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INTUITIONISTIC MULTI FUZZY SUBFIELDS OF A FIELD

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Abstract. In this paper, some properties of intuitionistic multi fuzzy subfield of a fieldare defined and studied. Also, some definitions, results and theorems are given.

Keywords. Fuzzy set, multi fuzzy subset,intuitionistic multi fuzzy subset, multi fuzzy subfield, intuitionistic multi fuzzy subfield.

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1. Introduction

After the introdution of fuzzy sets by L.A.Zadeh [13], several researchers explored on the generalization of the notion of fuzzy set.The concept of intuitionistic Multi fuzzy subset was introduced by K.T.Atanassov [1], as a generalization of the notion of fuzzy set. AzrielRosenfeld[2] defined a fuzzy groups. Vasu.M, Sivakumar.D&Arjunan.K[12] defined an anti-Multi fuzzy subfield of a field. We introduce the concept of intuitionistic Multi fuzzy subfield of a fieldand established some results.

2. Preliminaries

Definition 2.1.

Let X be a non-empty set. A fuzzy subset A of X is a function A : X $\rightarrow$[0, 1].

Definition 2.2. *[7]*.

A multi fuzzy subset A of a set X is defined as an object of the form A = { $\langle$ x, A1(x), A2(x), A3(x), …, An(x) $\rangle$ / x$\in$X}, where Ai : X$\rightarrow$[0, 1] for all i. Here A is called multi fuzzy subset of X with dimension n. It is denoted as A = $\langle$ A1, A2, A3, …, An$\rangle$.

Example 2.3 Let X = { a, b, c } be a set. Then A = {⟨ a, 0.4, 0.3, 0.7⟩, ⟨ b, 0.2, 0.7, 0.8⟩, ⟨ c, 0.4, 0.1, 0.5⟩ } is a multi fuzzy subset of X with the dimension three.

Definition 2.4

An intuitionistic fuzzy set (IFS) A in X is defined as an object having the form $A = \{(x, \mu A(x), \gamma A(x)) / x \in X\}$, where $\mu A: X \to [0,1]$ and $\gamma A: X \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and every x in X satisfying $0 \le \mu A(x) + \gamma A(x) \le 1$.

Example 2.5. An intuitionistic subset A={(a, 0.3, 0.6),(b, 0.3, 0.5),(c, 0.2, 0.5)}of a set X={ a,b,c}.

Definition 2.6.

An intuitionisticmulti fuzzy subset A of a set X is defined as an object of the form $A = \{ (x, \mu A1(x), \mu A2(x), .., \mu An(x), \gamma A1(x), \gamma A2(x), .., \gamma An(x))/ x \in X\}$, where $\mu Ai : X \to [0, 1]$ and $\gamma Ai : X \to [0, 1]$ for all i, define the degrees of membership and the degrees of non-membership of the element $x \in X$ respectively and every x in X satisfying $0 \le \mu Ai(x) + \gamma Ai(x) \le 1$ for all i. It is denoted as $A = (\mu A, \gamma A)$ where $\mu A = (\mu A1, \mu A2, \dots, \mu An)$ and $\gamma A = (\gamma A1, \gamma A2, \dots, \gamma An)$.

Example 2.7. Let X = { a, b, c } be a set. Then A = { (a, (0.4, 0.3, 0.2), (0.2, 0.4, 0.4)), (b, (0.2, 0.4, 0.3), (0.3, 0.5, 0.7)), (c, (0.4, 0.1, 0.5), (0.5, 0.6, 0.4) } is a intuitionistic multi fuzzy subset of X with the dimension three.

Definition 2.8.

Let A and B be any two intuitionistic multi fuzzy subset of X. We define the following relations and operations:

(i) $A \subseteq B$ if and only if $\mu Ai(x) \le \mu Bi(x)$ and $\gamma Ai(x) \ge \gamma Bi(x)$ for all $x \in X$ and for all i.

(ii) A = B if and only if $\mu Ai(x) = \mu Bi(x)$ and$\gamma Ai(x) = \gamma Bi(x)$ for all $x \in X$ and for all i.

(iii)$A \cap B$ if and only if $(A \cap B)(x)=\{\min\{\mu Ai(x),\mu Bi(x)\},\max\{\gamma Ai(x),\gamma Bi(x)\}\}$forall $x \in X$ and for all i.

(iv)$A \cup B$ if and only if $(A \cup B)(x)=\{\max\{\mu Ai(x),\mu Bi(x)\},\min\{\gamma Ai(x),\gamma Bi(x)\}\}$for all $x \in X$ and for all i.

