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REVISED VERSION OF ASM METHOD – THE BEST ONE FOR FINDING AN IBFS FOR TRANSPORTATION PROBLEMS

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Abstract. Abdul Quddoos *et al.* (July 2012) developedand published ASM-Method for obtaining the optimal solution for transportation problems (TP) directly in a lesser number of iterations with minimum attempt of mathematical calculations. Soon after, ASM-Method was used by many researchers for solving transportation and assignment problems. But during their further research, they encountered a few problems in which ASM-Method does not directly provide optimal solution to each and every problem (particularly in case of unbalanced TP), but at the same time it provides the best Initial Basic Feasible Solution (IBFS), whichis very close to optimal solution. To overcome this problem, the authors developed the Revised Version of ASM-Method (June 2016). In the history of Operations Research literature, more than five decades the Vogel's Approximation Method (VAM) was considered as the more efficient algorithm to find an IBFS of a TP. In this paper, we have tried to expose that the Revised Version of ASM-Method is the best one for finding an IBFS for any balanced transportation problem (BTP) as well as unbalanced transportation problem (UTP). To verify the performance of the method, 30 classical benchmark instances of balanced type and 10 of unbalanced type from the literature have been tested. Simulation results on BTPs confirm that the ASM method produces optimal solution to 27BTPs and near optimal solution to 3BTPs, where as VAM produces optimal solution to only 11 BTPs and near optimal solution to 19 BTPs. Another simulation results on UTPs substantiate that the ASM method produces optimal solution to 6 UTPs and near optimal solution to 4 UTPs, where as VAM produces optimal solution to no one and near optimal solution to 10UTPs. Therefore, it is established that the Revised Version of ASM method produces the best IBFS, in the sense that, which is either optimal or very close to optimal solution.Further,the most attractive feature of this method is that it requires only simplearithmetical and logical calculations and hence anyone can easily understand and use it far better than VAM. Also, this method will be more cost-effective for those decision makers who aredealing with logistics and supply chain problems.

1. Introduction

Transportation problems have been widely studied in Operations Research and Computer Science. They play an important role in logistics and supply-chain management for reducing the shipping cost and improving the service. Efficient algorithms have been developed for solving the transportation problems when the cost coefficients and the supply and demand quantities are known exactly. Quite few methods such as North West Corner (NWC) Method,Least Cost Method (LCM) and Vogel's Approximation Method(VAM) [24, 38, 41] have been established for finding the IBFS, where as Zero Suffix Method, Revised Version of ASM-Method [2] etc.have been introduced which directly attain the optimal solution. Also it can be said that those methods expose optimal solution without the disturbance of degeneracy condition. There requires least iterations to reach optimality, by applying the existing methods such as MODI method and Stepping Stone method available in the literature [24, 38, 41]. In Revised Version of ASM-Method much easier heuristic approach has been established for finding an optimal solution directly with lesser number of iterations and very easy computations. But from time to time there occur few troubles that, the optimal solution create by them are not actually optimal. In this paper, we have tried to expose that the Revised Version of ASM-Method is the best one for finding only an IBFS for any BTP as well as for any UTP by testing 30 and 10 benchmark problemsin BTP and UTP cases respectively.

The paper is organized as follows: Following theintroduction in Section 1, in Section 2.1, step-by-step algorithmof the VAM is obtainable and in Section 2.2, step-by-step algorithmof the Revised Version of ASM-Method is presented. In Section 3, one benchmark problem,each from balanced type and unbalanced type is illustrated by the method of Revised Version of ASM as well as by VAM. Section 4demonstrates the comparison of Revised Version of ASM-Method withVAM for 30 classical benchmark instances of balanced type and 10 of unbalanced type. Section 5 discusses about theadvantages of Revised Version of ASM-Method over VAM.Finally, in Section 6 conclusions are drawn.

2. Methodology

In this section, we describethe algorithms of Vogel's Approximation Method (VAM) and the Revised Version of ASM-Method.

2.1. Stepwise algorithm of vogel's approximation method (vam)

VAM is an iterative procedure for computing an IBFS of a transportation problem. This method is better than other two methods, namely,NWC and LCM, because the IBFS obtained by this method is nearer to the optimal solution. Solution procedure of this method is described below.

Step 1: Balance the transportation problem.

Step 2: Find the difference between the smallest and second smallest unit transportation costs along every row and column. This difference is known as penalty. Enter the column penalties below the corresponding columns and row penalties to the right of the corresponding rows.

Step-3: Select the highest penalty cost and observe the row or column along which this appears. If a tie occurs, choose any one of them randomly.

Step-4: Identify the cost cell $C_{ij}$for allocation which has the least cost in the selected row/column. Make allocation $X_{ij} = \min(S_i, D_j)$ to the cell (i,j).

Step-5: No further consideration is required for the row or column which is satisfied. If both the row and column are satisfied at a time, delete only one of the two, and the remaining row or column is assigned by a zero supply (or demand).

