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NON-ARCHIMEDEAN INTUITIONISTIC GENERALIZED FUZZY METRIC SPACES BY INTUITIONISTIC FUZZY $\psi-\Phi$ CONTRACTIVE MAPPINGS

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Abstract: We prove an intuitionistic generalized fuzzy Banach contraction theorem forM-complete non-Archimedean intuitionistic generalized fuzzy metric spaces. Also intuitionistic generalized fuzzy Elelstein contraction theorem for non-Archimedean intuitionistic generalizedfuzzy metric spaces by intuitionistic fuzzy ψ-ϕ contractive mappings are proved.

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1. Introduction

Theory of intuitionistic fuzzy set as a generalization of fuzzy set [8] was introduced by Atansov [7]. George and Veeramani [1] have pointed out that the definition of Cauchy sequence for fuzzy metric spaces given by Grabiec [9] is weaker and they gave one stronger definition of Cauchy sequence and termed as M-Cauchy sequence. The definition of Cauchy sequence given by Grabiec [9] has been termed as G-Cauchy sequence. With the help of fuzzy ψ-contractive mappings defined by Dorel Mihet[5], we define intuitionistic fuzzy ψ-ϕ contractive mappings. Our definition of intuitionistic fuzzy ψ-ϕ contractive mapping is more general than the definitions of intuitionistic generalized fuzzy contractive mapping given by Abdul Mohamad [2] and by this contraction we prove an intuitionistic fuzzy Banach contraction theorem for M-complete non-Archimedean intuitionistic generalized fuzzy metric spaces. We also prove intuitionistic generalized fuzzy Elelstein contraction theorem for non-Archimedean intuitionistic generalized fuzzy metric spaces by intuitionistic fuzzy ψ-ϕ contractive mappings.

2. Preliminaries

Definition: 2.1.

A binary operation$*$: $[0,1]\times[0,1]\rightarrow[0,1]$ is continuous t - norm if $*$ satisfies the following conditions:

(i) $*$is commutative and associative,

(ii) $*$is continuous,

(iii) $a*1 = a$, for all $a \in [0,1]$,

(iv) $a*b \le c*d$ whenever $a \le c$, $b \le d$ and a, b, c, d $\in[0, 1]$.

A few examples of continuous t-norm are $a*b = ab$, $a*b = \min\{a, b\}$, $a*b = \max\{a + b - 1, 0\}$.

Definition: 2.2.

A binary operation $\Diamond$: $[0,1]\times[0,1]\rightarrow[0,1]$ is continuous t-conorm if $\Diamond$ satisfies the following conditions:

(i) $\Diamond$ is commutative and associative,

(ii) $\Diamond$ is continuous,

(iii) $a\Diamond 0 = a$,for all $a \in[0,1]$,

(iv) $a\Diamond b \le c\Diamond d$ whenever $a \le c$, $b \le d$ and a, b, c, d $\in[0, 1]$.

A few examples of continuous t-conorm are $a\Diamond b= a + b - ab$, $a\Diamond b=\max\{a, b\}$, $a\Diamond b = \min\{a + b, 1\}$.

Definition: 2.3.

A 5-tuple $(X, \mu, \nu, *,\Diamond)$ is said to be an intuitionistic generalized fuzzy metric space if X is an arbitrary set,$*$is a continuous t-norm,$\Diamond$ is a continuous t-conorm, μ and ν are fuzzy sets on $X^3\times(0,\infty)$ and μ denotes the degree of nearness, ν denotes the degree of non-nearness between x and y relative to t satisfying the following conditions: for all $x, y, z \in X$, $s, t > 0$,

(i) $\mu(x, y, z, t) + \nu(x, y ,z, t) \le 1$;

(ii) $\mu(x, y, z,0) = 0$;

(iii) $\mu(x, y, z, t) = 1$ if and only if $x = y =z$;

(iv) $\mu(x, y, z, t) = \mu(p\{x, y, z\}, t)$, when p is the permutation function;

(v) $\mu(x, y, z, t + s) \ge \mu(x, y, a, t) *\mu(a, z, z, s)$;

(vi) $\mu(x, y, z, \cdot) : [0,\infty) \rightarrow [0, 1]$ is left-continuous;

(vii) $\nu(x, y, z, 0) = 1$;

(viii) $\nu(x, y, z, t) = 0$ if and only if $x = y = z$;

(ix) $\nu(x, y, z, t) = \nu(p\{x, y, z\}, t)$, when p is the permutation function;

(x) $\nu(x, y, z, t + s) \le \nu(x, y, a, t) \Diamond\nu(a, z, z, s)$;

(xi) $\nu(x, y, z, .) : [0,\infty) \rightarrow [0, 1]$ is right-continuous.