Definition 2.9

Let (F, +, ·) be a field. A multi fuzzy subset A of F is said to be a multi fuzzysubfield(MFSF) of F if the following conditions are satisfied:

(i)$Ai(x-y) \ge \min \{ Ai(x), Ai(y) \}$, for all $x, y \in F$, for all i,

(ii)$Ai(xy-1) \ge \min \{ Ai(x), Ai(y) \}$, for all $x, y \ne 0 \in F$,for all i, where 0 is the additive identity element of F.

Definition 2.10

Let (F, +, .) be a field. Anintuitionistic multi fuzzy subset A of F is said to be anintuitionistic multi fuzzysubfield(ILFSF) of F if it satisfies the following axioms:

(i) $\mu Ai(x-y) \geq \min\{\mu Ai(x), \mu Ai(y)\}$, for all x, y∈F, for all i,

(ii) $\mu Ai(xy\text{-}1) \geq \min\{\mu Ai(x), \mu Ai(y)\}$ for all x , y ≠ 0 in F, for all i

(iii) $\nu Ai(x-y) \leq \min\{\nu Ai(x), \nu A(y)\}$ for all x , y in F,for all i

(iv) $\nu Ai(xy\text{-}1) \leq \min\{\nu Ai(x), \nu A(y)\}$ for all x , y≠ 0 in F, for all iwhere 0 is the additive identity element of F.

3. Some properties

Theorem 3.1:

If A is an intuitionistic multi fuzzy subfield of a field (F, +, ·), then $\mu Ai(-x) = \mu Ai(x)$, for all x in F and$\mu Ai(x\text{-}1) = \mu Ai(x)$, for all x≠ 0 in F and $\nu Ai(-x) = \nu Ai(x)$, for all x in F and $\nu Ai(x\text{-}1) = \nu Ai(x)$, for all x≠ 0 in F and $\mu Ai(x) \leq \mu Ai(0)$, for all x in F and $\mu Ai(x) \leq \mu Ai(1)$, for all x≠ 0 in F and $\nu Ai(x) \geq \nu Ai(0)$, for all x in F and $\nu Ai(x) \geq \nu Ai(1)$, for all x≠ 0 in F, for all i, where 0 and 1 are identity elements in F.

Proof: For x in F and 0, 1 are identity elements in F.

Now, $\mu Ai(x) = \mu Ai(-(-x)) \geq \mu Ai(-x) \geq \mu Ai(x)$. Therefore, $\mu Ai(-x) = \mu Ai(x)$, for all x in F and for all i. Now, $\mu Ai(x) = \mu Ai((x\text{-}1)\text{-}1) \geq \mu Ai(x\text{-}1) \geq \mu Ai(x)$. Therefore, $\mu Ai(x\text{-}1) = \mu Ai(x)$, for all x≠ 0 in F and for all I, and, $\nu Ai(x) = \nu Ai(-(-x)) \leq \nu Ai(-x) \leq \nu A(x)$.

Therefore, $\nu Ai(-x) = \nu Ai(x)$, for all x in F and for all I, and, $\nu Ai(x) = \nu Ai((x\text{-}1)\text{-}1) \leq \nu Ai(x\text{-}1) \leq \nu Ai(x)$. Therefore, $\nu Ai(x\text{-}1) = \nu Ai(x)$, for all x≠ 0 in F and for all i.

Now, $\mu Ai(0) = \mu Ai(x-x) \geq \min\{\mu Ai(x), \mu Ai(-x)\} = \mu Ai(x)$. Therefore, $\mu Ai(0) \geq \mu Ai(x)$, for all x in F and for all i. Now, $\mu Ai(1) = \mu Ai(xx\text{-}1) \geq \min\{\mu Ai(x), \mu Ai(x\text{-}1)\} = \mu Ai(x)$. Therefore, $\mu Ai(1) \geq \mu Ai(x)$, for all x≠ 0 in F and for all I, and, $\nu Ai(0) = \nu Ai(x-x) \leq \max\{\nu Ai(x), \nu Ai(-x)\} = \nu Ai(x)$.

Therefore, $\nu Ai(0) \leq \nu Ai(x)$, for all x in F and for all I, and, $\nu Ai(1) = \nu Ai(xx\text{-}1) \leq \max\{\nu Ai(x), \nu Ai(x\text{-}1)\} = \nu Ai(x)$. Therefore, $\nu Ai(1) \leq \nu Ai(x)$, for all x≠ 0 in F and for all i.

