Advances in Mathematics: Scientific Journal 8 (2019), no.3, 458–468
(Special issue on ICRAPAM)
ISSN 1857-8365 printed version
ISSN 1857-8438 electronic version

EXACT SOLUTION OF CONVECTIVE HEATING ON ENTROPY GENERATION RATE

*V. Ananthaswamy[1], R. Thenmozhi[2]

[1]PG & Research Department of Mathematics, The Madura College, Madurai, Tamil Nadu, India

[2]Department of Mathematics(CA)(SFW),

Mannar Thirumalai Naickar College(Autonomus),Madurai,Tamilnadu,India

*Corresponding author e-mail: ananthu9777@rediffmail.com

Abstract. In this paper, we address the analytical expression of the entropy generation rate in a steady flow of an incompressible viscous fluid through a channel with permeable walls subjected to the combined effects of convective heat. The exact solutions are obtained for velocity and temperature profiles boundary value problem. Both the velocity and temperature profiles are utilized to compute the entropy generation number, Nusselt number, Bejan number. The variations of significant thermos physical parameters with respect to fluid velocity, temperature, entropy generation, Nusselt number and Bejan number are investigated, presented graphically, and discussed based on physical laws.

Keywords: Suction/Injection; Porous channel; Entropy generation; Bejan number; Exact solution.

1. Introduction

Entropy, the measure of a system's thermal energy per unit temperature that is unavailable for doing useful work. Because work is obtained from ordering a molecular motion, the amount of entropy is also a measure of the molecular disorder, or randomness, of a system. The concept of entropy provides deep insight into the direction of spontaneous change for many everyday phenomena. Because it is determined by the number of random microstates, entropy is related to the amount of additional information needed to specify the exact physical state of a system, given its macroscopic specification. For this reason, it is often said that entropy is an expression of the disorder, or randomness of a system, or of the lack of information about it.

The idea of entropy provides a mathematical way to encode the intuitive notion of which processes are impossible, even though they would not violate the fundamental law of conservation of energy. A flow, thermal and entropy generation characteristic inside a porous channel with viscous dissipation were investigated by Mahmud et al[4]. Osalusi, [5] studied entropy generation in a liquid film falling along an inclined porus heated plate.Eegunjobi[6] discussed combined effect of buoyancy force and Navier slip on entropy generation in a vertical porous channel. Effect of external oriented magnetic field on entropy generation in natural convection analysed by A. El Jery et al[11].Entropy generation and its minimization have been considered as an effective tool to improve the performance of any heat transfer process. Our results also indicated that the variations of significant thermos physical parameters with respect to fluid velocity, temperature, entropy generation, Nusselt number and Bejan number are investigated, presented graphically, and discussed based on physical laws.

2. Mathematical formulation

One possibility that must be consider about the combined effects of convective heating and suction/injection on the entropy generation rate in a steady flow of an incompressible viscous fluid through a channel with permeable walls. At the lower wall, the fluid is injected uniformly into the channel and fluid suction occurs at the upper wall. The channel lower wall is heated by convection from a hot fluid with temperature T_f which gives a heat transfer coefficient γ_1 at the same time as the upper wall losses heat to the ambient with heat transfer coefficient γ_2.

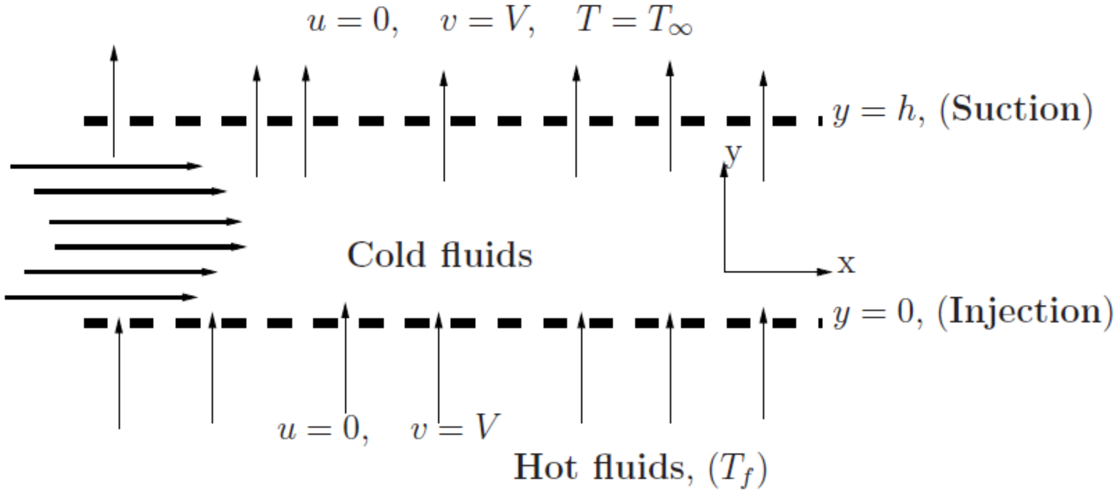

Figure1. Schematic diagram of the problem.

Under these assumptions, the governing equations for the momentum and energy balance in one dimension can be written as follows [6,7]:

$$V\frac{du}{dy}=-\frac{1}{\rho}\frac{dP}{dx}+\frac{\mu}{\rho}\frac{d^2u}{dy^2} \tag{1}$$

and

$$V\frac{dT}{dy}=\alpha\frac{d^2T}{dy^2}+\frac{\mu}{\rho c_p}\left(\frac{du}{dy}\right) \tag{2}$$

