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MINIMAL FUZZY SOFT OPEN SETS WITH RESPECT TO FUZZY SOFT IDEALS

***J.Ruth*[1], *S.Selvam*[2]**

[1]Part-time Ph.D., Research Scholar, PG & Research Department of Mathematics, Raja Doraisingam Government Arts College, Sivagangai-630561. (Affiliated to Alagappa University, Karaikudi, Tamilnadu), India.Email:josephineruth3@gmail.com

[2]Department of Mathematics, Assistant Professor, Government Arts and Science College,Tiruvadanai-623407.(Affiliated to Alagappa University, Karaikudi,Tamilnadu), India.Email: selvammaths123@gmail.com

Abstract: In this paper we introduce the conceptof minimal fuzzy soft open setsin fuzzy soft topological spaces defined in Lowen's sense.We also investigate the minimal fuzzy soft open sets with respect to fuzzy soft ideals and obtain some of itsproperties.

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1.Preliminaries and Introduction

The concept of fuzzy set theory was introduced by L.A.Zadeh [6] .R.Lowen [4] defined the fuzzy topology. The concept of soft set was initiated by Molodtsov [5]. Some structural properties of fuzzy soft topological spaces was discussed in Ruth [2]. In this paper we investigate the notion of minimal fuzzy soft open sets with respect to fuzzy soft ideals in Lowen's sense.Throughout this paper, X be an initial universe and E be the set of all parameters for X, I^X is the set of all fuzzy sets on X. (where,I=[0,1] and for $\lambda\in[0,1]$,$\overline{\lambda}(x)=\lambda$, for all $x\in X$.)

The following definitions and theorems are in Ruth [2] which are needed for our study.

Definition 1.1 *[1]*. Let $A\subseteq E$.f_A is called a fuzzy soft set on X, where f is a mapping from E into I^X.

i.e.,$f_e \triangleq f(e) \triangleq f_A(e)$ is a fuzzy set on X for each e ∈ A and$f_e = \bar{0}$if e ∉ A, where $\bar{0}$ is zero function on X.f_e for each e ∈ E, is called an element of the fuzzy soft set f_A.

FS(X,E) denotes the collection of all fuzzy soft sets on X and is called a fuzzy soft universe.If f_D∈FS(X,E) then we understand that D ⊆ E. In this paper to each parameter e∈ E,f_e is equivalent to $f_e(x)$ for all x ∈ X.

Definition 1.2 *[1]* For two fuzzy soft sets f_A and g_B on X, we say thatf_A is a fuzzy soft subset of g_B and write $f_A \sqsubseteq g_B$iff$f_e \leq g_e$, for each e ∈ E.

Definition 1.3. *[1]* Two fuzzy soft sets f_A and g_B on X are called equal iff$_A \sqsupseteq g_B$ and $g_B \sqsupseteq f_A$.

Defiition 1.4. *[1]*. Union of two fuzzy soft sets f_A and g_B on X is the fuzzy soft set
$h_C = f_A \sqcup g_B$, where C = A ∪ B and $h_e = f_e \vee g_e$, for each e∈ E. That is, $h_e = f_e \vee \bar{0} = f_e$ for each e∈A-B, $h_e = \bar{0} \vee g_e = g_e$ for each e∈B-A and $h_e = f_e \vee g_e$, for each e∈A ∪ B.

Definition 1.5. *[1]* Intersection of two fuzzy soft sets f_A and g_B on X isthe fuzzy soft set
$h_C = f_A \sqcap g_B$, where C = A ∩ B and $h_e = f_e \wedge g_e$, for each e∈E.

Definition 1.6. *[1]* The complement of a fuzzy soft setf_A is denoted by f_A^c, where $f^c : E \rightarrow I^X$ is a mapping given by$f_e^c = \bar{1} - f_e$, for each e ∈ E. Clearly $(f_A^c)^c = f_A$.

