is Contra pre-L-continuous and Contra semi-L- continuous. This proves(i).

Suppose g is contra M continuous. Let f be open and continuous. Let A be open in X. Since f is open, f(A) is open inY. Since g is contraM-continuous, g^{-1}(g(f(A))) is closed in Y. Since f is continuous, f^{-1}(g^{-1}(g(f(A)))) is closed in X. Since every closed set is both pre- closed and semi-closed,

$f^{-1}(g^{-1}(g(f(A))))$ is both pre-closed and semi-closed. Therefore, g ∘*f* is Contra pre-M-continuous and Contra semi-M-continuous. This proves (ii). (iii) follows from (i) and (ii)

Let *f*:X→Y be contra R-continuous and g:Y→Z be open and continuous. Let B be closed in Z. Then $(g\circ f)(g\circ f)^{-1}(B)=(g\circ f)(f^{-1}g^{-1}(B))=g(f(f^{-1}(g^{-1}(B)))$. Since g iscontinuous,$g^{-1}(B)$ is closed in Y. Since *f* is contraR-continuous, $ff^{-1}(g^{-1}(B))$ is open in Y. Since g is open, $g(f(f^{-1}(g^{-1}(B))))$ is open in Z. Since every open set is both pre-open and semi- open, $g(f(f^{-1}(g^{-1}(B))))$ is both pre-open and semi-open. Therefore, g°*f* is both Contra pre-R- continuous and Contra semi-R-continuous. This proves (iv).Let *f*:X→Y be contra S-continuous and g:Y→Z be closed and continuous. Let B be open in Z. Since g is continuous, $g^{-1}(B)$ is open in Y. Since *f* is contra S-continuous, $ff^{-1}(g^{-1}(B))$ is closed in Y. Since g is closed, $g(f(f^{-1}(g^{-1}(B))))$ is closed in Z. Since every closed set is both pre-closed and semi-closed, $g(f(f^{-1}(g^{-1}(B))))$ is both pre-closed and semi-closed. Therefore, g°*f* is Contra pre-S-continuous and Contra semi-S-continuous. This proves (v).(vi) follows from (iv) and (v).

Theorem 3.10

Let *f*:(X,τ)→Y be a function and A be a subset of X.Then the following hold.

(i) If *f*:(X,τ)→Y is contra M-continuous and if A is an open subspace of X then the restriction of *f* to A is both contra pre-M-continuous and contra semi-M- continuous.

(ii) If*f*:(X,τ)→Y is contra L-continuous and if A is a closed subspace of X then the restriction of *f* to A is both contra pre-L-continuous and contra semi-L-continuous

(iii) If *f*:(X,τ)→Y is contra LM-continuous and if A is a clopen subspace of then the restriction of *f* to A is both contra pre-LM-continuous and contra semi-LM- continuous.

Proof: Suppose *f*:(X,τ)→Y is contra M-continuous and if A is an open subspace of X. Let $h=f|_A$. Then $h=f\circ j$ where *j* is the inclusion map *j*:A→X. Since A is open, *j* is open and continuous. Since *f*:X→Y is contraM-continuous, using Theorem3.9(ii), *h* is both contra pre-M-continuous and contra semi-M-continuous. This proves

(i) Let *f*:(X,τ)→Y be contra L-continuous and if A be a closed subspace of X. Since A is closed, *j* is closed and continuous. Since *f*:(X,τ)→Y is contra L-continuous, using Theorem3.9 (i), *h* is both contra pre-L-continuous and contra semi-L-continuous. This proves (ii). (iii) follows from (i) and (ii).

Theorem 3.11

Let *f*:X→(Y,σ) be a function *f*(X)⊆Z⊆Y. Suppose *h*:X→Z is defined by h(x)=*f*(x) for all x∈X. Then the following hold.

(i) If *f*:X→(Y,σ) is contra R-continuous and *f*(X) be open in Z, then *h* is both contra pre-R-continuous and contra semi-R-continuous.

(ii) If *f*:X→(Y,σ) is contra S-continuous and *f*(X) is closed in Z, then *h* is both contra pre-S-continuousandcontrasemi-S-continuous.

(iii) If *f*:X→(Y,σ) is contra RS-continuous and *f*(X) is clopen in Z, then *h* is both contra pre-RS-continuous andcontra semi-RS-continuous.

Proof: By the Definition of h,we see that $h=j\circ f$ where *j*:*f*(X)→Z is an inclusion map. Suppose *f*:X→(Y,σ) is contra R-continuous and *f*(X) is open in Z, that implies the inclusion map *j* is both open and continuous.Then by applying Theorem3.9 (iv), *h* is both contra pre-R-continuous and contra semi-R-continuous.This proves (i).