Remark: 2.4.

If in the above definition the triangular inequalities (v) and (x) are replaced by

$\mu(x, y, z , \max\{t, s\}) \ge \mu(x, y, a, t) *\nu(a, z, z, s)$ and

$\nu(x, y, z , \max\{t, s\}) \le \nu(x, y, a, t) \Diamond\nu(a, z, z, s)$. Or, equivalently,

$\mu(x, y, z, t) \geq \mu(x, y, a, t) * \mu(a, z, z, t)$ and
$\nu(x, y, z, t) \leq \nu(x, y, a, t) \Diamond \nu(a, z, z, t)$.
Then $(X, \mu, \nu, *, \Diamond)$ is called non-Archimedean intuitionistic generalized fuzzy metric space.

Definition: 2.5.

Let $(X, \mu, \nu, *, \Diamond)$ be an intuitionistic generalized fuzzy metric space. A mapping $f : X \to X$ is intuitionistic fuzzy contractive if there exists $k \in (0, 1)$ such that
$\frac{1}{\mu(f(x),f(y),f(z),t)} - 1 \leq k\left(\frac{1}{\mu(x,y,z,t)} - 1\right)$ and $\frac{1}{\nu(f(x),f(y),f(z),t)} - 1 \geq 1/k\left(\frac{1}{\nu(x,y,z,t)} - 1\right)$
for all $x, y \in X$ and $t > 0$. (k is called contractive constant of f.

Definition: 2.6. [2]

Let $(X, \mu, \nu, *, \Diamond)$ be an intuitionistic generalized fuzzy metric space. We will say that the sequence $\{x_n\}_n$ in X is intuitionistic fuzzy contractive if there exists $k \in (0, 1)$ such that
$\frac{1}{\mu(x_{n+1},x_{n+2},x_{n+3},t)} - 1 \leq k\left(\frac{1}{\mu(x_n,x_{n+1},x_{n+2},t)} - 1\right)$ and
$\frac{1}{\nu(x_{n+1},x_{n+2},x_{n+3},t)} - 1 \geq 1/k\left(\frac{1}{\nu(x_n,x_{n+1},x_{n+2},t)} - 1\right)$ for all $t > 0$ and $n \in N$.

3. Intuitionistic Generalized Fuzzy ψ– Φ Contractive Mappings

Definition: 3.1.

Let $(X, \mu, \nu, *, \Diamond)$ be an intuitionistic generalized fuzzy metric space.

(i) A sequence $\{x_n\}_n$ in X is called M-Cauchy sequence, if for each $\epsilon \in (0, 1)$ and $t > 0$ there existsn$_0 \in N$ such that $\mu(x_n, x_m, x_m, t) > 1-\epsilon$ and $\nu(x_n, x_m, x_m, t) < \epsilon$ for all $m, n \geq n_0$.

(ii) A sequence $\{x_n\}_n$ in X is called G-Cauchy sequence if $\lim_{n\to\infty} \mu(x_n, x_{n+m}, x_{n+m}, t) = 1$ and $\lim_{n\to\infty} \nu(x_n, x_{n+m}, x_{n+m}, t) = 0$ for each $m \in N$ and $t > 0$.

Definition: 3.2.

A sequence $\{x_n\}_n$ in an intuitionistic generalized fuzzy metric space $(X, \mu, \nu, *, \Diamond)$ is said to converge to $x \in X$ if $\lim_{n\to\infty} \mu(x_n, x, x, t) = 1$ and $\lim_{n\to\infty} \nu(x_n, x, x, t) = 0$ for all $t > 0$.

Definition: 3.3.