Definition 1.7. *[1]* (Null fuzzy soft set) A fuzzy soft set f_E on X is calleda null fuzzy soft set and denoted by ∅ if $f_e = \bar{0}$, for each e ∈ E.

Definition 1.8. *[1]* (Absolute fuzzy soft set) A fuzzy soft set f_E on X iscalled an absolute fuzzy soft set and denoted by $\tilde{E}$, iff$_e = \bar{1}$, for each e ∈ E. Clearly, $(\tilde{E})^c = \emptyset$ and $\emptyset^c = \tilde{E}$.

Definition 1.9. *[1]* (λ -absolute fuzzy soft set) A fuzzy soft set f_E on X is called a λ-absolute fuzzy soft set and denoted by $\tilde{E}^\lambda$, if $f_e = \bar{\lambda}$, for each e ∈E. Clearly, $(\tilde{E}^\lambda)^c = \tilde{E}^{1-\lambda}$.

In this paper we write $\tilde{E}^\lambda$as$\bar{\lambda}_E$.

Theorem 1.10. *[1]* Let Δ be an index set and f_A, g_B, h_C, $(f_A)_i \triangleq (f_i)_{Ai}$,
$(g_B)_i \triangleq (g_i)_{Bi}$∈ FS(X,E), $\forall_i$ ∈Δ, then we have the following properties:

(1) $f_A \sqcap f_A = f_A$, $f_A \sqcup f_A = f_A$.

(2) $f_A \sqcap g_B = g_B \sqcap f_A$, $f_A \sqcup g_B = g_B \sqcup f_A$.

(3) $f_A \sqcup (g_B \sqcup h_C) = (f_A \sqcup g_B) \sqcup h_C$.

(4) $f_A = f_A \sqcap (f_A \sqcup g_B)$, $f_A = f_A \sqcup (f_A \sqcap g_B)$

(5) $f_A \sqcap (\sqcup_{i\in\Delta} (g_B)_i) = (\sqcup_{i\in\Delta}(f_A \sqcap (g_B)_i))$

(6) $f_A \sqcup (\sqcap_{i\in\Delta} (g_B)_i) = (\sqcap_{i\in\Delta}(f_A \sqcup (g_B)_i))$

(7) $\emptyset \sqsubseteq f_A \sqsubseteq \tilde{E}$.

(8) $(f_A^c)^c = f_A$.

(9) $(\sqcap_{i\in\Delta}(f_A)_i)^c = \sqcup_{i\in\Delta}((f_A)_i)^c$

(10) $(\sqcup_{i\in\Delta}(f_A)_i)^c = \sqcap_{i\in\Delta}((f_A)_i)^c$

(11) If $f_A \sqsubseteq g_B$, then $g_B^c \sqsubseteq f_A^c$

Definition 1.11. *(Fuzzy soft Ideal) [2]* Let A, B ⊆ E. A non empty family $\mathcal{J} \subseteq$ FS(X,E) of fuzzysoft sets is called fuzzy soft ideal on X if

i) $f_A \in \mathcal{J}$, $g_B \sqsubseteq f_A$ implies that $g_B \in \mathcal{J}$

ii) $f_A \in \mathcal{J}$, $g_B \in \mathcal{J}$ implies that $f_A \sqcup g_B \in \mathcal{J}$. (As $\mathcal{J}$ is not empty $\emptyset \in \mathcal{J}$)

Definition 1.12. *[2]* Let C, D and P⊆ E. A family $\tau \subseteq$FS(X,E) is called a fuzzy soft topology for X, if it satisfies the following axioms.

i) For all $\lambda \in [0,1]$, $\tilde{\lambda}_E \in \tau$.

ii) $f_C, g_D \in \tau$ implies that $f_C \sqcap g_D \in \tau$.

iii) If $\{f_{iP}\}_{i\in\Delta}$ is an indexed subfamily of τ , then $\sqcup_{i\in\Delta} f_{iP} \in \tau$.

The pair (X, τ) is called a fuzzy soft topological space. The members of τ are called fuzzy soft open sets.