The boundary conditions are

$$u(0)=0,\ k\frac{dT}{dy}(0)=\gamma_1\left(T_f-T(0)\right)$$

$$u(h)=0,\ k\frac{dT}{dy}(h)=\gamma_2\left(T(h)-T_\infty\right) \tag{3}$$

Where h is the channel width, u is the velocity of the fluid, P is the fluid pressure, V is the uniformsuction/injection velocity at the channel walls, μ is the fluid viscosity, α is the thermal diffusivity, k is the thermal conductivity coefficient, C_p is the specific heat at constant pressure, T is the temperature and T_∞ is the ambient temperature. We introduce the following non-dimensional quantities

$$\theta=\frac{T-T_\infty}{T_h-T_\infty},\ \ \alpha=\frac{k}{\rho c_p},\ \ w=\frac{u}{V},\ \ \overline{P}=\frac{hP}{\mu V},\ \ \eta=\frac{y}{h},\ \ X=\frac{x}{h},\ \ G=-\frac{d\overline{P}}{dX} \tag{4}$$

Substituting equation (4) into equations (1)-(3), we obtain

$$\frac{d^2w}{d\eta^2} - \mathrm{Re}\frac{dw}{d\eta} + G = 0 \tag{5}$$

$$\frac{d^2\theta}{d\eta^2} - RE\,\mathrm{Pr}\frac{d\theta}{d\eta} + Ec\,\mathrm{Pr}\left(\frac{dw}{d\eta}\right)^2 = 0 \tag{6}$$

with the boundary conditions,

$$w(0) = 0, \qquad \frac{d\theta}{d\eta}(0) = Bi_1(\theta(0) - 1) \tag{7}$$

$$w(1) = 0, \qquad \frac{d\theta}{d\eta}(1) = -Bi_2\theta(1) \tag{8}$$

Where

G is the pressure gradient parameter,

$$\begin{aligned}
&\mathrm{Re} = \frac{Vh\rho}{\mu} \text{ (Reynolds number)}\\
&Pe = \mathrm{Pr}\,\mathrm{Re} = \frac{Vh}{\alpha} \text{ (Peclet number)}\\
&\mathrm{Pr} = \frac{\mu}{\alpha\rho} \text{ (Prandtl number)}\\
&Ec = \frac{V^2}{c_p(T_h - T_0)} \text{ (Eckert number)}\\
&Bi_1 = \frac{\gamma_1 h}{k} \text{ (lower wall Biot number)}\\
&Bi_2 = \frac{\gamma_2 h}{k} \text{ (upper wall Biot number)}
\end{aligned} \tag{9}$$

Entropy Generation

Fluid flow and heat transfer processes inside a porous channel are irreversible. The non-equilibrium conditions happen due to the exchange of energy and momentum within the fluid, thus resulting in entropy generation is define as follows.

$$E_G = \frac{K}{T_\infty^2}\left(\frac{dT}{dy}\right)^2 + \frac{\mu}{T_\infty}\left(\frac{du}{dy}\right)^2 \tag{10}$$

Where the first term on the right side of eqn (10) is the irreversibility due to heat transfer and the second term is the entropy generation due to fluid friction. Using eqn (4), we express eqn (10) in dimensionless form as

$$N_S = \frac{T_\infty^2 h^2 E_G}{K(T_f - T_\infty)^2} = \left(\frac{d\theta(\eta)}{d\eta}\right)^2 + \frac{Br}{\Omega}\left(\frac{dw(\eta)}{d\eta}\right)^2 \tag{11}$$

Where $\Omega = (T_h - T_\infty)/T_\infty$ is the temperature difference parameter, $Br = Ec\,\mathrm{Pr}$ is the Brinkman number and

$$N_1 = \left(\frac{d\theta(\eta)}{d\eta}\right)^2 \tag{12}$$

$$N_2 = \frac{Br}{\Omega}\left(\frac{dw(\eta)}{d\eta}\right)^2 \tag{13}$$

The Bejan number is given as

$$Be = \frac{N_1}{N_s} \tag{14}$$

Clearly, Bejan number ranges from 0 to 1 and $Be = 0$ is the limit where the irreversibility is dominated by fluid friction effects and $Be = 1$ is the limit where their reversibility due to heat transfer dominates the flow system by virtue of finite temperature differences.

3. Mathematical solution

Moreover, solving eqns (5) and (6) using the eqn (7) and (8).We can obtain the exact solutions of velocity and temperature are as follows,

$$w = \frac{G(e^{\mathrm{Re}\,\eta} + (1 - e^{\mathrm{Re}})\eta - 1)}{(1 - e^{\mathrm{Re}})\,\mathrm{Re}} \tag{15}$$

$$\theta = A + Be^{\mathrm{Re}\,\mathrm{Pr}\,\eta} + \frac{Ec\,\mathrm{Pr}\,G^2}{\mathrm{Re}^2}\left[\frac{\eta}{\mathrm{Re}\,\mathrm{Pr}} - \frac{e^{\mathrm{Re}\,\eta}}{\left(1 - e^{\mathrm{Re}}\right)(1 - \mathrm{Pr})}\left(\frac{e^{\mathrm{Re}\,\eta}}{2\left(1 - e^{\mathrm{Re}}\right)} + \frac{2}{\mathrm{Re}}\right)\right] \tag{16}$$

Where:

$$A = Je^{2\,\mathrm{Re}}\left(\frac{1}{2\,\mathrm{Re}} + \frac{1}{Bi_2}\right) + Ke^{\mathrm{Re}}\left(\frac{1}{\mathrm{Re}} + \frac{1}{Bi_2}\right) - L\left(1 + \frac{1}{Bi_2}\right) - Be^{\mathrm{Re}\,\mathrm{Pr}}\left(1 + \frac{\mathrm{Re}\,\mathrm{Pr}}{Bi_2}\right) \tag{17}$$

$$B = \frac{1}{M}\left[\begin{array}{l} \frac{J}{2\,\mathrm{Re}}\left(e^{2\,\mathrm{Re}}Bi_1\left(Bi_2 + 2\,\mathrm{Re}\right) - Bi_2\left(Bi_1 - 2\,\mathrm{Re}\right)\right) + \\ \frac{K}{\mathrm{Re}}\left(e^{\mathrm{Re}}Bi_1\left(Bi_2 + \mathrm{Re}\right) - Bi_2\left(Bi_1 - \mathrm{Re}\right)\right) - L\left(Bi_1 + Bi_2 + Bi_1 Bi_2\right) + Bi_1 Bi_2 \end{array}\right] \tag{18}$$

