ADV MATH SCI JOURNAL

Advances in Mathematics: Scientific Journal 8 (2019), no.3, 484–492
(Special issue on ICRAPAM)
ISSN 1857-8365 printed version
ISSN 1857-8438 electronic version

DOMINATION ON INTUITIONISTICANTI FUZZY GRAPH

R. Muthuraj[1], V.V. Vijesh[2], S. Sujith [3]

[1]*PG & Research Department of Mathematics, H.H.The Rajah's College, Pudukkottai – 622 001, Tamilnadu, Affiliated to Bharathidasan University, Tiruchirappalli,India. Email:rmr1973@yahoo.co.in*

[2]*Department of Mathematics, Sree Narayana Gurukulam College of Engineering, Kadayiruppu, Ernakulam – 682 311, Kerala, Research Scholar, PG and Research Department of Mathematics,H.H.The Rajah's College, Pudhukottai, Affiliated to Bharathidasan University, Tiruchirappalli,Tamilnadu,India.Email:v.vijesh14@gmail.com*

[3]*Department of Applied Science, College of Engineering, Vadakara, Calicut – 673 105, Kerala,Co-Guide, PG and Research Department of Mathematics,H.H.The Rajah's College, Pudhukottai, Affiliated to Bharathidasan University, Tiruchirappalli,Tamilnadu,India. Email:suji.svn@gmail.com*

Abstract: In this paper, we define the notion of anti-vertex cover and domination in intuitionistic anti-fuzzy graph with their cardinality. Also, the definition of dominating set, domination number, isolated vertex, bipartite intuitionistic anti-fuzzy graph, anti-independent vertices, anti-independent set and the cardinality on intuitionistic anti-fuzzy graphs are given. Based on these ideas some results of intuitionistic anti fuzzy graphs are derived.

Keywords: Intuitionistic anti-fuzzy graphs, anti-vertex cover, dominating set, domination number, isolated vertex, anti-independent set, cardinality of intuitionistic anti-fuzzy graphs.

1. Introduction

The concept of fuzzy graph was introduced by Kaufmann [2] from the fuzzy relation introduced by L. A. Zadeh [16]. The study of fuzzy graph theory started in the year 1975 after the phenomenal work published by Rosenfeld [11]. Rosenfeld introduced several fuzzy analogues of fuzzy graph theoretic concepts such as paths, cycles and connectedness. J. N. Mordeson and P. S. Nair [3] introduced the concept of operations on fuzzy graphs, but this concept was extended by M. S. Sunitha and A. Vijayakumar [13]. Muhammad Akram [4] introduced the concept of connected anti fuzzy graphs, self centroid anti fuzzy graphs, regularity and irregularity, constant and totally constant anti fuzzy graphs with some of their properties.

Intuitionistic fuzzy sets are generalization of fuzzy sets [16]. Atanassov [1] introduced the concept of intuitionistic fuzzy relation, which has both membership grades and non-membership grades. The concept of domination in fuzzy graphs was investigated by A. Somasundaram and S. Somasundaram [12] in 1998. R. Parvathi and G. Thamizhendhi [10] introduced the concept of domination on intuitionistic fuzzy graph in 2010. In their paper, they introduced dominating set, domination number, independent set, total dominating set, total domination number in intuitionistic fuzzy graphs. R. Muthuraj, Sujith S. and Vijesh V. V.[7] introduced the notion of intuitionistic anti fuzzy graphs and their operations such as anti-union,anti-join together with anti-complement of intuitionistic anti fuzzy graphs. Usually, the concept of domination is used to find the minimization of a model. But in some cases, we need to get the maximization and maximum value with minimal objects. This situation leads to the concept of domination in intuitionistic anti fuzzy graphs. In this paper, we introduce the concept of dominating set and domination number on intuitionistic anti-fuzzy graphs and derived some theorems and results on them.