Let Ψ be the class of all mappings $\psi: [0, 1] \to [0, 1]$ such that ψ is continuous, non-decreasing and $\psi(t) < t$, for all $t \in (0, 1)$. Let Ψ be the class of all mappings $\phi: [0, 1] \to [0, 1]$ such that ϕ is continuous, non-decreasing and $\phi(t) > t$, for all $t \in (0, 1)$. Let $(X, \mu, \nu, *, \Diamond)$ be an intuitionistic generalized fuzzy metric space and $\psi \in \Psi$ and $\phi \in \Phi$. A mapping $f: X \to X$ is called an intuitionistic generalized fuzzy ψ-ϕ-contractive mapping if the following implications hold:

$\mu(x, y, z, t) > 0 \Rightarrow \psi(\mu(f(x), f(y), f(z), t)) \geq \mu(x, y, z, t)$
$\nu(x, y, z, t) < 1 \Rightarrow \phi(\nu(f(x), f(y), f(z), t)) \leq \nu(x, y, z, t)$.

Example:3.4.

Let $(X, \mu, \nu, *, \Diamond)$ be an intuitionistic generalized fuzzy metric space and $f:X \to X$ satisfies

$$\frac{1}{\mu(f(x),f(y),f(z),t)} - 1 \leq k\left(\frac{1}{\mu(x,y,z,t)} - 1\right) \text{ and } \frac{1}{\nu(f(x),f(y),f(z),t)} - 1 \geq 1/k\left(\frac{1}{\nu(x,y,z,t)} - 1\right)$$

for all $x, y, z \in X$ and $t > 0$.Then for some $k \in (0, 1)$ with $k > 1 - t$, f is an intuitionistic fuzzy ψ-ϕ-contractive mapping, with $\psi(t) = 1 - k$, $\phi(t) = t/(1 - k)t + k$.

Example: 3.5.

Let X be a non-empty set with at least two elements. If we define the intuitionistic generalized fuzzy set $(X, \mu, \nu, *, \Diamond)$ by $\mu(x, x, x, t) = 1$ and $\nu(x, x, x, t) = 0$ for all $x \in X$ and $t > 0$ and
$\mu(x, y, z, t) = \begin{cases} 0, & if\ t \leq 1 \\ 1, & if\ t > 1 \end{cases}$, $\nu(x, y, z, t) = \begin{cases} 1, & if\ t \leq 1 \\ 0, & if\ t > 1 \end{cases}$

for all x, y $\in$X, $x \neq y \neq z$, then (X, μ, ν, *, ◊) is an M-complete non-Archimedean intuitionistic generalized fuzzy metric space under any continuous t-norm * and continuous t-conorm ◊.

Now, let $\mu(x, y, z, t) < 1$

$\Rightarrow \mu(x, y, z, t) = 0$

$\Rightarrow \psi\ (\mu(f(x), f(y), f(z), t)) \geq \mu(x, y, z, t) = 0$ and

$\nu(x, y, z, t) > 0$

$\Rightarrow \nu(x, y, z, t) = 1$

$\Rightarrow \phi\ (\nu(f(x), f(y), f(z), t)) \leq \nu(x, y, z, t) = 1$

Therefore every mapping f: (X, μ, ν, *, ◊) → (X, μ, ν, *, ◊) is an intuitionistic generalized fuzzy ψ-ϕ-contractive mapping.

Definition: 3.6.

An intuitionistic fuzzy ψ-ϕ-contractive sequence in an intuitionistic generalized fuzzy metric space (X, μ, ν, *, ◊) is any sequence $\{x_n\}_n$ in X such that $\psi\ (\mu\ (x_{n+1}, x_{n+2}, x_{n+3}, t)) \geq \mu(x_{n+2}, x_{n+1}, x_n, t)$ and $\phi\ (\nu\ (x_{n+1}, x_{n+2}, x_{n+3}, t)) \leq \nu(x_{n+2}, x_{n+1}, x_n, t)$. An intuitionistic generalized fuzzy metric space (X, μ, ν, *, ◊) is called M-complete (or, G-complete) if every M-Cauchy (or, G-Cauchy) sequence is convergent in X.

4. Main Results

Theorem: 4.1.

Let (X, μ, ν, *, ◊) be an M-complete non-Archimedean intuitionistic generalized fuzzy metric space and f: X → X be an intuitionistic fuzzy ψ – ϕ - contractive mapping. If there exists x $\in$X such that $\mu\ (x, f(x), f(x), t) > 0$ and $\nu\ (x, f(x), f(x), t) < 1$ for all $t > 0$, then f has a unique fixed point.