$$M = \mathrm{Re}\,\mathrm{Pr} - Bi_1 + e^{\mathrm{Re}\,\mathrm{Pr}}\left(\mathrm{Re}\,\mathrm{Pr} + Bi_2\right) \tag{19}$$

$$J = \frac{Ec\,\mathrm{Pr}\,G^2}{\mathrm{Re}\left(1 - e^{\mathrm{Re}}\right)^2 (2 - \mathrm{Pr})} \tag{20}$$

$$K = \frac{2Ec\,\mathrm{Pr}\,G^2}{\mathrm{Re}^2\left(1 - e^{\mathrm{Re}}\right)(1 - \mathrm{Pr})} \tag{21}$$

$$L = \frac{Ec\,G^2}{\mathrm{Re}^3} \quad . \tag{22}$$

The analytical expression for entropy generation using eqn(11), (12) and (13) is as follow

$$N_S = N_1 + \frac{Br}{\Omega}N_2 \tag{23}$$

Where

$$N_1 = \left(\mathrm{Re}\,\mathrm{Pr}\,Be^{\mathrm{Re}\,\mathrm{Pr}\,\eta} + \frac{Ec\,\mathrm{Pr}\,G^2}{\mathrm{Re}^2}\left[\frac{1}{\mathrm{Re}\,\mathrm{Pr}} - \frac{\mathrm{Re}\,e^{\mathrm{Re}\,\eta}}{\left(1 - e^{\mathrm{Re}}\right)(1 - \mathrm{Pr})}\left(\frac{e^{\mathrm{Re}\,\eta}}{\left(1 - e^{\mathrm{Re}}\right)} + \frac{2}{\mathrm{Re}}\right)\right]\right)^2 \tag{24}$$

$$N_2 = \frac{G^2}{\mathrm{Re}^2\left(1 - e^{\mathrm{Re}}\right)^2}\left[\mathrm{Re}^2\,e^{2\,\mathrm{Re}\,\eta} + 2\,\mathrm{Re}\left(1 - e^{\mathrm{Re}}\right)e^{\mathrm{Re}\,\eta} + \left(1 - e^{\mathrm{Re}}\right)^2\right] \tag{25}$$

Also the analytical expression for Bejan number using eqns (23) & (24) is

$$Be = \frac{N_1}{N_s} \tag{26}$$

4. Result and discussion

In Fig. 1, it is essentially to note that the fluid suction takes place at the upper wall while the fluid injection occurs at the lower wall simultaneously. In Fig 2(a) shows the effects of Reynolds number (Re) increases, on the axial velocity profile. As Re increase fluid injection of hot fluids into the cold fluid flowing inside the channel increases at the same time as the rate of fluid suction increases at the upper permeable wall. This leads to a decrease in the fluid velocity at the lower channel wall region. Figure 3(a) we observed that the effect of an increase in Reynolds number (Re) on temperature profile, the fluid temperature increases. The fluid temperature at the lower wall is higher than the fluid temperature at the upper wall due to combined effects of convective heating and hot fluid injection at the lower wall. In Figure 3(b) we noticed that the effects of increasing Eckert number (Ec) on the fluid temperature. It is observed that fluid temperature increases due to viscous heating on both walls.Fig.3(c) represents the increase the value of Bi_1 ,on the temperature profile is also increase in suction and also in injection.Fig.3(d) depicts the effects of increase the value of Bi_2 on temperature profile is decrease in suction and injection. From Figure 3(e), we noticed that if increase the value of Prandtl number (Pr) then temperature profile is also increases in suction and injection.

InFig.4(a) demonstrates the entropy generation rate when Reynolds number is increasing then entropy production decreases at the injection walls, on the other hand, entropy generation rate at suction wall increases.Figs.4 (b)&4(c) represent entropy generation rate when the lower wall Biot number Bi_1 and upper wall Biot number Bi_2 are increases then entropy generation rate increase at the injection wall and also increase entropy generation rate at the suction wall. The Fig.4(d) represents entropy generation rate when group parameter $Br\Omega^{-1}$ increases, the entropy generation rate at both walls increases but more at suction wall. The entropy generation is not affect by group parameter $Br\Omega^{-1}$ within the channel center line state.

In Fig.5(a), we noticed that Reynolds number is increase, the Bejan number decreases on injection wall and increases on suction wall. Clearly shows that the irreversibility due to fluid friction dominates the flow system at injection wall. At the same time as heat transfer irreversibility dominate at the suction wall. Figs.5(b)-(c) demonstrates the effects of lower wall Biot number Bi_1 and upper wall Biot number Bi_2 .The Bejan number is increases at injection wall while it also increases at the suction wall. As a result, the central effects of heat transfer irreversibility increases.Fig.5(d) represents the effects of increasing group parameter; the Bejan number is decrease at both walls important to increasing influence of fluid friction irreversibility.