2. Preliminaries

Definition 2.1: An intuitionistic anti-fuzzy graph is of the form $G = \langle V, E\rangle$, where:

(i) $V = \{v_1, v_2, \ldots\ldots v_n\}$ such that $\mu_1: V \to [0,1]$ and $\gamma_1: V \to [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively and

$0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$ for every $v_i \in V$, (i = 1, 2, …n) ………… (1)

(ii) $E \subseteq V \times V$ where $\mu_2: V \times V \to [0,1]$ and $\gamma_2: V \times V \to [0,1]$ are such that:

$\mu_2(v_i, v_j) \ge \max\{\mu_1(v_i), \mu_1(v_j)\}$ ……….(2),

$\gamma_2(v_i, v_j) \ge \min\{\gamma_1(v_i), \gamma_1(v_j)\}$ ……….. (3)

and $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$ for every $(v_i, v_j) \in E$, (i, j = 1, 2, …n) …………(4)

Note: If one of the inequalities (1) or (2) or (3) or (4) is not satisfied, then the graph G is not an intuitionistic anti-fuzzy graph.

Note: An intuitionistic anti-fuzzy graph $\langle V, E\rangle$ is denoted by $G_A\langle V, E\rangle$.

Note: If $\mu_{2ij} = \gamma_{2ij} = 0$, for some i and j, then there is no edge between the vertices $\mathbf{v_i}$ and $\mathbf{v_j}$. Otherwise, there exists an edge between $\mathbf{v_i}$ and $\mathbf{v_j}$.

Definition 2.2: An intuitionistic anti-fuzzy graph $H_A(V', E')$ is an intuitionistic anti-fuzzy subgraph of $G_A\langle V, E\rangle$ if $V' \subseteq V$, $E' \subseteq E$ such that $\mu_{1i}' \le \mu_{1i}$, $\gamma_{1i}' \ge \gamma_{1i}$ and $\mu_{2ij}' \le \mu_{2ij}$, $\gamma_{2ij}' \ge \gamma_{2ij}$.

Definition 2.3: Let $G_A = \langle V, E\rangle$ be an intuitionistic anti-fuzzy graph. Then the vertex cardinality of G_A is defined by $|V| = \sum_{v_i \in V}\left(\frac{1+\mu_1(v_i)-\gamma_1(v_i)}{2}\right)$.

Definition 2.4: Let $G_A = \langle V, E\rangle$ be an intuitionistic anti-fuzzy graph. Then the edge cardinality of G_A is defined by $|E| = \sum_{(v_i,v_j)\in E}\left(\frac{1+\mu_2(v_i,v_j)-\gamma_2(v_i,v_j)}{2}\right) = \sum_{e_i \in E}\left(\frac{1+\mu_2(e_i)-\gamma_2(e_i)}{2}\right)$.

Definition 2.5: Let $G_A = \langle V, E\rangle$ be an intuitionistic anti-fuzzy graph. Then the cardinality of G_A is defined by $|G_A| = \big||V| + |E|\big| = \left|\sum_{v_i \in V}\left(\frac{1+\mu_1(v_i)-\gamma_1(v_i)}{2}\right) + \sum_{(v_i,v_j)\in E}\left(\frac{1+\mu_2(v_i,v_j)-\gamma_2(v_i,v_j)}{2}\right)\right|$.

***Definition 2.6*:** The number of vertices in an intuitionistic anti-fuzzy graph G_A is called the order of G_A and is denoted by $o(G_A)$ or p_A.

***Definition 2.7*:** The number of edges in an intuitionistic anti-fuzzy graph G_A is called the size of G_A and is denoted by $s(G_A)$orq_A.

Example 2.8: Consider an intuitionistic anti-fuzzy graph $G_A = \langle V, E\rangle$ such that V = {v_1, v_2, v_3, v_4} and E = {(v_1, v_2), (v_2, v_3), (v_1, v_4), (v_1, v_3)}