Theorem 3.2:

If A is an intuitionistic multi fuzzy subfield of a field (F, +, ·), then for all i,

(i) $\mu Ai(x-y) = \mu Ai(0)$ gives $\mu Ai(x) = \mu Ai(y)$, for all x and y in F,

(ii) $\mu Ai(xy\text{-}1) = \mu Ai(1)$ gives $\mu Ai(x) = \mu Ai(y)$, for all x and y≠ 0 in F,

(iii) $\nu Ai(x-y) = \nu Ai(0)$ gives $\nu Ai(x) = \nu Ai(y)$, for all x and y in F and

(iv) $\nu Ai(xy\text{-}1) = \nu Ai(1)$ gives $\nu Ai(x) = \nu Ai(y)$, for all x and y≠ 0 in F, where 0 and 1 are identity elements in F.

Proof: Let x and y in F and 0, 1 are identity elements in F.

(i) Now, $\mu Ai(x) = \mu Ai(x-y+y) \geq \min\{\mu Ai(x-y), \mu Ai(y)\} = \min\{\mu Ai(0), \mu Ai(y)\} = \mu Ai(y)$
$= \mu Ai(x-(x-y)) \geq \min\{(\mu Ai(x-y), \mu Ai(x)\} = \min\{\mu Ai(0), \mu Ai(x)\} = \mu Ai(x)$. Therefore, $\mu Ai(x) = \mu Ai(y)$, for all x, y in F and for all i.

(ii) Now. $\mu Ai(x) = \mu Ai(xy\text{-}1y) \geq T(\mu Ai(xy\text{-}1), \mu Ai(y)) = T(\mu Ai(1), \mu Ai(y)) = \mu Ai(y)$
$= \mu Ai((xy\text{-}1)\text{-}1x) \geq T(\mu Ai(xy\text{-}1), \mu Ai(x)) = T(\mu Ai(1), \mu Ai(x)) = \mu Ai(x)$. Therefore, $\mu Ai(x) = \mu Ai(y)$, for all x, y ≠ 0 in F and for all i.

(iii) Now, $\nu Ai(x) = \nu Ai(x-y+y) \leq S(\nu Ai(x-y), \nu Ai(y)) = S(\nu Ai(0), \nu Ai(y)) = \nu Ai(y)$
$= \nu Ai(x-(x-y)) \leq S(\nu Ai(x-y), \nu Ai(x)) = S(\nu Ai(0), \nu Ai(x)) = \nu Ai(x)$.

Therefore, $\nu Ai(x) = \nu Ai(y)$, for all x, y in F and for all i.

(iv) Now, $\nu Ai(x) = \nu Ai(xy\text{-}1y) \leq S(\nu Ai(xy\text{-}1), \nu Ai(y)) = S(\nu Ai(1), \nu Ai(y)) = \nu Ai(y)$
$= \nu Ai((xy\text{-}1)\text{-}1x) \leq S(\nu Ai(xy\text{-}1), \nu Ai(x)) = S(\nu Ai(1), \nu Ai(x)) = \nu Ai(x)$.

Therefore, $\nu Ai(x) = \nu Ai(y)$, for all x≠ 0, y≠ 0 in F and for all i.

Theorem 3.3:

Let A be anintuitionistic multi fuzzy subset of a field (F, +, ·). If for all i, μAi(e) = μAi(eꞁ) = 1 and νAi(e) =νAi(eꞁ) = 0 and μAi(x–y) ≥min{(μAi(x), μAi(y) } for all x and y in F andμAi(xy-1) ≥min{μAi(x), μAi(y) } for all x and y≠ e in F and νAi(x–y) ≤max{νAi(x), νAi(y)} for all x and y in F and νAi(xy-1) ≤max{νAi(x), νAi(y)}for all x and y≠ e in F, then A is an intuitionistic multi fuzzy subfield of F, where e and eꞁ are identity elements of F.

Proof: It is trivial.

Theorem 3.4:

If A is an intuitionistic multi fuzzy subfield of a field (F, +, ·), then H = { x / x∈F: μAi(x) = 1, νAi(x) = 0 for all i } is either empty or a subfield of F.

Proof: If no element satisfies this condition, then H is empty. If x, y in H, thenμAi(x–y) ≥T(μAi(x), μAi(–y)) ≥T(μAi(x),μAi(y)) =T(1, 1) = 1. Therefore, μAi(x–y) = 1, for all x, y in H and for all I, and μAi(xy-1) ≥T(μAi(x), μAi(y-1)) ≥T(μAi(x),μAi(y)) = T(1, 1) = 1. Therefore, μAi(xy-1) = 1, for all x, y≠ e in H and for all i. And νAi(x–y) ≤S(νAi(x), νAi(–y)) ≤ S(νAi(x),νAi(y)) = S(0, 0) = 0. Therefore νAi(x–y) = 0, for all x, y in H and for all I, and νAi(xy-1) ≤ S(νAi(x),νAi(y-1)) ≤ S(νAi(x),νAi(y)) = S(0, 0) = 0. Therefore νAi(xy-1) = 0, for all x, y≠ e in H and for all i.
We get x–y, xy-1 in H. Therefore H is a subfield of F. Hence H is either empty or a subfield of F.