Suppose f:X→(Y,σ) is contra S-continuous and f(X) is closed in Z. Since f(X) is closed in Z, the inclusion map j is closed and continuous. Then by applying Theorem3.9(v), h is both contra pre-S-continuous and contra semi-S-continuous. This proves (ii).(iii) follows from (i) and (ii).

4. Pasting Lemma

We establish the pasting Lemmas for contra pre-R-continuous, contra pre-S- continuous, contra semi-R-continuous and contra semi-S-continuous functions.

Theorem 4.1

Let X=A∪B. Let f:A→(Y,σ) and g:B→(Y,σ) be contra R-continuous (resp.contraS-continuous) functions.If f(x)=g(x) for every x∈A∩B, function h:X→Y defined by $h(x)=\begin{cases} f(x), & x\in A \\ g(x), & x\in B \end{cases}$ is contra pre-R-continuous(resp.contrapre-S-continuous).

Proof: Let C be a open(resp.closed) set inY. Therefore,

$$\begin{aligned} h\circ h^{-1}(C) &= h(f^{-1}(C)\cup g^{-1}(C)) \\ &= h(f^{-1}(C))\cup h(g^{-1}(C)) \\ &= f(f^{-1}(C))\cup g(g^{-1}(C)) \end{aligned}$$

Since f is contra R-continuous(resp.contra S-continuous), $f(f^{-1}(C))$ is open (resp.closed) in Y and since g is contra R-continuous (resp.contra S-continuous), $g(g^{-1}(C))$ is open (resp.closed) in Y. Therefore, $h\circ h^{-1}(C)$ is also open (resp.closed) in Y. Since every open set is pre-open (every closed set is pre-closed), $h\circ h^{-1}(C)$ is also pre-open (resp.pre-closed) in Y. This shows that h is contra pre-R-continuous (resp.contra pre-S-continuous).

Theorem 4.2

Let X=A∪B. Let f:A→(Y,σ) and g:B→(Y,σ) be contra R-continuous (resp. contraS-continuous) functions. If f(x)=g(x) for every x∈A∩B, function h:X→Y defined by

$h(x)=\begin{cases} f(x), & x\in A \\ g(x), & x\in B \end{cases}$ is contra semi-R-continuous (resp.contra semi-S-continuous).

Proof: Let C be a open (resp.closed)set in Y.Therefore,

$$\begin{aligned} h\circ h^{-1}(C) &= h(f^{-1}(C)\cup g^{-1}(C)) \\ &= h(f^{-1}(C))\cup h(g^{-1}(C)) \\ &= f(f^{-1}(C))\cup g(g^{-1}(C)) \end{aligned}$$

Since f is contra R-continuous(resp.contra S-continuous), $f(f^{-1}(C))$ is open (resp.closed) in Y and since g is contra R-continuous (resp.contra S-continuous), $g(g^{-1}(C))$ is open (resp.closed) in Y. Therefore ,$h\circ h^{-1}(C)$ is also open (resp.closed) in Y. Since every open set is semi-open (every closed set is semi-closed), $h\circ h^{-1}(C)$ is also semi-open (resp.semi-closed) inY. This shows that h is contra semi-R-continuous(resp.contra semi-S-continuous).

5.Characterizations

In this section we characterize contraρ-continuity and almost contraρ-continuity functions by the hash functions $f^{\#}$of f:X→Y.

Theorem5.1

The function f:(X,τ)→Y is contra pre-L-continuous (contra semi-L-continuous) if andonly if $f^{-1}(f^{\#}(G))$ is pre-closed (semi-closed) in X for every open subset G of X.

Proof:

Suppose f is contra pre-L-continuous (contrasemi-L-continuous). Let G be open in X. Then A=X\G is closed in X. By Lemma 2.6 (i) ,$f^{-1}(f^{\#}(G))=X\setminus f^{-1}(f(A))$. Since f is contra pre-L-continuous (contra semi-L-continuous) and since A is closed in X, $f^{-1}(f(A))$ is pre-open (semi-open) in X. Hence $f^{-1}(f^{\#}(G))$ is pre-closed (semi-closed) in X. Conversely assume that $f^{-1}(f^{\#}(G))$ is pre-closed (semi-closed) in X for every open subset G of X. Let A be closed in X. By Lemma 2.6 (ii),$f^{-1}(f(A))=X\setminus f^{-1}(f^{\#}(G))$ where G=X\A. By our assumption, $f^{-1}(f^{\#}(G))$ is pre-closed (semi closed) and hence $f^{-1}(f(A))$ is pre-open (semi-open) in X. Therefore f is contra pre-L-continuous (contra semi-L-continuous).