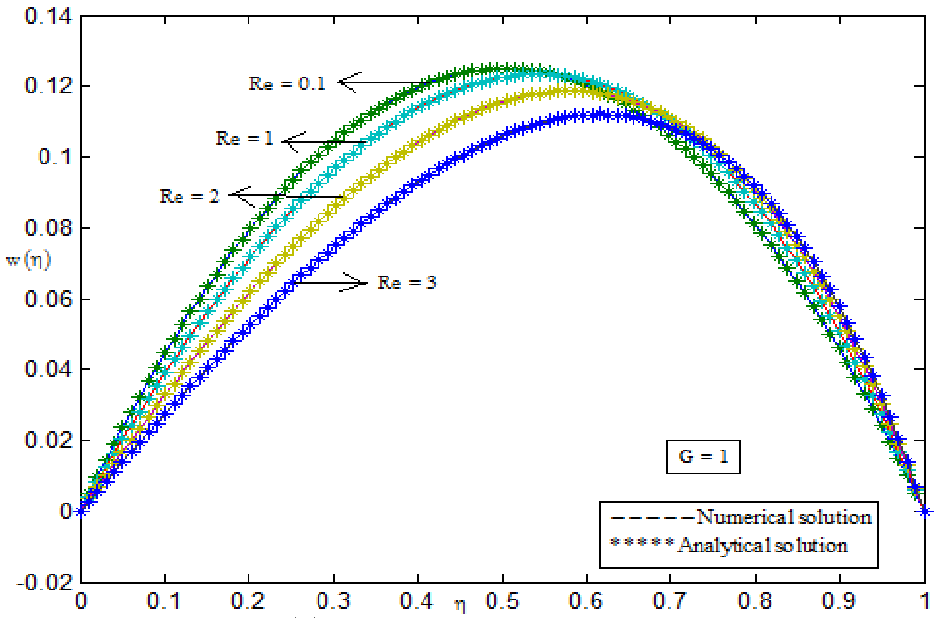

Fig.2(a) : Dimensionless velocity $w(\eta)$ versus dimensionless distance η . The graph is depicted of eqn. (15) for various values of Reynolds number Re and in fixed value of Pressure gradient increase the value of are velocity is decreases and injection at the same point.

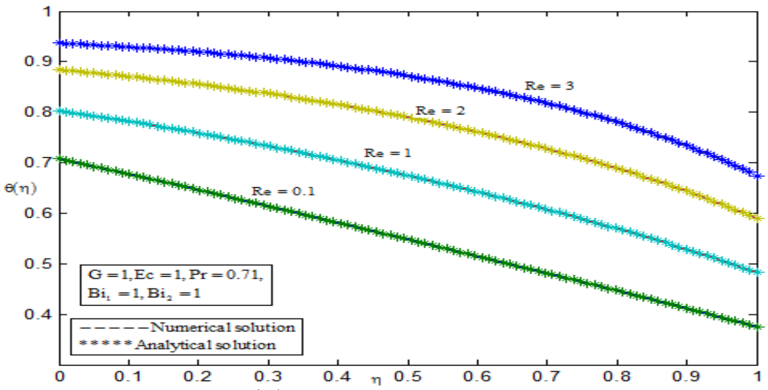

Fig 3(a): Dimensionless temperature $\theta(\eta)$ versus dimensionless distance η .The graph is depicted of eqn. (16) for various values of Reynolds number Re and in some fixed values of other dimensionless parameters.

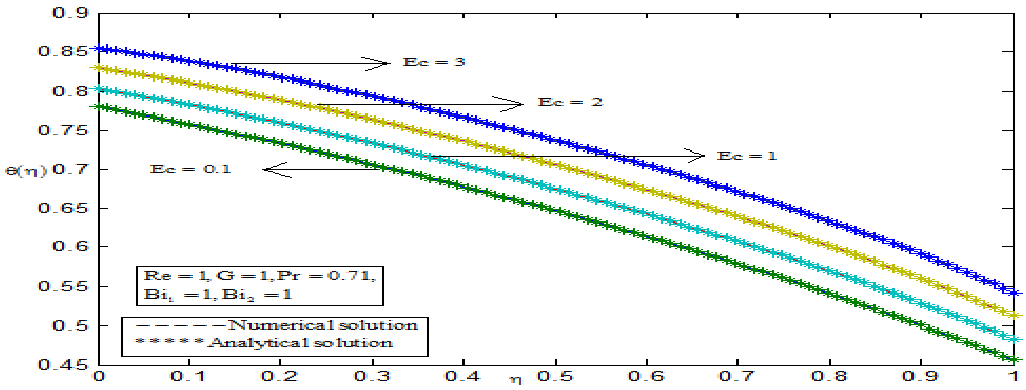

Fig3(b):Dimensionlesstemperature $\theta(\eta)$ versus dimensionless distance η .The graph is depicted of eqn.(16) for various valuesof Eckert number Ec and in some fixed values of other dimensionless parameters.

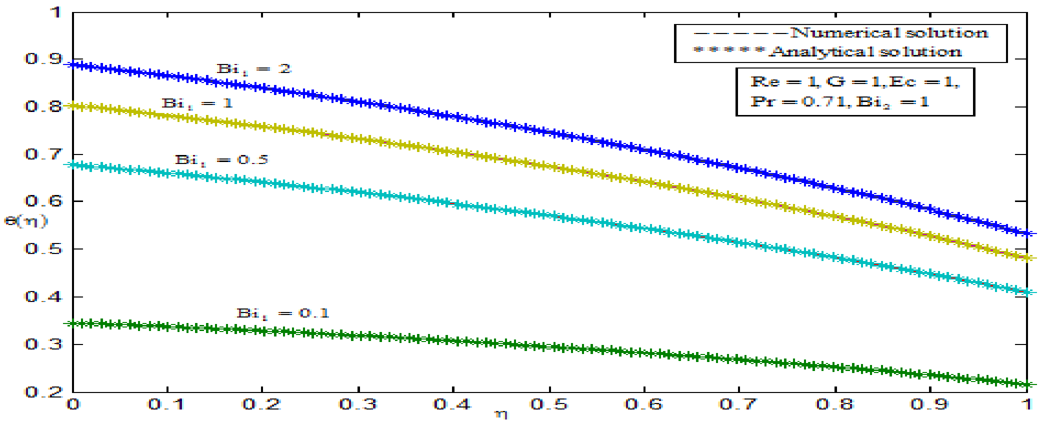

Fig3(c):Dimensionlesstemperature $\theta(\eta)$ versus dimensionless distance η .The graph is depicted of eqn. (16) for various valuesof lower wall Biot number Bi_1 and in some fixed values of other dimensionless parameters.