Proof: Let x $\in$X be such that $\mu(x, f(x), f(x), t) > 0$ and $\nu(x, f(x), f(x), t) < 1$, $t > 0$ and $x_n = f^n(x)$, n$\in\mathbb{N}$, we have for all $t > 0$,

$\mu(x_0, x_1, x_2, t) \leq \psi(\mu(x_1, x_2, x_3, t)) < \mu(x_1, x_2, x_3, t)$

$\nu(x_0, x_1, x_2, t) \geq \phi(\nu(x_1, x_2, x_3, t)) > \nu(x_1, x_2, x_3, t)$ and

$\mu(x_1, x_2, x_3, t) \leq \psi(\mu(x_2, x_3, x_4, t)) < \mu(x_2, x_3, x_4, t)$

$\nu(x_1, x_2, x_3, t) \geq \phi(\nu(x_2, x_3, x_4, t)) > \nu(x_2, x_3, x_4, t)$

Hence by induction for all $t > 0$, $\mu(x_n, x_{n+1}, x_{n+2}, t) < \mu(x_{n+1}, x_{n+2}, x_{n+3}, t)$ and $\nu(x_n, x_{n+1}, x_{n+2}, t) > \nu(x_{n+1}, x_{n+2}, x_{n+3}, t)$.

Therefore, for every $t > 0$, $\{\mu(x_n, x_{n+1}, x_{n+2}, t)\}$ is a non-decreasing sequence of numbers in (0, 1] and $\{\nu(x_n, x_{n+1}, x_{n+2}, t)\}$ is a non-increasing sequence of numbers in [0, 1).

Fix $t > 0$. Denote $\lim_{n\to\infty} \mu(x_n, x_{n+1}, x_{n+2}, t)$ by l and $\lim_{n\to\infty} \nu(x_n, x_{n+1}, x_{n+2}, t)$ by m.

Then we have $l \in [0, 1]$ and $m \in [0, 1]$. Since $\psi\ (\mu\ (x_{n+1}, x_{n+2}, x_{n+3}, t)) \geq \mu(x_n, x_{n+1}, x_{n+2}, t)$ and ψ is continuous, $\psi(l) \geq l$. This implies $l = 1$. Also, since $\phi\ (\nu\ (x_{n+1}, x_{n+2}, x_{n+3}, t)) \leq \nu(x_n, x_{n+1}, x_{n+2}, t)$ and ϕ is continuous, $\phi(m) \leq m$. This implies $m = 0$. Therefore, $\lim_{n\to\infty} \mu(x_n, x_{n+1}, x_{n+2}, t) = 1$ and $\lim_{n\to\infty} \nu(x_n, x_{n+1}, x_{n+2}, t) = 0$. If $\{x_n\}_n$ is not a M-Cauchy sequence then there are $\epsilon \in (0, 1)$ and $t > 0$ such that for each k $\in$N there exist m(k), n(k) $\in$N with $m(k) > n(k) \geq k$ and $\mu(x_{m(k)}, x_{n(k)}, x_{n(k)}, t) \leq 1 - \epsilon$ and $\nu(x_{m(k)}, x_{n(k)}, x_{n(k)}, t) \geq \epsilon$. Let for each k, m(k) be the least positive integer exceeding n(k) satisfying the above property, that is, $\mu(x_{m(k)-1}, x_{n(k)}, x_{n(k)}, t) \geq 1 - \epsilon$ and $\mu(x_{m(k)}, x_{n(k)}, x_{n(k)}, t) \leq 1 - \epsilon$. Also, $\nu(x_{m(k)-1}, x_{n(k)}, x_{n(k)}, t) \leq \epsilon$ and $\nu(x_{m(k)}, x_{n(k)}, x_{n(k)}, t) \geq \epsilon$. Then for each positive integer k, $1 - \epsilon \geq \mu(x_{m(k)}, x_{n(k)}, x_{n(k)}, t) \geq \mu(x_{m(k)-1}, x_{n(k)}, x_{n(k)}, t) * \mu(x_{m(k)-1}, x_{m(k)}, x_{m(k)}, t) \geq (1 - \epsilon) * \mu(x_{m(k)-1}, x_{m(k)}, x_{m(k)}, t)$ and $\epsilon \leq \nu(x_{m(k)}, x_{n(k)}, x_{n(k)}, t) \leq \nu(x_{m(k)-1}, x_{n(k)}, x_{n(k)}, t) \Diamond \nu(x_{m(k)-1}, x_{m(k)}, x_{m(k)}, t) \leq \epsilon \Diamond \nu(x_{m(k)-1}, x_{m(k)}, x_{m(k)}, t)$.