Step-6: Calculate fresh penalty costs for the remaining sub-matrix as in Step-2 and allocate following the procedure of Steps 3, 4 and 5. Continue the process until all rows and columns are satisfied.

Step-7: Finally calculate the total transportation cost which is the sum of the product of unit transportation cost and corresponding allocated value.

2.2 Stepwise algorithm of theasm-method

Step 1 : Construct the transportation tableau fromgiven TP. Check whether the problem is balanced ornot. If the problem is balanced, go to Step 4, otherwisego to Step 2.

Step 2 : If the problem is not balanced, then anyone of the following two cases may arise:
a) If total supply exceeds total demand, introducean additional dummy column to the transportation tableto absorb the excess supply. The unit transportationcost for the cells in this dummy column is set to 'M',where M > 0 is a very large but finite positive quantity.

or

b) If total demand exceeds total supply, introducean additional dummy row to the transportation table tosatisfy the excess demand. The unit transportation costfor the cells in this dummy row is set to 'M', whereM>0 is a very large but finite positive quantity.

Step 3 : a) In case (a) of Step 2, identify thelowest element of each row and subtract it from eachelement of the respective row and then, in the resultingtableau, identify the lowest element of each columnand subtract it from each element of the respectivecolumn and go to Step 5.

or

b) In case (b) of Step 2, identify the lowest elementof each column and subtract it from each element ofthe respective column and then, in the resulting tableau,identify the lowest element of each row and subtract itfrom each element of the respective row and go toStep 5.

Step 4 : Identify the lowest element of each rowand subtract it from each element of the respectiverow and then, in the resulting tableau, identify the lowestelement of each column and subtract it from eachelement of the respective column.

Step 5 : In the reduced tableau, each row and eachcolumn contains at least one zero. Now, select the firstzero (say zero) and count the number of zeros(excluding the selected one) in the row and columnand record as a subscript of selected zero. Repeat thisprocess for all zeros in the transportation tableau.

Step 6 : Now, choose the cell containing zero forwhich the value of subscript is minimum and supplymaximum possible amount to that cell. If tie occurs forsome zeros in Step 5, choose the cell of that zero forbreaking tie such that the sum of all the elements in therow and column is maximum. Supply maximum possibleamount to that cell.

Step 7 : Delete that row (or column) for furtherconsideration for which the supply from a given sourceis exhausted (or the demand for a given destination issatisfied). If, at any stage, the column demand iscompletely satisfied and row supply is completelyexhausted simultaneously, then delete only one column(or row) and the remaining row (or column) is assigneda zero supply (or demand) in further calculation.

Step 8 : Now, check whether the reduced tableaucontains at least one zero in each row and each column. If this does not happen, repeat Step 4 otherwise go toStep 9.

Step 9 : Repeat Step 5 to Step 8 till all the demandsare satisfied and all the supplies are exhausted.

3. Numerical illustration

Two algorithms for finding an IBFS of TPs are illustrated by the following two benchmark problems from the literature.

3.1 Illustration 1: (Utpal Kanti Das et al., 2014, [43]) Consider the following cost minimizing TP with four sources and six destinations:

Table 3.1: The given BTP

Sources	D1	D2	D3	D4	D5	Supply
S1	10	8	9	5	13	100
S2	7	9	8	10	4	80
S3	9	3	7	10	6	70
S4	11	4	8	3	9	90
Demand	60	40	100	50	90	

3.1.1 Solution by Revised Version of ASM-Method

First the given BTP is solved using the procedure of ASM-Method as follows:
Constructing the Reduced Cost Matrix:

(a) Perform Row Minimum Subtraction

Table 3.2: The Resultant Matrix after Row Minimum Subtraction

Sources	D1	D2	D3	D4	D5	Supply
S1	5	3	4	0	8	100
S2	3	5	4	6	0	80
S3	6	0	4	7	3	70
S4	0	1	5	0	6	90
Demand	60	40	100	50	90	

(b) Perform Column Minimum Subtraction

Table 3.3: The Resultant Matrix after Column Minimum Subtraction

Sources	D1	D2	D3	D4	D5	Supply
S1	2	3	0	0	8	100
S2	0	5	0	6	0	80
S3	3	0	0	7	3	70
S4	5	1	1	0	6	90
Demand	60	40	100	50	90	

The Reduced Cost Matrix (RCM-1)

Making the Allocations one by one

Making the First Allocation

(i) Zero entry cells in order (row-wise)	(ii) No. of zeros in its row and col. (excluding the selected zero) [Minimum]	(iii) Sum of all the elements in the row and col. [Maximum]
(1, 3)	3	
(1, 4)	2	
(2, 1)	2	
(2, 3)	4	
(2, 5)	2	
(3, 2)	1*	22†
(3, 3)	3	
(4, 4)	1*	26†

Note: The minimum entry in column (ii) is marked with the symbol * and the maximum entry in column (iii) is marked with the symbol †.