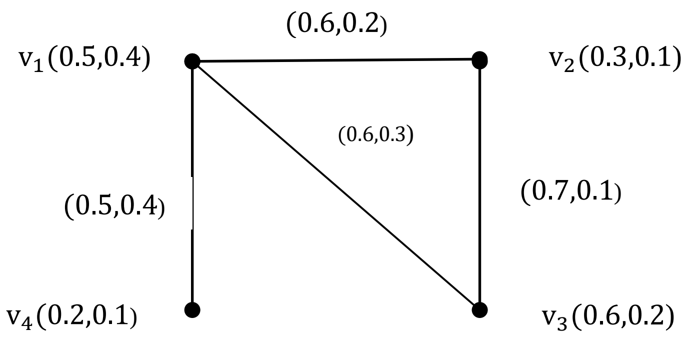

Figure 2.1: Intuitionistic anti-fuzzy graph $\mathbf{G_A}$

The Vertex Cardinality of $\mathbf{G_A}$ in Figure 2.1 is 2.4

The Edge Cardinality of $\mathbf{G_A}$ in Figure 2.1 is 2.7

The Cardinality of $\mathbf{G_A}$ in Figure 2.1 is 5.1

***Definition 2.9*:** The number of vertices in an intuitionistic anti-fuzzy graph G_A is called the order of G_A and is denoted by $o(G_A)$ or p_A.

Definition 2.10: The number of edges in an intuitionistic anti-fuzzy graph G_A is called the size of G_A and is denoted by $s(G_A)$or q_A.

***Definition 2.11*:** An edge $e = (u, v)$ of intuitionistic anti-fuzzy graph $G_A = \langle V, E\rangle$ is said to be an effective edge if $\mu_2(u, v) = \max\{\mu_1(u), \mu_1(v)\}$ and $\gamma_2(u, v) = \min\{\gamma_1(u), \gamma_1(v)\}$

Definition 2.12: An intuitionistic anti-fuzzy graph $G_A = \langle V, E\rangle$ is said to be complete if
$\mu_{2ij} = \max\left\{\mu_{1i}, \mu_{1j}\right\}$ and $\gamma_{2ij} = \min\ \{\gamma_{1i}, \gamma_{1j}\}$, $\forall v_i, v_j \in V$.

Note: The underlying graph of a complete intuitionistic anti-fuzzy graph is complete.

Definition 2.13 An intuitionistic anti-fuzzy graph $G_A = \langle V, E\rangle$ is said to be bipartite intuitionistic anti-fuzzy graph if the vertex set V can be partitioned into two non empty sets V_1 and V_2 such that

(i) $\mu_{2ij} = 0$ and $\gamma_{2ij} = 0$, if $v_i, v_j \in V_1$ or $v_i, v_j \in V_2$

(ii) $\mu_{2ij} > 0$ *and* $\gamma_{2ij} > 0$, *if* $v_i \in V_1$ or $v_j \in V_2$ for some i and j

(or) $\mu_{2ij} = 0$ and $\gamma_{2ij} > 0$, *if* $v_i \in V_1$ or $v_j \in V_2$ for some i and j

(or) $\mu_{2ij} > 0$ *and* $\gamma_{2ij} = 0$, if $v_i \in V_1$ or $v_j \in V_2$ for some i and j.

3. Domination on intuitionistic anti fuzzy graphs

In this section, we introduce the notion of domination number and dominating set on intuitionistic anti-fuzzy graphs.

***Definition 3.1*:** Let $G_A = \langle V, E \rangle$ be an intuitionistic anti-fuzzy graph. Let $u, v \in V$, we say that u dominates v in G_A if $\mu_2(u, v) = \max\{\mu_1(u), \mu_1(v)\}$ and $\gamma_2(u, v) = \min\{\gamma_1(u), \gamma_1(v)\}$. That is if (u, v) is an effective edge of G_A.

***Definition 3.2*:** A subset D of V is called a dominating set in an intuitionistic anti-fuzzy graph G_A if, for every vertex $v \notin D$, there exists $u \in D$ such that u dominates v.

Remark: If $\mu_2(u, v) > \max\{\mu_1(u), \mu_1(v)\}$ or $\gamma_2(u, v) > min\{\gamma_1(u), \gamma_1(v)\}$ for all $u, v \in V$, then obviously the only dominating set in G_A is V.

***Definition 3.3*:** A dominating set D of an intuitionistic anti-fuzzy graph $G_A = \langle V, E \rangle$ is said to be a minimal dominating set if no proper subset of D is a dominating set of G_A.

***Definition 3.4*:** The maximum fuzzy cardinality taken over all minimal dominating set in an intuitionistic anti-fuzzy graph G_A is called the domination number of G_A and it is denoted by $\gamma(G_A)$ or γ_A.