2.Minimal fuzzy soft open sets

Definition 2.1. Let (X,τ) be a fuzzy soft topological space ,$\lambda \in (0,1]$,$e_0 \in E$, $f_A \in \tau$, where A⊆E such that $\bar{\lambda} \le f_A(e_0)$, f_A is called a local minimal fuzzy soft open set at e_0,
w.r.t.λ, If $g_A \in \tau$ and $g_A(e_0) \ge \bar{\lambda}$, then $g_A \not\subset f_A$

Example 2.2. Let (X,τ) be a fuzzy soft topological space with indiscrete fuzzy soft topology τ in Ruth [2]. Let $e_0 \in E$ then to each $\lambda \in (0,1]$,$\tilde{\lambda}_E$ is a local minimal fuzzy soft open set at e_0.

Definition 2.3. Let (X,τ) be a fuzzy soft topological space with fuzzy soft ideal I on X ,$e_o \in E$ and $f_A \in \tau$ such that $f_A(e_0) \neq \bar{0}$. Then f_A is called a $\mathcal{J}$ – local minimal fuzzy soft open set at,e_0 if $g_A \in \tau$ with $g_A(e_0) \neq \bar{0}$ and $g_A \sqsubseteq f_A$ then $(f_A(e) - (g_A(e))_{e\in E} \in \mathcal{J}$.
The set of all $\mathcal{J}$– localminimal fuzzy soft open set at $e_0$will be denoted by min $(X,\tau,\mathcal{J},e_0)$.

Definition 2.4. Let (X,τ) be a fuzzy soft topological space with fuzzy soft ideal $\mathcal{J}$on X, λ∈ (0,1] ,e ∈ E, $f_A \in \tau$ where A⊆E such that $\bar{\lambda} \leq f_A(e)$.$f_A$is called $\mathcal{J}$– localminimal fuzzy soft open set at e,w.r.t.λ,if $g_A \sqsubseteq f_A$,$g_A \in \tau$,and$\bar{\lambda} \leq (g_A(e))$ then ($f_A(e) - (g_A(e))_{e\in E}$) ∈ $\mathcal{J}$.
The set of all $\mathcal{J}$ – localminimal fuzzy soft open sets at e ,w.r.t.λ is denoted by min(X,τ,$\mathcal{J}$,e,, λ).

Remarks: 1. If$f_E \in \tau$ then [f_E]={g_E ∈FS(X,E)/(($|f_e - g_e|)_{e \in E}$)∈ $\mathcal{J}$}.
2. If f_E,h_E ∈min (X,τ,$\mathcal{J}$,e_0)⇒ (($|f_e - g_e|)_{e \in E}$) ∈ $\mathcal{J}$, h_E ∈[f_E]

Theorem 2.5. Let (X,τ) be a fuzzy soft topological spacewith fuzzy soft ideal $\mathcal{J}$on X,$f_E \in \mathcal{J}$ and e_0 ∈ E such that f(e_0) ≠ $\bar{0}$.Then f_E is called $\mathcal{J}$– localminimal fuzzy soft open set at e_0 iff for each $g_E \in \tau$ with g(e_0)≠ $\bar{0}$, ($|f_e - g_e|)_{e \in E}$)∈ $\mathcal{J}$.
Proof: Supposef_E is a$\mathcal{J}$– local minimal fuzzy soft open set.Let $g_E \in \mathcal{J}$ with g(e_0)≠ $\bar{0}$. Then g(e_0)∧f(e_0)≠ $\overline{0}$. As$f_E \sqcap g_E \sqsubseteq f_E$,and $f_E \sqcap g_E \in \tau$,(f∧ g)(e_0)≠ $\bar{0}$,
we get: (($f_e - (f_e \wedge g_e))_{e \in E}$)∈ $\mathcal{J}$. This implies that (($f_e - g_e)_{e \in E}$) ∈ $\mathcal{J}$.
Conversely, suppose $g_E \sqsubseteq f_E$,$g_E \in \tau$, g(e_0)≠ $\bar{0}$ and (($f_e - g_e)_{e \in E}$) ∈ $\mathcal{J}$.
Then f_E is a$\mathcal{J}$– local minimal fuzzy soft open set.