In the identified cell (4, 4), the maximum possible allocation value of 50 is allocated. Now delete the D4column of the RCM-1 and adjust the supply of the S4 row with 90 – 50 = 40. Observe that the resultant cost matrix does not possess at least one zero in each row and in each column. So, we go for constructing the RCM further. The further RCM is shown in Table 3.4

Table 3.4: Further Reduced Cost Matrix (RCM-2)

Sources	D1	D2	D3	D5	Supply
S1	2	3	0	8	100
S2	0	5	0	0	80
S3	3	0	0	3	70
S4	4	0	0	5	40
Demand	60	40	100	90	

Making the Second Allocation

(i) Zero entry cells in order (row-wise)	(ii) No. of zeros in its row and col. (excluding the selected zero) [Minimum]	(iii) Sum of all the elements in the row and col. [Maximum]
(1, 1)	3	
(2, 1)	2*	14
(2, 3)	5	
(2, 5)	2*	21†
(3, 2)	2*	14
(3, 3)	4	
(4, 2)	2*	17
(4, 3)	4	

In the identified cell (2, 5), the maximum possible allocation value of 80 is allocated. Now delete the S2 row of the RCM-2and adjust the supply of D5 column with 90 – 80 = 10.Observe that the resultant cost matrix does not possess at least one zero in each row and in each column. So, we go for constructing the RCM further. The further RCM is shown in Table 3.5

Table 3.5: Further Reduced Cost Matrix (RCM-3)

Sources	D1	D2	D3	D5	Supply
S1	0	3	0	5	100
S3	1	0	0	0	70
S4	2	0	0	2	40
Demand	60	40	100	10	

Making the Third Allocation

(i) Zero entry cells in order (row-wise)	(ii) No. of zeros in its row and col. (excluding the selected zero) [Minimum]	(iii) Sum of all the elements in the row and col. [Maximum]
(1, 1)	1*	
(1, 3)	3	
(3, 2)	3	
(3, 3)	4	
(3, 5)	2	
(4, 2)	2	
(4, 3)	3	

In the identified cell (1, 1), the maximum possible allocation value of 60 is allocated. Now delete the D1column of the RCM-3 and adjust the supply of S1 row with 100 – 60 =40. Observe that the resultant cost matrix possesses at least one zero in each row and in each column. So, we go for the next allocation.

Making the Fourth Allocation

(i) Zero entry cells in order (row-wise)	(ii) No. of zeros in its row and col. (excluding the selected zero) [Minimum]	(iii) Sum of all the elements in the row and col. [Maximum]
(1, 3)	2*	8†
(3, 2)	3	
(3, 3)	4	
(3, 5)	2*	7
(4, 2)	2*	2
(4, 3)	3	

In the identified cell (1, 3), the maximum possible allocation value of 40 is allocated. Now delete the S1row of the RCM-3 and adjust the demand ofD3 column. Observe that the resultant cost matrix possesses at least one zero in each row and in each column. So, we go for the next allocation.

Making the Fifth Allocation

(i) Zero entry cells in order (row-wise)	(ii) No. of zeros in its row and col. (excluding the selected zero) [Minimum]	(iii) Sum of all the elements in the row and col. [Maximum]
(3, 2)	3	
(3, 3)	3	
(3, 5)	2*	2†
(4, 2)	2*	2†
(4, 3)	2*	

Since tie occurs in column (iii), we can choose any cell. We arbitrary choose the cell (3, 5). [The optimal solution will not be affected if we choose the cell (4, 2) instead of (3, 5) also]. In the identified cell (3, 5), the maximum possible allocation value of 10 is allocated. Now delete the D5column of the RCM-3 and adjust the supply of the S3 row. Observe that the resultant cost matrix possesses at least one zero in each row and in each column. So, we go for the next allocation.

Making the Sixth, Seventh and Eight Allocations

(i) Zero entry cells in order (row-wise)	(ii) No. of zeros in its row and col. (excluding the selected zero) [Minimum]	(iii) Sum of all the elements in the row and col. [Maximum]
(3, 1)	2	
(3, 3)	1*	2†
(4, 1)	1*	2†

In the cells (3, 2), (3, 3) and (4, 3), the possible allocation values of 40, 20, and 40 respectively are allocated. Now the allocation process is complete. The final allocation table obtained through Revised Version of ASM method is shown in Table 3.6.

Table 3.6: Allocation table due to the Revision Version ASM Method

Sources	D1	D2	D3	D4	D5	Supply
S1	**60** 10	8	**40** 9	5	13	100
S2	7	9	8	10	**80** 4	80
S3	9	**40** 3	**20** 7	10	**10** 6	70
S4			**40**	**50**		

	11	4	8	3	9	90
Demand	60	40	100	50	90	

Writing the Allocation Values:

X11 = 60, X13 = 40, X25= 80, X32 = 40, X33 = 20, X35 = 10, X43 = 40, X44 = 50, and all other Xij = 0. Note that the generated solution is a non-degenerate one as it contains exactly eight (m+n-1 = 4+5-1= 8) allocations.