Taking limit as $k \to \infty$ we have:

$\lim_{k\to\infty} \{(1 - \epsilon) * \mu(x_{m(k)-1}, x_{m(k)}, x_{m(k)}, t)\} = (1 - \epsilon)$, $\lim_{k\to\infty} \mu(x_{m(k)-1}, x_{m(k)}, x_{m(k)}, t) = (1 - \epsilon)$, $1 = (1 - \epsilon)$ and

$\lim_{k\to\infty} \{\epsilon \Diamond \nu(x_{m(k)-1}, x_{m(k)}, x_{m(k)}, t)\} = \epsilon \Diamond \lim_{k\to\infty} \nu(x_{m(k)-1}, x_{m(k)}, x_{m(k)}, t) = \epsilon \Diamond 0 = \epsilon$.

It follows that $\lim_{k\to\infty} \mu(x_{m(k)}, x_{n(k)}, x_{n(k)}, t) = 1 - \epsilon$ and $\lim_{k\to\infty} \nu(x_{m(k)}, x_{n(k)}, x_{n(k)}, t) = \epsilon$.

Now, $\mu(x_{m(k)}, x_{n(k)}, x_{n(k)}, t) \le \psi(\mu(x_{m(k)+1}, x_{n(k)+1}, x_{n(k)+1}, t))$ and $\nu(x_{m(k)}, x_{n(k)}, x_{n(k)}, t) \ge \phi(\nu(x_{m(k)+1}, x_{n(k)+1}, x_{n(k)+1}, t))$.
Since ψ and ϕ are continuous taking limit as $k \to \infty$ we have:

$$1 - \epsilon \le \psi(1 - \epsilon) < 1 - \epsilon \text{ and } \epsilon \ge \phi(\epsilon) > \epsilon,$$

which are contradictions. Thus $\{x_n\}_n$ is a M- Cauchy sequence.
If $\lim_{n\to\infty} x_n = y$ then from $\psi(\mu(f(y), f(x_n), f(x_n), t)) \ge \mu(y, x_n, x_n, t)$ and $\phi(\nu(f(y), f(x_n), f(x_n), t)) \le \nu(y, x_n, x_n, t)$ it follows that $x_{n+1} \to f(y)$. Therefore, we have:

$$\mu(y, f(y), f(y), t) \ge \mu(y, x_n, x_n, t) * \mu(x_n, x_{n+1}, x_{n+1}, t) * \mu(x_{n+1}, f(y), f(y), t) \to 1 \text{ as } n \to \infty.$$

This implies $\mu(y, f(y), f(y), t) = 1$.

$$\nu(y, f(y), f(y), t) \le \nu(y, x_n, x_n, t) \lozenge \nu(x_n, x_{n+1}, x_{n+1}, t) \lozenge \nu(x_{n+1}, f(y), f(y), t) \to 0 \text{ as } n \to \infty.$$

This implies $\nu(y, f(y), f(y), t) = 0$. Hence, $f(y) = y$.
If x, y, zare fixed points of f then

$$\mu(f(x), f(y), f(z), t) = \mu(x, y, z, t) \le \psi(\mu(f(x), f(y), f(z), t)) \text{and}$$
$$\nu(f(x), f(y), f(z), t) = \nu(x, y, z, t) \ge \phi(\nu(f(x), f(y), f(z), t)), \text{ for all } t > 0.$$

If $x \ne y \ne z$ then $\mu(x, y, z, s) < 1$ and $\nu(x, y, z, s) > 0$ for some $s > 0$ i.e., $0 < \mu(x, y, z, s) < 1$and $0 < \nu(x, y, z, s) < 1$ hold, implying $\mu(f(x), f(y), f(z), s) \le \psi(\mu(f(x), f(y), f(z), s)) < \mu(f(x), f(y), f(z), s)$ and $\nu(f(x), f(y), f(z), s) \ge \phi(\nu(f(x), f(y), f(z), s)) > \nu(f(x), f(y), f(z), s)$, which are contradictions.
Thus $x = y = z$. This completes the proof.