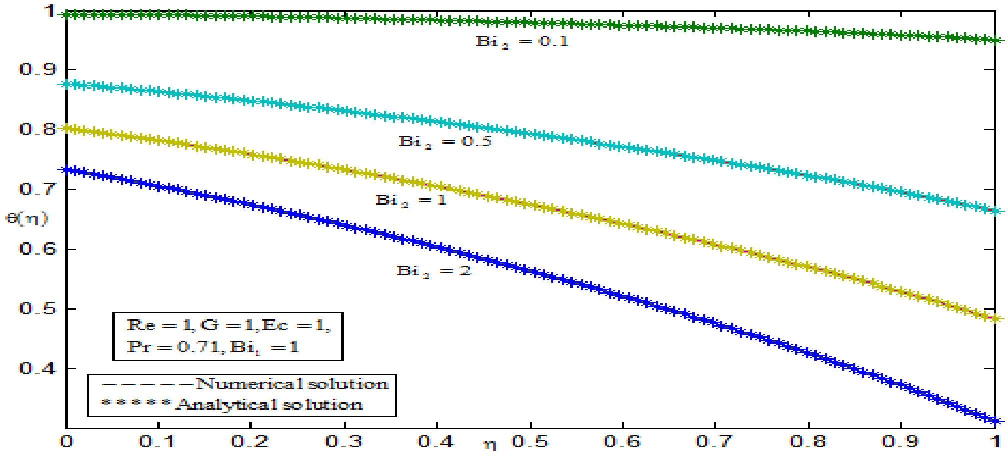

Fig 3(d): Dimensionlesstemperature $\theta(\eta)$ versus dimensionless distance η .The graph is depicted of eqn. (16) for various valuesof upper wall Biot number Bi_2 and in some fixed values of other dimensionless parameters.

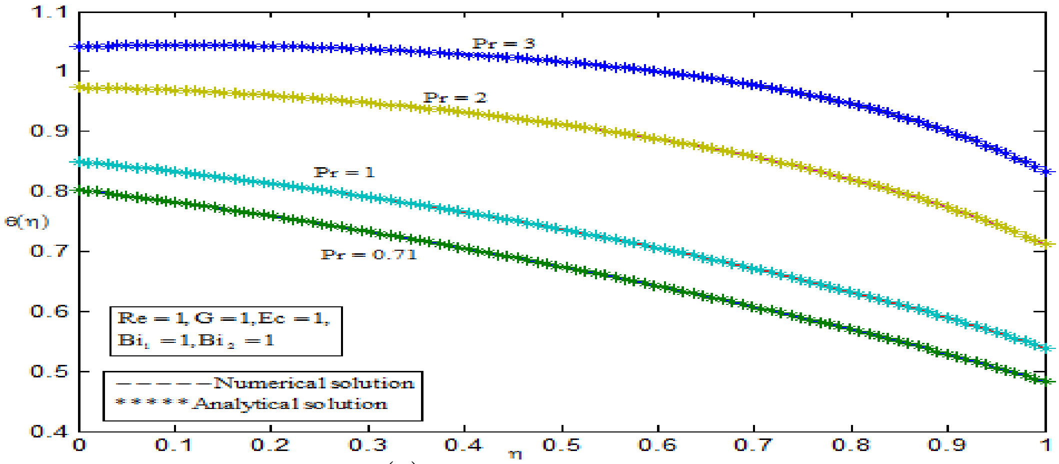

Fig3(e):Dimensionlesstemperature $\theta(\eta)$ versus dimensionless distance η .The graph is depicted of eqn.(16) for various aaluesof Prandtl number $\Pr$ and in some fixed values of other dimensionless parameters.

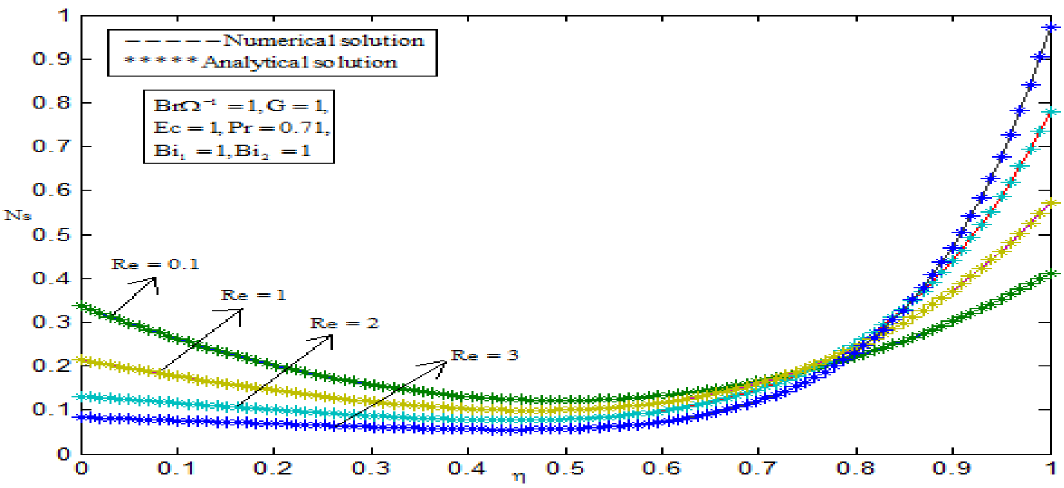

Fig4(a):Nusselt number Ns versus dimensionless distance η .The graph is depicted of eqn.(23) for various values of Reynolds number Re and in some fixed values of other dimensionless parameters.