Computing the Total Transportation Cost:

$Z = (60 \times 10) + (40 \times 9) + (80 \times 4) + (40 \times 3) + (20 \times 7) + (10 \times 6) + (40 \times 8) + (50 \times 3)$

$= 600 + 360 + 320 + 120 + 140 + 60 + 320 + 150$= \$2070.

It can be easily verified by MODI method that the IBFS generated by the Revised Version of ASM-Method is the optimal solution to the given BTP.

3.1.2 Solution by VAM

Next the given BTP is solved using the procedure of VAM and the resulting solution obtained is shown in Table 3.7.

Writing the Allocation Values:

X11 = 50, X14 = 50, X25= 80, X31 = 10, X33 = 50, X35 = 10, X42 = 40, X43 = 50, and all other Xij = 0. Note that the generated solution is a non-degenerate one as it contains exactly eight (m+n-1 = 4+5-1= 8) allocations.

Computing the Total Transportation Cost:

$Z = (50 \times 10) + (50 \times 5) + (80 \times 4) + (10 \times 9) + (50 \times 7) + (10 \times 6) + (40 \times 4) + (50 \times 8)$

$= 500 + 250 + 320 + 90 + 350 + 60 + 160 + 400$

= \$2130.

Observation:It is noted that the IBFS generated by VAM is more than that of bythe Revised Version of ASM-Method.

Table 3.7: Allocation table due to VAM

Sources	D1	D2	D3	D4	D5	Supply
S1	**50** 10	8	9	**50** 5	13	100
S2	7	9	8	10	**80** 4	80
S3	**10** 9	3	**50** 7	10	**10** 6	70
S4	11	**40** 4	**50** 8	3	9	90
Demand	60	40	100	50	90	

3.2 Illustration 2: (Abdul Quddoos et al., 2016, [2])

Consider the following cost minimizing TP of unbalanced type:

Table 3.8: The given unbalanced transportation problem (UTP)

Sources	D1	D2	D3	Supply
S1	4	8	8	76
S2	13	24	16	82
S3	8	16	24	77
Demand	72	102	41	

3.2.1 Solution by Revised Version of ASM-Method

Conversion to Balanced TP (BTP)

Table 3.9: Balanced form of the given UTP

Sources	D1	D2	D3	D4	Supply
S1	4	8	8	M	76
S2	13	24	16	M	82
S3	8	16	24	M	77
Demand	72	102	41	20	

Constructing the Reduced Cost Matrix

(a) Perform Row Minimum Subtraction

Table 3.10: The Resultant Matrix after Row Minimum Subtraction

Sources	D1	D2	D3	D4	Supply
S1	0	4	4	M - 4	76
S2	0	11	3	M -13	82
S3	0	8	16	M - 8	77
Demand	72	102	41	20	

(b) Perform Column Minimum Subtraction

Table 3.11: The Resultant Matrix after Column Minimum Subtraction

Sources	D1	D2	D3	D4	Supply
S1	0	0	1	9	76
S2	0	7	0	0	82
S3	0	4	13	5	77
Demand	7	102	41	20	

The Reduced Cost Matrix (RCM-1)

Making the Allocations One by One

Making the First Allocation

Zero entry cells in order (row-wise) (i)	No. of zeros in its row and column (excluding the selected zero) [Minimum] (ii)	Sum of all the elements in the row and column [Maximum] (iii)
(1, 1)	3	
(1, 2)	1*	
(2, 1)	4	
(2, 3)	2	
(2, 4)	2	
(3, 1)	2	

Note: The minimum entry in column (ii) is marked with the symbol * and the maximum entry in column (iii) is marked with the symbol †.

In the identified cell (1, 2), the maximum possible allocation value of 76 is allocated. Now delete the 1st row of the RCM-1 and adjust the supply of the 2nd column. Observe that the resultant cost matrix does not possess at least one zero in each row and in each column. So, we go for constructing the RCM further. The further RCM is shown in Table 3.12.

Table 3.12: Further Reduced Cost Matrix (RCM-2)

Sources	D1	D2	D3	D4	Supply

S2	0	3	0	0	82
S3	0	0	13	5	77
Demand	72	26	41	20	

Making the Second Allocation

Zero entry cells in order (row-wise) (i)	No. of zeros in its row and column (excluding the selected zero) [Minimum] (ii)	Sum of all the elements in the row and column [Maximum] (iii)
(2, 1)	3	
(2, 3)	2	
(2, 4)	2	
(3,1)	2	
(3, 2)	1*	

In the identified cell (3, 2), the maximum possible allocation value of 26 is allocated. Now delete the 2nd column of the RCM-2and adjust the supply of the 3rd row. Observe that the resultant cost matrix possesses at least one zero in each row and in each column. So, we go for the next allocation.