Example 3.5: Consider the following intuitionistic anti-fuzzy graph $\mathbf{G_A}$:

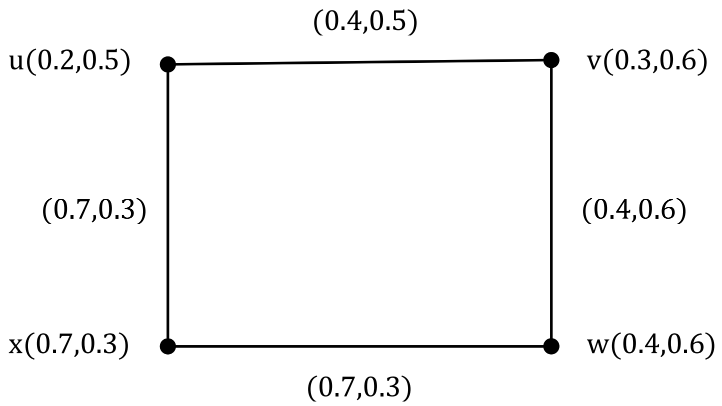

Figure 3.1: Intuitionistic anti-fuzzy graph $\mathbf{G_A}$

In Figure 3.1, x is an effective neighbour to w and u, w is an effective neighbour to v and x, u is an effective neighbour to x, v is an effective neighbour to w.

Here, $\mu_2(u, v) > \max\{\mu_1(u), \mu_1(v)\}$ and $\gamma_2(u, v) > \min\{\gamma_1(u), \gamma_1(v)\}$. Hence the edge (u, v) is a weak edge. So u and v are not effective neighbors to each other. Thus u and v cannot dominate each other. The possible minimal dominating sets are $D_1 = \{x, w\}$, $D_2 = \{x, v\}$ and $D_3 = \{w, u\}$. The corresponding domination number is $|D_1| = 1.1$, $|D_2| = 1.05$, $|D_3| = 0.75$. Therefore, the domination number $\gamma(G_A) = 1.1$

***Definition 3.6*:** The set of v in V such that the edge (u, v) is an effective edge of intuitionistic anti-fuzzy graph $G_A = \langle V, E \rangle$ is called the neighbourhood of u.

That is, neighbourhood of a vertex v in an intuitionistic anti-fuzzy graph $G_A = \langle V, E \rangle$ is $N(v) = \{v \in V: e = (u, v) \text{ is an effective edge}\}$.

The closed neighbourhood of v in an intuitionistic anti-fuzzy graph $G_A = \langle V, E\rangle$ is defined as $N[v] = N(v) \cup \{v\}$.

Remark: For any vertex v ∈ V, N(v) is the set of all vertices which are dominated by v.

***Definition 3.7*:** A vertex u of an intuitionistic anti-fuzzy graph $G_A = \langle V, E\rangle$ is said to be an isolated vertex if $\mu_2(u, v) \neq \max\{\mu_1(u), \mu_1(v)\}$ and $\gamma_2(u, v) \neq \min\{\gamma_1(u), \gamma_1(v)\}$, for all $v \in V - \{u\}$. In particular case if $\mu_2(u, v) = 0$ and $\gamma_2(u, v) = 0$, for all $v \in V - \{u\}$.

***Definition 3.8*:** Two vertices $u, v \in V$ in an intuitionistic anti-fuzzy graph $G_A = \langle V, E\rangle$ are said to be anti-independent if $\mu_2(u, v) \neq \max\{\mu_1(u), \mu_1(v)\}$ and $\gamma_2(u, v) \neq \min\{\gamma_1(u), \gamma_1(v)\}$

***Definition 3.9*:** A subset S of V in an intuitionistic anti-fuzzy graph $G_A = \langle V, E\rangle$ is said to be anti-independent set of G_A if $\mu_2(u, v) \neq \max\{\mu_1(u), \mu_1(v)\}$ and $\gamma_2(u, v) \neq \min\{\gamma_1(u), \gamma_1(v)\}$, for all $u, v \in S$

Definition 3.10: An anti-independent set S of an intuitionistic anti-fuzzy graph $G_A = \langle V, E\rangle$ is said to be maximal anti-independent set, if for every vertex $v \in V \setminus D$, the set $S \cup \{v\}$ is not anti-independent.