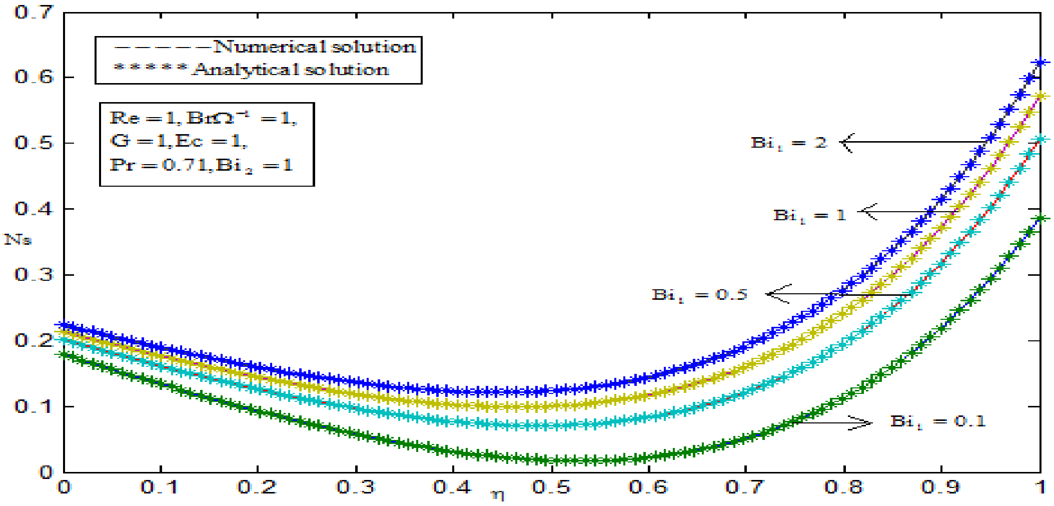

Fig 4(b): Nusselt number *Ns* versus dimensionless distance η .The graph is depicted of eqn. (23) for various values of Bi_1 and in some fixed values of other dimensionless parameters.

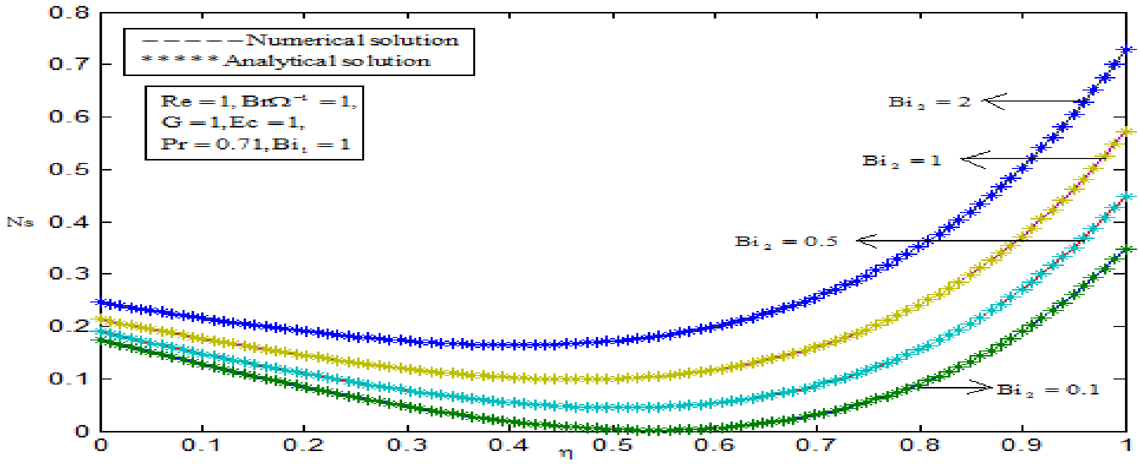

Fig 4(c):Nusselt number *Ns* versus dimensionless distance η .The graph is depicted of eqn. (23) for various values of Bi_2 and in some fixed values of other dimensionless parameters.

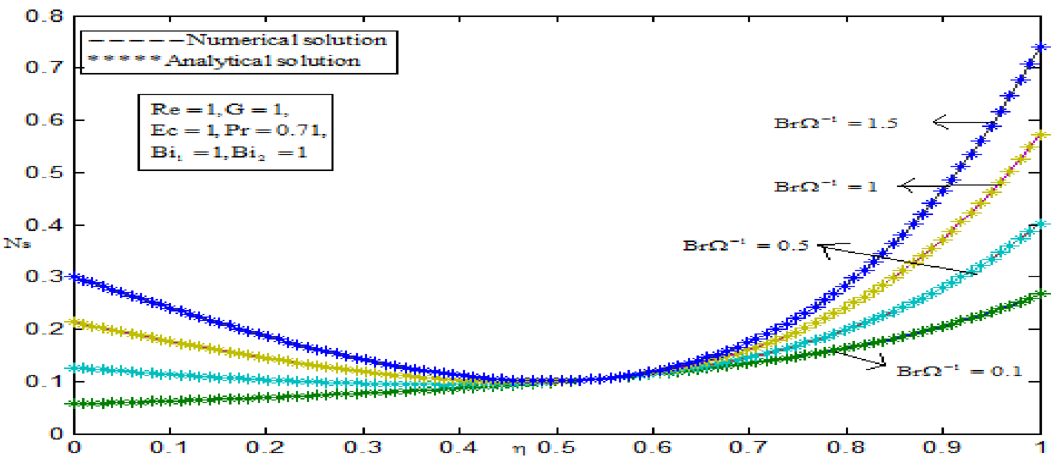

Fig4(d): Nusselt number *Ns* versus dimensionless distance η .The graph is depicted of eqn.(23)for various values of $Br\Omega^{-1}$ and in some fixed values of other dimensionless parameters.