Making the Third Allocation

Zero entry cells in order (row-wise) (i)	No. of zeros in its row and column (excluding the selected zero) [Minimum] (ii)	Sum of all the elements in the row and column [Maximum] (iii)
(2, 1)	3	
(2, 3)	2	
(2, 4)	2	
(3, 1)	1*	

In the identified cell (3, 1), the maximum possible allocation value of 51 is allocated. Now delete the 3rd row of the RCM-2and adjust the supply of 1st column. Observe that the resultant cost matrix does not possess at least one zero in each row and in each column.So, we go for constructing the RCM further. The further RCM is shown in Table 3.13.

Table 3.13: Further Reduced Cost Matrix (RCM-3)

Sources	D1	D3	D4	Supply
S2	0	0	0	82
Demand	21	41	20	

Making the Fourth, Fifth, and Sixth Allocations

Zero entry cells in order (row-wise) (i)	No. of zeros in its row and column (excluding the selected zero) [Minimum] (ii)	Sum of all the elements in the row and column [Maximum] (iii)
(2, 1)	0*	0†
(2, 3)	0*	0†
(2, 4)	0*	0†

The possible allocation values 21, 41 and 20 are allocated in the cells (2, 1), (2, 3) and (2, 4) respectively. Now all the allocation process is over. The final allocation table obtained through the Revised Version of ASM method is shown in Table 3.14.

Table 3.14: Allocation table due to the Revised Version of ASM Method

Sources	D1	D2	D3	D4	Supply
S1	4	**76** 8	8	M	76
S2	**21** 13	24	**41** 16	**20** M	82
S3	**51** 8	**26** 16	24	M	77
Demand	72	102	41	20	

Writing the Allocation Values

X12 = 76, X21 = 21, , X23 = 41, X24 = 20,X31 = 51, X32 = 26, and all other Xij = 0.

Computing the Total Transportation Cost

Z= (76×8) + (21 × 13) + (41 × 16) + (51 × 8) + (26 × 16)

= 608 + 273 + 656 + 408 + 416 =$2361

It is also verified by the MODI method that the IBFS generated by the Revised Version of ASM Method is the optimal solution to the given UTP.

3.2.2 Solution by VAM

Next the given UTP is solved using the procedure of VAM and the resulting solution obtained is shown in Table 3.15

Table 3.15: Allocation table due to the Revised Version of ASM Method

Sources	D1	D2	D3	D4	Supply
S1	4	**76** 8	8	M	76
S2	13	**21** 24	**41** 16	**20** M	82
S3	**72** 8	**5** 16	24	M	77
Demand	72	102	41	20	

Writing the Allocation Values

X12 = 76, X22 = 21, X23 = 41, X24 = 20,X31 = 72,X32 = 5, and all other Xij = 0.

Computing the Total Transportation Cost

Z= (76×8) + (21 × 24) + (41 × 16) + (72 × 8) + (5 × 16)

= 608 + 504 + 656 + 576 + 80

=$2424

Observation:It is noted that the total transportation cost of $2424 by the IBFS generated by VAM is more than that of ($2461) bythe Revised Version of ASM-Method.

4. Result analysis

4.1 Analysis for Balanced Case

The comparison of the results for 30 benchmark problems of balanced case has been studied in this research to measure the effectiveness of the Revised ASM-Method over VAM. This comparison is shown in following Table 4.1.

Table 4.1: Performance Measure of Revised Version of ASM-Method for Classical BTPs