Definition 3.10: The maximum fuzzy cardinality among all maximal anti-independent set in G_A is called the upper anti-independence number of G_A and is denoted by $I(G_A)$.

***Definition 3.11*:** The minimum fuzzy cardinality among all maximal anti-independent set in G_A is called the lower anti-independence number of G_A and is denoted by $i(G_A)$.

Example 3.12:

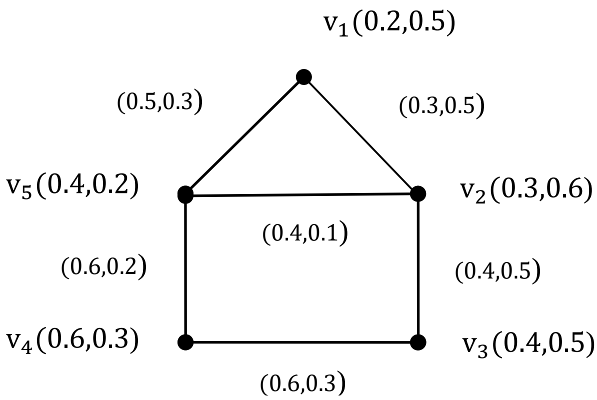

Figure 3.3: Intuitionistic anti-fuzzy graph $\mathbf{G_A\langle V, E\rangle}$

In figure 3.3, G_A is an intuitionistic anti-fuzzy graph with vertex set V = $\{v_1, v_2, v_3, v_4, v_5\}$ and E = $\{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_5), (v_1, v_5), (v_2, v_5)\}$. $S_1 = \{v_1, v_3, v_5\}$, $S_2 = \{v_1, v_4\}$ and $S_3 = \{v_2, v_4\}$ are minimal anti-independent sets with $I(G_A) = \max\{|S_1|, |S_2|, |S_3|\} = \max\{1.4, 1, 1\} = 1.4$ and $i(G_A) = \min\{|S_1|, |S_2|, |S_3|\} = 1$

4. Main results on domination

Theorem 4.1:

Domination on intuitionistic anti-fuzzy graph $\mathbf{G_A = \langle V, E\rangle}$ is a symmetric relation on V.

Proof: Let $G_A = \langle V, E\rangle$ be an intuitionistic anti-fuzzy graph.

Let $u, v \in V$ such that u dominates v in G_A. Then, $\mu_2(u,v) = \max\{\mu_1(u), \mu_1(v)\}$ and $\gamma_2(u,v) = \min\{\gamma_1(u), \gamma_1(v)\}$. So, $\mu_2(v,u) = \max\{\mu_1(v), \mu_1(u)\}$ and $\gamma_2(v,u) = \min\{\gamma_1(v), \gamma_1(u)\}$.

That is, v dominates u in G_A.

Hence domination on intuitionistic anti-fuzzy graph $\mathbf{G_A} = \langle \mathbf{V}, \mathbf{E} \rangle$ is a symmetric relation on V. ■

Theorem 4.2:

An intuitionistic anti-fuzzy graph $G_A = \langle V, E \rangle$ is complete intuitionistic anti-fuzzy graph if and only if D = {v} is a dominating set for all v ∈ V.

Proof: Let $G_A = \langle V, E \rangle$ be a complete intuitionistic anti-fuzzy graph.

So, $\mu_{2ij} = \max\{\mu_{1i}, \mu_{1j}\}$ and $\gamma_{2ij} = \min\ \{\gamma_{1i}, \gamma_{1j}\},\ \forall v_i, v_j \in V$.

Consider an arbitrary vertex v ∈ V. By the definition of complete intuitionistic anti-fuzzy graph, the arbitrary vertex v is incident with every vertex of u ∈ V − {v} and satisfy

$$\mu_2(u,v) = \max\{\mu_1(u), \mu_1(v)\} \text{ and } \gamma_2(u,v) = \min\{\gamma_1(u), \gamma_1(v)\}.$$

Thus u is dominated by an element v ∈ V. Hence D = {v} is a dominating set, for all v ∈ V.