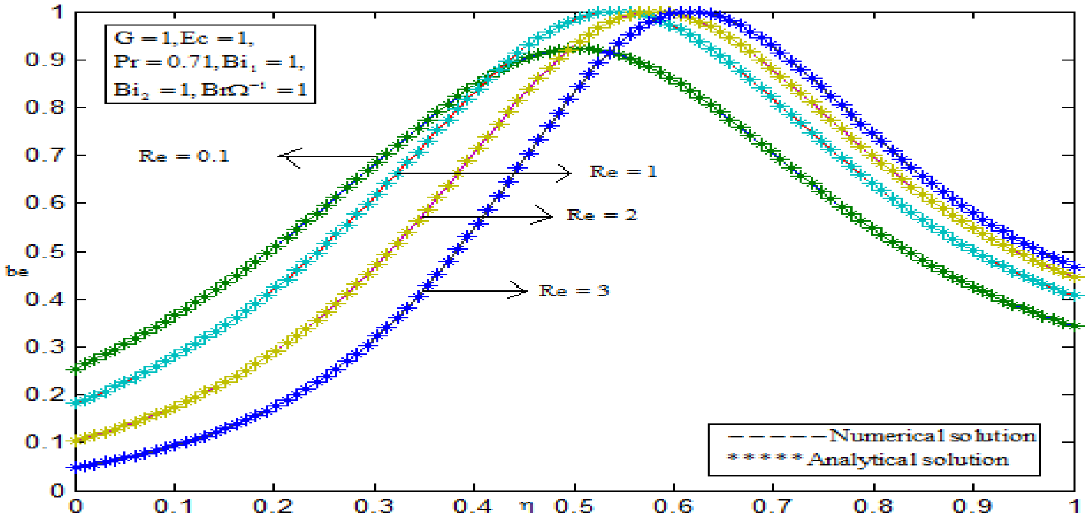

Fig5(a):Bejan number Be versus dimensionless distance η .The graph is depicted of eqn.(26) for various valuesof Re and in some fixed values of other dimensionless parameters.

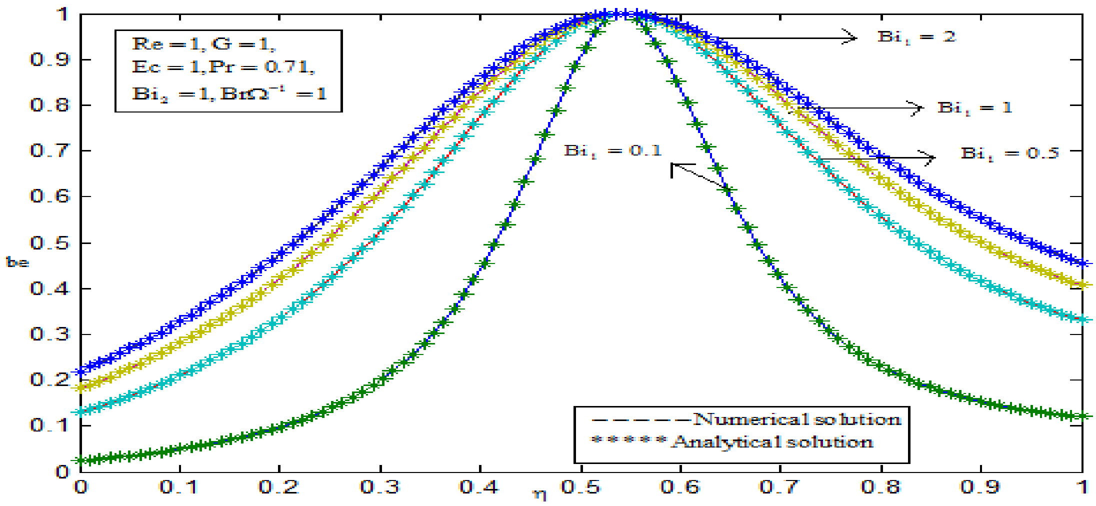

Fig5(b):Bejan number Be versus dimensionless distance η .The graph is depicted of eqn.(26) for various values of Bi_1 and in some fixed values of other dimensionless parameters.

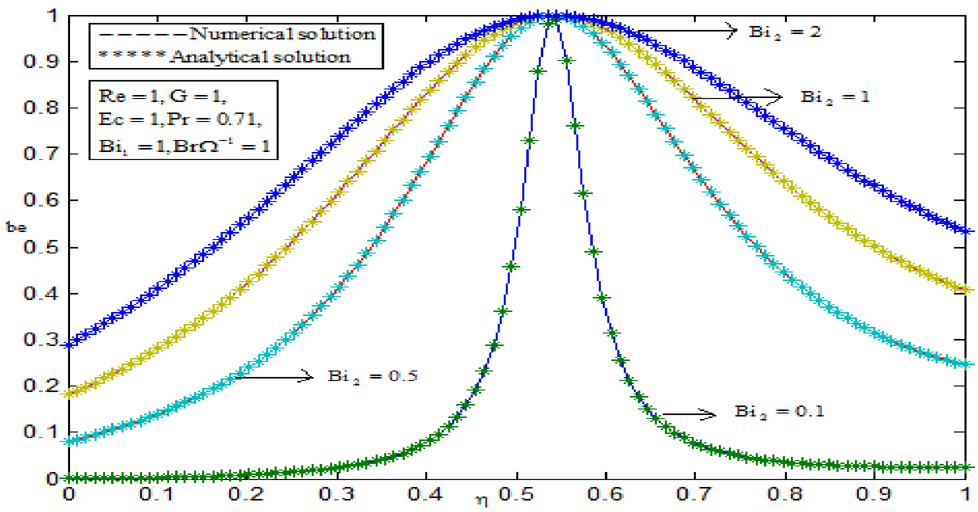

Fig5(c):Bejan number Be versus dimensionless distance η .The graph is depicted of eqn.(26) for various values of Bi_2 and in some fixed values of other dimensionless parameters.