Problem No.,(Author(s), Year)	Optimal Solution	Solution by	
		ASM	VAM
Problem 1(Ramadan and Ramadan, 2012, [32]) [Cij] 3×3= [32 40 120; 60 68 104; 200 80 60] [Si] 3×1= [20, 30, 45] [Dj] 1×3= [30, 35, 30]	5600	5600	5600
Problem 2(Srinivasan and Thompson, 1977, [40]) [Cij] 3×4 = [3 6 3 4; 6 5 11 15; 1 3 10 5] [Si] 3×1 = [80, 90, 55] [Dj] 1×4= [70, 60, 35, 60]	880	880	955
Problem 3(Schrenket al., 2011, [36]) [Cij] 3×4= [3 6 1 5; 7 9 2 7; 2 4 2 1] [Si] 3×1= [6, 6, 6] [Dj] 1×4= [4, 5, 4, 5]	59	59	59
Problem 4 (Samuel, 2012, [35]) [Cij] 3×4= [1 2 3 4;4 3 2 0; 0 2 2 1] [Si] 3×1= [6, 8, 10] [Dj] 1×4= [4, 6, 8, 6]	28	28	28
Problem 5(Imam et al. , 2009, [20]) [Cij] 3×4= [10 2 20 11;12 7 9 20; 4 14 16 18] [Si] 3×1= 15, 25, 10] [Dj] 1×4= [5, 15, 15, 15]	435	435	475
Problem 6(Ahmed M.M., et al., 2014, [6]) [Cij] 4×3= [2 7 4; 3 3 1; 5 4 7; 1 6 2] [Si] 4×1= [5, 8, 7, 14][Dj] 1×3= [7, 9, 18]	76	76	80
Problem 7(Mollah M Ahmed et al. 2016, , [30]) [Cij] 4×4= [7 5 9 11;4 3 8 6;3 8 10 5;2 6 7 3] [Si] 4×1= [30, 25, 20, 15] [Dj] 1×4= [30, 30, 20, 10]	410	410	470
Problem 8(JumanandHoque M.A., 2015, [23]) [Cij] 3×4= [19 30 50 12;70 30 40 60;40 10 60 20] [Si] 3×1= [7, 10, 18][Dj] 1×4= [5, 7, 8, 15]	809	809	859
Problem 9 (Juman and Hoque M.A., 2015, [23]) [Cij] 3×4= [13 18 30 8;55 20 25 40;30 6 50 10] [Si] 3×1= [8, 10, 11] [Dj] 1×4= [4, 6, 7, 12]	417	417	476
Problem 10 (Aminur R. Khan, 2012, [7]) [Cij] 3×4= [6 1 9 3;11 5 2 8;10 12 4 7] [Si] 3×1= [70, 55, 90] [Dj] 1×4= [85, 35, 50, 45]	1160	1160	1220

Problem 11 (Aminur R. Khan, 2012, [7]) [Cij] 4×6= [7 10 7 4 7 8;5 1 5 5 3 3;4 3 7 9 1 9; 4 6 9 0 0 8] [Si] 4×1= [5, 6, 2, 9] [Dj] 1×6= [4, 4, 6, 2, 4, 2]	68	68	68
Problem 12 (Adlakha and Kowalski, 2009, [3]) [Cij] 4×5= [2 1 3 2 2; 3 2 1 1 1; 5 4 2 1 3; 7 5 5 3 1] [Si] 4×1= [20, 70, 30, 60] [Dj] 1×5= [50, 30, 30, 50, 20]	390	390	390
Problem 13 (Abdul Hakim,AchiyaKhatun, 2018, [18]) [Cij] 3×4= [5 3 6 2 ; 4 7 9 1; 3 4 7 5] [Si] 3×1= [19, 37, 34] [Dj] 1×4= [16, 18, 31, 25]	355	355	355
Problem 14 (Abdul Hakim and et al. , 2018, [18]) [Cij] 4×4= [4 6 5 2;6 4 1 4;5 2 3 1;4 6 7 8] [Si] 4×1= [6, 10, 12, 14] [Dj] 1×4= [9, 16, 10, 7]	111	111	114
Problem 15 (Ray and Hossain, 2007, [33]) [Cij] 4×3= [4 3 4;10 7 5;8 8 3;5 6 6] [Si] 4×1= [11, 12, 10, 7] [Dj] 1×3= [16, 10, 14]	183	183	199
Problem 16 (Opera Jude et al., 2017, [31]) [Cij] 4×4= [45 52 63 57;58 48 56 54;52 55 62 58; 65 48 44 54] [Si] 4×1= [15500, 12000, 14400, 11600] [Dj] 1×4= [12600, 12500, 13000, 15400]	2655600	2655600	2657000
Problem 17 (Opera Jude et al., 2017, [31]) [Cij] 4×4= [2 5 6 3;9 6 2 1;5 2 3 6;7 7 2 4] [Si] 4×1= [6, 9, 7, 12] [Dj] 1×4= [10, 4, 6, 14]	83	83	92
Problem 18 (Opera Jude et al., 2017, [31]) [Cij] 3×3= [4 3 5; 6 5 4; 8 10 7] [Si] 3×1= [90, 80, 100] [Dj] 1×3= [70, 120.80]	1390	1390	1500
Problem 19 (Md. AshrafulBabu et al., 2013, [9]) [Cij] 3×4= [19 30 50 12; 70 30 40 60; 40 10 60 20] [Si] 3×1= [7 10 18][Dj] 1×4= [5, 8, 7, 15]	799	799	859
Problem 20 (Md. AshrafulBabu et al., 2014, [8]) [Cij] 4×4= [5 3 6 10;6 8 10 7;3 1 6 7;8 2 10 12] [Si] 4×1= [30, 10, 20, 10] [Dj] 1×4= [20, 25, 15, 10]	285	285	285
Problem 21 (Mhlanga A, 2014, [29]) [Cij] 4×5= [4 9 8 10 12;6 10 3 2 3;3 2 7 10 3;3 5 5 4 8]	316	322*	316