Conversely, suppose D = {v} is a dominating set, for all v ∈ V.

Therefore, v dominates u, for all u ∈ V \ D. So, there exists an effective edge from u to v, for all u ∈ V \ D. Therefore,

$$\mu_2(u,v) = \max\{\mu_1(u), \mu_1(v)\} \text{ and } \gamma_2(u,v) = \min\{\gamma_1(u), \gamma_1(v)\}, \text{ for all } u \in V \setminus D.$$

Thus, $\mu_2(u,v) = \max\{\mu_1(u), \mu_1(v)\}$ and $\gamma_2(u,v) = \min\{\gamma_1(u), \gamma_1(v)\}$, for all $u, v \in V$.

Hence $G_A = \langle V, E \rangle$ is a complete intuitionistic anti-fuzzy graph. ■

Result: The domination number of a complete intuitionistic anti-fuzzy graph is

$$\gamma = \max_{u \in V}\left(\frac{1+\mu_1(u)-\gamma_1(u)}{2}\right).$$

Example 4.3: Consider the complete intuitionistic anti-fuzzy graph $G_A = \langle V, E \rangle$

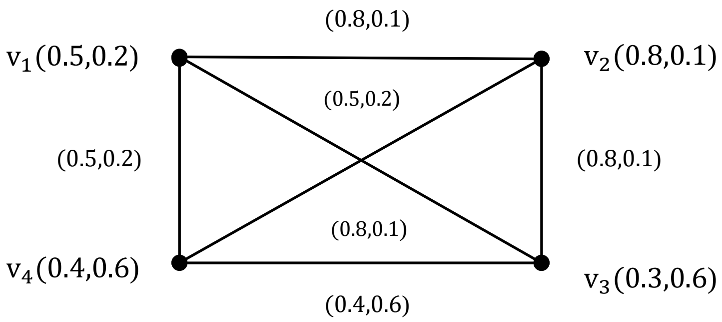

Figure 4.1: Complete intuitionistic anti-fuzzy graph $\mathbf{G_A}\langle \mathbf{V}, \mathbf{E} \rangle$

Here each vertex of G_A constitute the dominating set and which are minimal dominating sets.

$$\gamma(G_A) = \max_{v_i \in V}\left(\frac{1+\mu_1(v_i)-\gamma_1(v_i)}{2}\right)$$

$$= \max\left\{\frac{1+\mu_1(v_1)-\gamma_1(v_1)}{2}, \frac{1+\mu_1(v_2)-\gamma_1(v_2)}{2}, \frac{1+\mu_1(v_3)-\gamma_1(v_3)}{2}, \frac{1+\mu_1(v_4)-\gamma_1(v_4)}{2}\right\}$$

$= 0.85$

Theorem 4.4:

If a connected intuitionistic anti-fuzzy graph $G_A = \langle V, E \rangle$ has a minimal dominating set D such that V \ D is non empty, then $N(u) \cap D = \{v\}$, where $u \in V \setminus D$ and $v \in D$.

Proof: Let $G_A = \langle V, E \rangle$ be a connected intuitionistic anti-fuzzy graph. Let D be the minimal dominating set such that $V \setminus D$ is non empty.Let $u \in V \setminus D$. Then u is incident with some element $v \in D$ and u is dominated by v. So (u, v) is an effective edge.Since D is the minimal dominating set, u cannot be dominated by an element of D other than v.Thus the neighbourhood of u contains only one element v of D. That is,$N(u) \cap D = \{v\}$. ■

Theorem 4.5:

A dominating set D of an intuitionistic anti-fuzzy graph G_A is a minimal dominating set if and only if, for each $d \in D$, any of the following conditions hold:
(i) $N(d) \cap D = \varphi$.
(ii) There exists a vertex $u \in V \setminus D$ such that $N(u) \cap D = \{d\}$.

Proof: Let $G_A = \langle V, E \rangle$ be an intuitionistic anti-fuzzy graph and D be a minimal dominating set of G_A.

Let $d \in D$ and $D' = D - \{d\}$.Since D is the minimal dominating set of G_A, D' is not a dominating set. Hence there exists an element $u \in V \setminus D'$ such that u is not dominated by any element of D'.