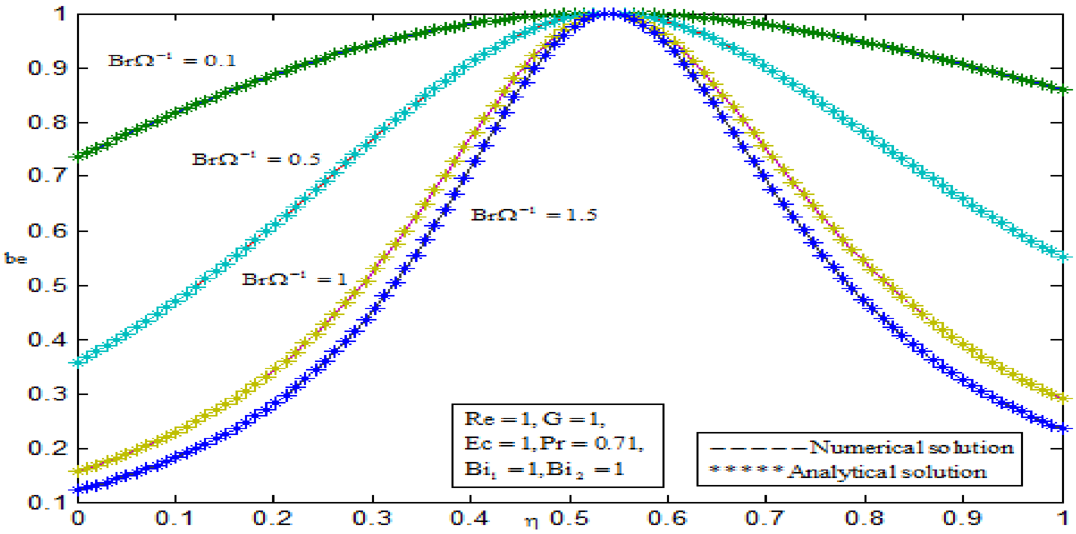

Fig5(d):Bejan number *Be* versus dimensionless distance η .The graph is depicted of eqn.(26) for various valuesof $Br\Omega^{-1}$ and in some fixed values of other dimensionless parameters.

5. Conclusion

The thermal convection is the fundamental physical process in a variety of engineering circumstances, such as heat accumulator system, comfortable environment, grain drying unit, electron cooling, etc. In this vital time, scientists and engineers concentrate most important area of research in Entropy generation minimization. In this paper the combined effect of convective heat transfer and suction/injection on entropy generation rate in a steady flow of fluid through a channel with permeable walls has been considered. The model problems are tackled analytically. Our results exposed among others the presence of flow reversal at the channel's upper wall due to suction. The heat transfer irreversibility controls the flow process within the channel center line section, at the same time as persuade of fluid friction irreversibility can be noticed at the channel walls. Generally, we observed that heat transfer irreversibility dominates the flow process with Bejan number very close to 1 within the channel center line region, while the little influence of fluid friction irreversibility can be observed at the channel walls. This perfect agreement is noticed in our paper. Moreover, with suitable combination of thermos physical parameter values, entropy minimization can be attained in a flow process.

Reference

[1] Bejan, Entropy Generation through Heat and Fluid Flow, John Wiley & Sons, New York, NY, USA, 1982.

[2] M.D.Spasojević, M. R. Janković, and D.D.Djaković,"A anew aapproach to entropy production minimization in diabatic distillation column with trays," Thermal Science, vol. 14, no. 2, pp. 317–328, 2010

[3] Makinde, O.D. Computational modelling of MHD unsteady flow and heat transfer toward a flat plate with Navier slip and Newtonian heating. Braz. J. Chem. Eng. 2012, 29,159–166.

[4] Mahmud, S.; Fraser, R.A. Flow, thermal and entropy generation characteristic inside a porouschannel with viscous dissipation. Int. J. Therm. Sci. 2005, 44, 21–32.

[5] Makinde, O.D.; Osalusi,E.Entropy generation in a liquid film falling along an inclined porous heated plate. Mech. Res. Commun. 2006, 33, 692–698

[6] Eegunjobi, A.S.; Makinde, O.D. Combined effect of buoyancy force and Navier slip on entropy

generation in a vertical porous channel. Entropy 2012, 14, 1028–1044.
[7] Chauhan, D.S.; Kumar, V. Heat transfer and entropy generation during compressible fluid flow in a channel partially filled with porous medium. Int. J. Energ. Tech. 2011, 3, 1–10.
[8] Eegunjobi, A.S.; Makinde, O.D. Effects of Navier slip on entropy generation in a porous channel with suction/injection. J.Therm.Sci.Technol. 2012, 7, 522–535.
[9] Tasnim,S.M.; Mahmud, S.; Mamum, M.A.H.Entropy generation in a porous channel withhydromagetic effect. Int. J. Exergy2002, 3, 300–308.
[10] M. Magherbi, H. Abbassi, N. Hidouri, and A.Ben Brahim, “Second law analysis in convective heat and mass transfer,” Entropy, vol. 8, no. 1, pp. 1–17, 2006.
[11] A. El Jery, N.Hidouri, MMagherbi, and A. B.Ben Brahim, “Effect of external oriented magnetic field on entropy generation in natural convection,” Entropy, vol. 12, no. 6, app.1391–1417, 2010
[12] H. I.Andersson, J.B.Aarseth, and B.S.Dandapat, “Heat transfer in a liquid film on an unsteady stretching surface,” International Journal of Heat and Mass Transfer, vol. 43, no. 1, pp. 69–74, 2000.
[13] R. Giles (2016). Mathematical Foundations of Thermodynamics: International Series of Monographs on Pure and Applied Mathematics. Elsevier Science. ISBN978-1-4831-8491-3.
.

Nomenclature

Symbol	Meaning
h	Channelwidth
u	velocity of the fluid
P	fluid pressure
V	uniform suction /injection velocity
μ	fluid viscosity
α	Thermaldiffusivity
k	thermal conductivity coefficient
C_P	specific heat at constantpressure
T_h	Temperature at suction wall
$T(0)$	Temperature at injection wall
T_f	Hot fluid temperature
T_∞	Ambient temperature
γ_1 and γ_2	slip coefficients
G	pressure gradient parameter
Re	Reynolds number
Pe	Peclet number
Pr	Prandtl number
Br	Brinkmann number
Bi_1	Lower wall Biot number
Bi_2	Upper wall Biot number
E_G	volumetric entropy generation rate
Ω	temperature difference parameter
N_1	Entropy generation due to heat transfer
N_2	Entropy generation due to viscous dissipation
Be	Bejan number