[Si] 4×1= [24, 18, 20, 16] [Dj] 1×5= [10, 20, 10, 18, 20]			
Problem 22 (Deshmukh N.M,, 2012, [16]) [Cij] 3×5= [4 1 2 4 4; 2 3 2 2 3; 3 5 2 4 4] [Si] 3×1= [60, 35, 40] [Dj] 1×5= [22, 45, 20, 18, 30]	290	290	290
Problem 23 (Deshmukh N.M,, 2012, [16]) [Cij] 3×4= [19 30 50 10;70 30 40 60;40 8 70 20] [Si] 3×1= [7, 9, 18] [Dj] 1×4= [5, 8, 7, 14]	743	743	779
Problem 24 (Deshmukh N.M,, 2012, [16]) [Cij] 4×6= [9 12 9 6 9 10;7 3 7 7 5 5;6 5 9 11 3 11;6 8 11 2 2 10] [Si] 4×1= [5, 6, 2, 9][Dj] 1×6= [4, 4, 6, 2, 4, 2]	112	112	112
Problem 25 (Russell E.J., 1969, [34]) [Cij] 5×5= [73 40 9 79 20; 62 93 96 8 13; 96 65 80 5065; 57 58 29 12 87; 56 23 87 18 12] [Si] 5×1= [8, 7, 9, 3, 5] [Dj] 1×5= [6, 8, 10, 4, 4]	1102	1103*	1104
Problem 26 (Shweta Sing et al., 2012, [39]) [Cij] 5×5= [68 35 4 74 15; 57 88 91 3 8; 91 60 75 45 60; 52 53 24 7 82; 51 18 82 13 7] [Si] 5×1= [18, 17, 19, 13, 15] [Dj] 1×5= [16, 18, 20, 14, 14]	2202	2324*	2224
Problem 27 (WagenerU.A., 1965, [45]) [Cij] 5×6= [5 3 7 3 8 5; 5 6 12 5 7 11; 2 8 3 4 8 2; 9 6 10 5 10 9; 5 3 7 3 8 5] [Si] 5×1= [3, 4, 2, 8, 3] [Dj] 1×6= [3, 4, 6, 2, 1, 4]	112	112	112
Problem 28 (UtpalKanti Das et al., 2014, [43]) [Cij] 4×5= [10 8 9 5 13; 7 9 8 10 4; 9 3 7 10 6; 11 4 8 3 9] [Si] 4×1 = [100, 80, 70, 90] [Dj]1×5= [60, 40, 100, 50, 90]	2070	2070	2130
Problem 29 (UtpalKanti Das et al., 2014, [43]) [Cij] 5×7 = [12 7 3 8 10 6 6;6 9 7 12 8 12 4;10 12 8 4 99 3; 8 5 11 6 7 9 3;7 6 8 11 9 5 6] [Si] 5×1 = [60, 80, 70, 100, 90] [Dj]15×7 = [20, 30, 40, 70, 60, 80, 100]	1900	1900	1930
Problem 30 (Khan A.R. et al., 2015, [26]) [Cij] 6×6= [12 4 13 18 9 2; 9 16 10 7 15 11; 4 9 10 8 9 7; 9 3 12 6 4 5;7 11 5 18 2 7; 16 8 4 5 1 10] [Si] 6×1= [120, 80, 50, 90, 100, 60]	2170	2170	2310

[Dj] 1×6= [75, 85, 140, 40, 95, 65]			

From Table 4.1, we scrutinize that the Revised Version of ASM method has produced optimal solution to 27(i.e. 90% of) problems and near optimal solution to 3 problems (namely Problem Nos. 21, 25 and 26), where as VAM has produced optimal solution to only 11 (i.e. 36.6% of) problems and near optimal solution to 19 problems.

4.2 Analysis for Unbalanced Case

The evaluation of the results for 10 benchmark problems of unbalanced case has been studied in this research to measure the effectiveness of the Revised ASM-Method over VAM. This comparison is shown in following Table 4.2.

Table 4.2: Performance Measure of Revised Version of ASM-Method for Classical UTPs

Problem No.,(Author(s), Year)	Optimal Solution	Solution by ASM-Method	Solution by VAM
Problem 1 (Sen et al., 2010, [37]) [Cij] 5×4 = [60 120 75 180; 58 100 60 165; 62 110 65 170; 65 115 80 175; 70 135 85 195] [Si] 5×1= [8000, 9200, 6250, 4900, 6100] [Dj] 1×4= [5000, 2000, 10000, 6000]	21,46,750	2164000*	2164000
Problem 2 (Kulkarni and Datar, 2010, [27]) [Cij] 4×3= [3 4 6; 7 3 8; 6 4 5; 7 5 2] [Si] 4×1= [100, 80, 90, 120] [Dj] 1×3= [110, 110, 60}	840	840	880
Example 3 (Deshmukh, 2012, [16]) [Cij] 3×4= [19 30 50 10; 70 30 40 60; 40 8 70 20] [Si] 3×1= [7, 9, 18] [Dj] 1×4= [40, 8, 7, 14]	743	779*	779
Example 4 (T.Geetha and N.Anandhi, 2015, [17]) [Cij] 3×4= [6 1 9 3; 11 5 2 8; 10 12 4 7] [Si] 3×1= [70, 55, 70] [Dj] 1×4= [85, 35, 50, 45]	960	960	1010
Example 5 (T.Geetha and N.Anandhi, 2015, [17]) [Cij] 4×3= [5 6 9; 3 5 10; 6 7 6; 6 4 10] [Si] 4×1= [100, 75, 50, 75] [Dj] 1×3= [70, 80, 120]	1465	1465	1555
Example 6 (T.Geetha and N.Anandhi, 2015, [17]) [Cij] 3×4= [10 15 12 12; 8 10 11 9; 11 12 13 10] [Si] 3×1= [200, 150, 120] [Dj] 1×4= [140, 120, 80, 220]	4720	4720	5020
Example 7 (T.Geetha and N.Anandhi, 2015, [17]) [Cij] 3×4= [7 8 11 10; 10 12 5 4; 6 11 10 9] [Si] 3×1= [30, 45, 35]	606	628*	620