If $u = d$, then u is not a neighbor of any vertex in D. So $N(d)$ and D has no vertex in common.

Thus $N(d) \cap D = \varphi$.

If $u \neq d$, then u is not dominated by D'. But u is dominated by D. Thus the vertex u is a neighbor to d only in D. Hence,$N(u) \cap D = \{d\}$.

Conversely, assume that D is a dominating set and for each $d \in D$, any of the following conditions hold:

(i) $N(d) \cap D = \varphi$.

(ii) There exists a vertex $u \in V \setminus D$ such that $N(u) \cap D = \{d\}$.

Suppose D is not a minimal dominating set. So there exists a vertex $d \in D$ such that D' is a dominating set. Hence d is a neighbor to at least one vertex in D'. Therefore, $N(d) \cap D \neq \varphi$, which contradict to (i).

If D' is a dominating set then every vertex in $V \setminus D$ is a neighbor to at least one vertex in D'. The second condition does not hold, which contradict our assumption that at least one of these conditions holds. So D is a minimal dominating set. ■

***Theorem 4.6*:**

An isolated vertex of an intuitionistic anti-fuzzy graph G_A does not dominate any other vertex in G_A.

Proof: Let u be an isolated vertex of an intuitionistic anti-fuzzy graph$G_A = \langle V, E \rangle$.

Thus for any vertex $v \in V - \{u\}$,

$$\mu_2(u, v) \neq \max\{\mu_1(u), \mu_1(v)\} \text{ and } \quad \gamma_2(u, v) \neq \min\{\gamma_1(u), \gamma_1(v)\}$$

So there does not exists an effective edge from u to any other vertex $v \in V - \{u\}$.

Thus, $N(u) = \varphi$. Hence the isolated vertex u does not dominate any other vertex in intuitionistic anti-fuzzy graphG_A. ■

***Theorem 4.7*:**

If D is a minimal dominating set of an intuitionistic anti-fuzzy graph $G_A = \langle V, E \rangle$ without isolated vertices, then $V \setminus D$ is a dominating set of G_A.

Proof: Let $G_A = \langle V, E \rangle$ be an intuitionistic anti-fuzzy graph without isolated vertices. Let D be a minimal dominating set of G_A.

Consider an arbitrary vertex $u \in D$. Since G_A has no isolated vertex, there exists a vertex $v \in N(u)$ such that $\mu_2(u,v) = \max\{\mu_1(u), \mu_1(v)\}$ and $\gamma_2(u,v) = \min\{\gamma_1(u), \gamma_1(v)\}$.

So $v \in V \setminus D$ and which is dominated by u. Thus every element of D is dominated by some element of $V \setminus D$. Since G_A has no isolated vertex, an arbitrary element v of $V \setminus D$ is incident with some element of D such that $\mu_2(u,v) = \max\{\mu_1(u), \mu_1(v)\}$ and $\gamma_2(u,v) = \min\{\gamma_1(u), \gamma_1(v)\}$.That is, every element of $V \setminus D$ is dominated by some element of D. Hence $V \setminus D$ is a dominating set of G_A. ■

***Theorem 4.8*:**

An anti-independent set is a maximal anti-independent set of intuitionistic anti-fuzzy graph $G_A = \langle V, E \rangle$ if and only if it is anti-independent and dominating set.

Proof: Let D be a maximal anti-independent set in an intuitionistic anti-fuzzy graph $G_A = \langle V, E \rangle$ and hence for every vertex $v \in V \setminus D$, the set $D \cup \{v\}$ is not anti-independent. For every $v \in V \setminus D$, there is a vertex $u \in D$ such that $\mu_2(u,v) = \max\{\mu_1(u), \mu_1(v)\}$and $\gamma_2(u,v) = \min\{\gamma_1(u), \gamma_1(v)\}$. That is, u is an effective neighbour to v. Thus D is a dominating set. Hence D is both dominating and anti-independent set.