***Theorem 3.*5:**

Let A be a (T, S)-intuitionistic multi fuzzy subfield of a field (F, +, ·). Then for all i,

(i) if μAi(x–y) = 1, then μAi(x) = μAi(y), for all x and y in F and if μAi(xy-1) = 1, then μAi(x) = μAi(y), for all x and y≠ e in F,

(ii) if νAi(x–y) = 0, then νAi(x) = νAi(y), for all x and y in F and if νAi(xy-1) = 0, then νAi(x) =νAi(y), for all x and y≠ e in F, where e and eꞁ are identity elements of F.

Proof: Let x and y in F.
(i) Now, μAi(x) = μAi(x–y+y)≥T(μAi(x–y), μAi(y)) = T(1, μAi(y)) = μAi(y)= μAi(–y) = μAi(–x+x–y)≥T(μAi(–x), μAi(x–y)) = T(μAi(–x), 1) = μAi(–x) = μAi(x). Therefore μAi(x) = μAi(y), for all x, y in F and for all I, and μAi(x) = μAi(xy-1y)≥T(μAi(xy-1), μAi(y)) = T(1, μAi(y)) = μAi(y)= μAi(y-1) = μAi(x-1xy-1)≥T(μAi(x-1), μAi(xy-1)) = T(μAi(x-1), 1) = μAi(x-1)= μAi(x). Therefore μAi(x) = μAi(y), for all x≠ e, y≠ e in F and for all i.
(ii) Now, νAi(x) = νAi(x–y+y)≤S(νAi(x–y), νAi(y)) = S(0, νAi(y)) = νAi(y)= νAi(–y) = νAi(–x+x–y)≤S(νAi(–x), νAi(x–y)) = S(νAi(–x), 0) = νAi(–x) = νAi(x). Therefore νAi(x) = νAi(y), for all x, y in F and for all I, and νAi(x) = νAi(xy-1y)≤S(νAi(xy-1), νAi(y)) = S(0, νAi(y)) = νAi(y)= νAi(y-1) = νAi(x-1xy-1)≤S(νAi(x-1), νAi(xy-1)) = S(νAi(x-1), 0) = νAi(x-1) = νAi(x). Therefore, νAi(x) = νAi(y), for all x≠ e, y≠ e in F and for all i.

***Theorem 3.*6:**

If A be a (T, S)-intuitionistic multi fuzzy subfield of a field(F, +, ·) , then for all i,

(i) if μAi(x–y) = 0, then either μAi(x) = 0 or μAi(y) = 0, for all x and y in F and if μAi(xy-1) = 0, then either μAi(x)= 0 or μAi(y)= 0, for all x and y≠ e in F,

(ii) if νAi(x–y) =1, then either νAi(x) = 1or νAi(y) = 1, for all x and y in F and if νAi(xy-1) =1, then either νAi(x) = 1or νAi(y) = 1, for all x and y≠ e in F, where e and eꞁ are identity elements of F.

Proof: Let x and y in F.
(i) By the definition μAi(x–y) ≥T(μAi(x), μAi(y)), which implies that 0 ≥T(μAi(x), μAi(y)). Therefore, either μAi(x) = 0 or μAi(y) = 0, for all x, y in F and for all I, and,by thedefinition μAi(xy-1) ≥T(μAi(x), μ Ai(y)), which implies that 0 ≥T(μAi(x), μ Ai(y)).
Therefore, either μAi(x) = 0 or μAi(y) = 0, for all x, y≠ e in F and for all i.

(ii) By the definition $\nu Ai(x-y) \leq S(\nu Ai(x), \nu Ai(y))$, which implies that $1 \leq S(\nu Ai(x), \nu Ai(y))$.
Therefore, either $\nu Ai(x) = 1$or $\nu Ai(y) = 1$, for all x, y in F and for all I, and by the definition $\nu Ai(xy-1) \leq S(\nu Ai(x), \nu A(iy))$, which implies that $1 \leq S(\nu Ai(x), \nu Ai(y))$.
Therefore, either $\nu Ai(x) = 1$or $\nu Ai(y) = 1$, for all x, $y \neq e$ in F and for all i.

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