[Dj] 1×4= [20, 28, 19, 33]			
Example 8 (Abdul Quddoos et al., 2016, [2]) [Cij] 4×3= [2 7 14; 3 3 1; 5 4 7; 1 6 2] [Si] 4×1= [5, 8, 7, 15] [Dj] 1×3= [7, 9, 18]	75	79*	76
Example 9 (Abdul Quddoos et al., 2016, [2]) [Cij] 4×4= [4 6 8 13; 13 11 19 8; 14 4 10 13; 9 11 13 8] [Si] 4×1= [50, 70, 30, 50] [Dj] 4×3= [25, 35, 105, 20]	1545	1545	1855
Example 10 (Abdul Quddoos et al., 2016) [Cij] 3×3= [4 8 8; 13 24 16; 8 16 24] [Si] 3×1= [76, 82, 77] [Dj] 1×1= [72, 102,41]	2361	2361	2424

From Table 4.2, we detect that the Revised Version of ASM method has produced optimal solution to 6 (i.e. 60% of) problems and near optimal solution to 4 problems (namely Problem Nos. 1, 3, 7 and 8), where as VAM has produced optimal solution no one (i.e. 0% of) problem and near optimal solution to 10 problems.

4.3 Effectiveness of Revised Version of ASM over VAM

The overall analysis of the results produced by the Revised Version of ASM method and VAM reflects their efficiency. The efficiency of the Revised Version of ASM method is shown in Table 4.3 and that by the VAM is shown inTable 4.4.

Table 4.3 Effectiveness of Revised Version of ASM-Method

Type of TP	No. of Problems Tested	No. of Problems produced Optimal Solution	% of Problems produced Optimal Solution	No. of Problems produced Near Optimal Solution	% of Problems produced Near Optimal Solution
Balanced	30	27	90%	03	10%
Unbalanced	10	06	60%	04	40%

Table 4.4 Effectivenessof VAM

Type of TP	No. of Problems Tested	No. of Problems produced Optimal Solution	% of Problems produced Optimal Solution	No. of Problems produced Near Optimal Solution	% of Problems produced Near Optimal Solution
Balanced	30	11	36.7%	19	63.3%
Unbalanced	10	Nil	0%	10	100%

5. Advantages of the revised version of ASM-Method

The Revised Version of ASM-Method is originating to have the following advantages over VAM:

1. It is an excellent method to find the best IBFS, which is either optimal directly or very close to the optimal solution.
2. It has produced optimal solution to 90% of the BTPs and 60% of the UTPs..

3. It always provides non-degenerate solution to the TP.
4. It is based on making allocations to zero entry cell of reduced cost matrix.
5. It is very easy to understand and apply.
6. Mathematical calculations involved in this method are very easy, so no expertise inmathematics is needed to use this method.

6. Conclusion

In this paper, we have tried to expose that the Revised Version of ASM-Method is the best one for finding an IBFS for any transportation problem. To verify the performance of the method, 30 classical benchmark instances of balanced kind and 10 that of from unbalanced kind from the literature have been tested. Simulation results substantiate that the method produces optimal solution to 27(i.e. 90% of) BTPs and near optimal solution to 3BTPs, where as VAM produces optimal solution to only 11(i.e. 36.6%) BTPs and near optimal solution to 19 BTPs. Another testing result on unbalanced kind authenticates that the method produces optimal solution to 6UTPs and near optimal solution to 4UTPs, where as VAM produces optimal solution to no one (i.e. 0% of) UTP and near optimal solution to 10UTPs.Therefore, it is established and recognized that the Revised Version of ASM-Method produces the best IBFS, which is either optimal or very close to optimal solution.Further,the most attractive feature of this method is that it requires only simplearithmetical and logical calculations and hence anyone can easily understand and apply it far better than VAM. Also, this method will be more cost-effective for those decision makers who are trading with logistics and supply chain problems.

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