Conversely, assume that the set D is both anti-independent and dominating. Suppose D is not maximal anti-independent. Hence there exists a vertex $v \in V \setminus D$ such that $D \cup \{v\}$ is anti-independent. So there is no vertex in D which is an effective neighbour to v. Thus D cannot be a dominating set. This is a contradiction to the assumption that D is dominating. Hence D is a maximal anti-independent set. ■

***Theorem 4.9*:**

Every maximal anti-independent set D in an intuitionistic anti-fuzzy graph $G_A = \langle V, E \rangle$is a minimal dominating set.

Proof: Let D be a maximal anti-independent set in an intuitionistic anti-fuzzy graph $G_A = \langle V, E \rangle$. By Theorem 4.7, D is a dominating set. Suppose D is not a minimal dominating set, then there exists at least one vertex $v \in D$ for which $D \setminus \{v\}$ is a dominating set. But if $D \setminus \{v\}$ dominates $V - \{D \setminus \{v\}\}$, then at least one vertex in $D \setminus \{v\}$ must be effective neighbor to v. This contradicts the fact that D is an anti-independent set of G_A. Therefore, D must be a minimal dominating set. ■

4. Conclusion

The notion of domination in graph theory is wealthy in theoretical and which has many applications in real world such as radar stations, nuclear power plants and communication networks. In this paper, we have introduced the concept of dominating sets, domination number, anti-independent set and isolated vertex on intuitionistic anti-fuzzy graphs. Some absorbing and significant properties and results on these new concepts are proved. Further, the authors proposed to extend these results on anti-vertex cover of intuitionistic anti-fuzzy graphs.

References

[1] Atanassov K.T., Intuitionistic fuzzy sets: theory and applications. Physica, New York, 1999.

[2] Kaufmann A., Introduction to the theory of fuzzy subsets, Vol.1, Academic press, New York, 1975.

[3] Mordeson J.N. and Nair P.S., Fuzzy Graphs and Fuzzy Hyper graphs, v. 46. Physica-Verlag, Heidelberg, New York (2000).

[4] Muhammad Akram, Anti Fuzzy Structures on graphs, Middle East Journal of Scientific Research 11 (12), (2012) 1641 – 1648.

[5] Muthuraj R. and Sasireka A., On anti fuzzy graph, Advances in Fuzzy Mathematics, Vol.12(5) (2017), 1123 – 1135.

[6] Muthuraj R. and Sasireka A., Domination on anti fuzzy graph, International Journal of Mathematical Archive – 9(5), (2018), 82 – 92.

[7] Muthuraj R., Sujith S. and Vijesh V. V., Operations on intuitionistic anti fuzzy graphs, International Journal of Recent Technology and Engineering (IJRTE),ISSN: 2277-3878, Vol.8, Issue IC2, (2019), 1098 – 1103.

[8] Nagoor Gani A. and Begum S. S. Degree, order and size in intuitionistic fuzzy graphs, Int. J. Algorithms, Computing and Mathematics, 3(3) (2010) 11 – 16.

[9] Parvathi R., Karunambigai M. G. and Atanassov K., operations on intuitionistic fuzzy graphs, Proceedings of IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), (2009) 1396 – 1401.

[10] Parvathi R. andThamizhendhi G., Domination on intuitionistic fuzzy graphs, NIFS Vol.16, (2010), 2, 39 – 49.

[11] Rosenfeld A. “Fuzzy graphs,” in Fuzzy Sets and Their Applications, L. A. Zadeh, K. S. Fu, and M. Shimura, Eds., pp. 77–95, Academic Press, New York, NY, USA, 1975.

[12] Somasundaram A. and Somasundaram S., Domination in fuzzy graphs – I, Pattern Recognition Letters, 19, (1998), 787 – 791.

[13] Sunitha M. S. and Vijayakumar A. Complement of a fuzzy graph, Indian J. Pure Appl. Math., 33(9) (2002) 1451 – 1464.

[14] Velammal S. and Karthikeyan S., Dominations in intuitionistic fuzzy graphs, IJAIR, (2012), 123 – 130.

[15] Vijesh V. V. and Muthuraj R., Some Characteristics on Join of Intuitionistic Fuzzy Graphs, IOSR Journal of Mathematics, e-ISSN: 2278 – 5728, p-ISSN: 2319-765X, 23 – 31.

[16] Zadeh L. A., Fuzzy Sets, Information Sciences, 8 (1965) 338 – 353.