Theorem 2.6. Let (X,τ) be a fuzzy soft topological spacewith fuzzy soft ideal$\mathcal{J}$on X, ,$f_E \in \tau$
And e∈ E.Then the following are equivalent.

i) min (X,τ,$\mathcal{J}$,e_0)=[f_E]

ii) for each λ∈ (0,1] with $f_E(e_0)$≥ $\bar{\lambda}$,min(X,τ,$\mathcal{J}$,e_0,λ)=[f_E].

Proof: (i)⇒(ii) Let $g_E \in \tau$ with g(e_0)≥ $\bar{\lambda}$, by above theorem (($f_e - g_e)_{e \in E}$) ∈ $\mathcal{J}$ therefore min(X,τ,$\mathcal{J}$,e_0,λ)=[f_E].
(ii)⇒(i) suppose for all λ∈ (0,1], $\bar{\lambda}$ ≤f(e_0) and min(X,τ,$\mathcal{J}$,e_0,λ)=[f_E]. Let $g_e \in \tau$ with g(e_0) ≠ $\bar{0}$.Therefore g(e_0) ≥ $\lambda > \bar{0}$ for some $\bar{\lambda}$ ≤f(e_0). As min(X,τ, $\mathcal{J}$,e_0,λ)=[f_E],
(($f_e - g_e)_{e \in E}$) ∈ $\mathcal{J}$.Thereforef_E ∈min(X,τ, $\mathcal{J}$,e_0,λ). Therefore min(X,τ, $\mathcal{J}$,e_0,λ) ∈ [f_E].

Theorem 2.7. *L*et Let (X,τ) be a fuzzy soft topological spacewith fuzzy soft ideal$\mathcal{J}$onX,
$f_E \in \tau$ ande_0 ∈ E and λ∈ (0,1] with$\bar{\lambda} \leq$ f(e_0) .Then the following are equivalent.

i)min (X, τ, $\mathcal{J}$,e_0,λ)=[f_E]
ii)min (X, τ $\mathcal{J}$,e_0,β)=[f_E] for anyβ ∈ (0,1] with $\bar{\beta}$ ≤f(e_0),$\bar{\lambda} \leq \bar{\beta}$.
Proof (i)⇒(ii)

Let β=[f_E] for any β ∈ (0,1] with $\bar{\beta}$ ≤f(e_0),$\bar{\lambda} \leq \bar{\beta}$.

Let $g_E \in \tau$ with $\bar{\beta}$ ≤g(e_0).Then $\bar{\lambda} \leq \bar{\beta}$ ≤g(e_0) and so $\bar{\lambda}$ ≤g(e_0), therefore by (i),
it follows that (($f_e - g_e)_{e \in E}$) ∈ $\mathcal{J}$.
This implies that min (X, τ, $\mathcal{J}$,e_0,β)= [f_E].
Proof (ii)⇒(i)

(ii)⇒(i) is obvious.

References

[1] A.Aygunoglu.et.al, An introduction to fuzzy soft topological spaces, Hacettepe Journal of Mathematics and Statistics, Volume 43(2)(2014), 193-204.
[2] J.Ruth"New Fuzzy Soft Topologies Via Fuzzy Soft Ideals", #JSP-6219-IJMAA ,(2018).
[3] S.Selvam,"A Study on fuzzy topologies",Ph.D(thesis),Alagappa University Karaikudi,(2007).
[4] R.Lowen," Fuzzy topological spaces and fuzzy compactness", J.Math.Appl., 56(1976), 621-633.
[5] D.Molodtsov, "Soft set theory "- results, Computers and Mathematics with applications 37(1999) 19-31.
[6] L.A.Zadeh," Fuzzy sets", Information and Control,8(1965),338-353.