Theorem 5.2

The function f:(X,τ)→Y is contra pre-M-continuous (contra semi-M-continuous) if and only if $f^{-1}(f^{\#}(A))$ is pre-open (semi-open) in X for every closed subset A of X.

Proof: Suppose f is contra pre-M-continuous (contra semi-M-continuous). Let A be closed in X. Then G=X\A is open in X. By Lemma2.6 (i),$f^{-1}(f^{\#}(A))=X\setminus f^{-1}(f(G))$. Since f is contra pre-M-continuous (contra semi-M-continuous) and since G is open in X, $f^{-1}(f(G))$ is pre-closed (semi-closed) in X. Hence $f^{-1}(f^{\#}(A))$ is pre-open (semi-open) in X. Conversely assume that $f^{-1}(f^{\#}(A))$ is pre-open (semi-open) in X for every closed subset A of X. Let G be open in X. By Lemma 2.6 (ii), $f^{-1}(f(G))=X\setminus f^{-1}(f^{\#}(A))$ where A=X\G. By our assumption, $f^{-1}(f^{\#}(A))$ is pre-open (semi-open) and hence $f^{-1}(f(G))$ is pre-closed (semi-closed) in X. Therefore f is contra pre-M-continuous(contra semi-M-continuous).

Theorem5.3

The function$f$:X→(Y,σ) is contra pre-R-continuous (contra semi-R-continuous)if and only if $f^{\#}(f^{-1}(G))$ is pre-closed(semi-closed) in Y for every open subset G of Y.

Proof: Suppose f is contra pre-R-continuous(contra semi-R-continuous). Let G be open in Y. Then A=Y\G is closed in Y. By Lemma 2.7(i), $f^{\#}(f^{-1}(G))=Y\setminus f(f^{-1}(A))$.Since f is contra pre-R-continuous (contrasemi-R-continuous) and since A is closed in Y, $f(f^{-1}(A))$ is pre-open (semi-open) in Y. Hence $f^{\#}(f^{-1}(G))$ is pre-closed (semi-closed) in Y.Conversely assume that $f^{\#}(f^{-1}(G))$ is pre-closed (semi-closed) in Y for every open subset G of Y. Let A be closed in Y. By Lemma 2.7(ii),$f(f^{-1}(A))=Y\setminus f^{\#}(f^{-1}(G))$ where G=Y\A.B your assumption, $f^{\#}(f^{-1}(G))$ is pre-closed (semi-closed) and hence $f(f^{-1}(A))$ is pre-open (semi-open) inY. Therefore f is contrapre-R-continuous(contra semi-R-continuous).

Theorem5.4

The function f:X→(Y,σ) is contra pre-S-continuous (contra semi-S-continuous) if and only if $f^{\#}(f^{-1}$(A)) is pre-open (semi-open) in Y for every closed subset A of Y.

Proof: Suppose f is contra pre-S-continuous (contrasemi-S-continuous). Let A be closed inY. Then G=Y\A is open inY. By Lemma2.7(i), $f^{\#}(f^{-1}$(A))=X\\$f(f^{-1}$(G)). Since f is contra pre- S-continuous (contrasemi-S-continuous) and since G is open inY, ff^{-1}(G)) is pre-closed (semi-closed) in Y. Hence $f^{\#}(f^{-1}$(A)) is pre-open (semi-open)inY. Conversely assume that $f^{\#}(f^{-1}$(A)) is pre-open (semi-open) inY for every closed subset A of Y.Let G be open in Y. By Lemma2.7 (ii), $f(f^{-1}$(G))=X\\$f^{\#}(f^{-1}$(A)) where A=Y\G. By our assumption, $f^{\#}(f^{-1}$(A)) is pre-open (semi-open) in Y and hence $f(f^{-1}$(G)) is pre-closed (semi-closed) in Y. Therefore f is contra pre-S-continuous (contra semi-S-continuous).

6. Conclution

In this paper the notions of Contra pre-ρ-continuous and Contra Semi-ρ-continuous where ρ∈{L,M,R,S}of a function f:X→Y between a topological space and a non empty set are introduced. Here we discuss their links with pre-open (semi-open), pre-closed (semi-closed) sets.Also we establishpasting lemmas for Contra pre-ρ-continuous and Contra Semi-ρ-continuous where ρ∈ {R,S} functions and obtain some characterizations for,Contrapre-ρ- continuous and ContraSemi-ρ-continuous where ρ∈{L,M,R,S}.W ehave put forward some examples to illustrate our notions.

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