Lemma: 4.2.

Let $(X, \mu, \nu, *, \lozenge)$ be a non-Archimedean intuitionistic generalized fuzzy metric space. If $\{x_n\}_n$, $\{y_n\}_n$ and $\{z_n\}_n$ be two sequences in X converges to x, y and z respectively then
$\lim_{n\to\infty} \mu(x_n, y_n, z_n, t) = \mu(x, y, z, t)$ and $\lim_{n\to\infty} \nu(x_n, y_n, z_n, t) = \nu(x, y, z, t)$.

Proof: Since $(X, \mu, \nu, *, \lozenge)$ be a non-Archimedean intuitionistic generalized fuzzy metric space, therefore $\mu(x_n, y_n, z_n, t) \ge \mu(x_n, x, y, t) * \mu(x, y, z, t) * \mu(z, y_n, z_n, t)$

$$\Rightarrow \lim_{n\to\infty} \mu(x_n, y_n, z_n, t) \ge 1 * \mu(x, y, z, t) * 1 = \mu(x, y, z, t) \text{ and}$$
$$\nu(x_n, y_n, z_n, t) \le \nu(x_n, x, y, t) \lozenge \nu(x, y, z, t) \lozenge \nu(z, y_n, z_n, t)$$
$$\Rightarrow \lim_{n\to\infty} \nu(x_n, y_n, z_n, t) \le 0 \lozenge \nu(x, y, z, t) \lozenge 0 = \nu(x, y, z, t).$$

Also, $\mu(x, y, z, t) \ge \mu(x, y, x_n, t) * \mu(x_n, y_n, z_n, t) * \mu(z, y_n, z_n, t)$

$$\Rightarrow \mu(x, y, z, t) \ge 1 * \lim_{n\to\infty} \mu(x_n, y_n, z_n, t) * 1 = \lim_{n\to\infty} \mu(x_n, y_n, z_n, t) \text{and}$$
$$\nu(x, y, z, t) \ge \nu(x, y, x_n, t) \lozenge \nu(x_n, y_n, z_n, t) \lozenge \nu(z, y_n, z_n, t)$$
$$\Rightarrow \nu(x, y, z, t) \ge 0 \lozenge \lim_{n\to\infty} \nu(x_n, y_n, z_n, t) \lozenge 0 = \lim_{n\to\infty} \nu(x_n, y_n, z_n, t).$$

Hence the proof.

We prove the following theorem without the continuity condition.

Theorem: 4.3.

Let $(X, \mu, \nu, *, \lozenge)$ be a compact non-Archimedean intuitionistic generalized fuzzy metric space. Let $f: X \to X$ be an intuitionistic fuzzy ψ-ϕ-contractive mapping. Then f has a unique fixed point.
Proof: Let $x \in X$ and $x_n = f^n(x)$, $n \in N$. Assume $x_n \ne x_{n+1}$ for each n (if not $f(x_n) = x_n$).
Now assume $x_n \ne x_m$ $(n \ne m)$, otherwise for $m < n$ we get

$$\mu(x_n, x_{n+1}, x_{n+2}, t) = \mu(x_m, x_{m+1}, x_{m+2}, t) \le \psi(\mu(x_{m+1}, x_{m+2}, x_{m+3}, t))$$
$$< \mu(x_{m+1}, x_{m+2}, x_{m+3}, t) < \cdot\cdot\cdot < \mu(x_n, x_{n+1}, x_{n+2}, t) \text{ and}$$
$$\nu(x_n, x_{n+1}, x_{n+2}, t) = \nu(x_m, x_{m+1}, x_{m+2}, t) \ge \phi(\nu(x_{m+1}, x_{m+2}, x_{m+3}, t))$$
$$> \nu(x_{m+1}, x_{m+2}, x_{m+3}, t) > \cdot\cdot\cdot > \nu(x_n, x_{n+1}, x_{n+2}, t), \text{ a contradiction.}$$

Since X is compact, $\{x_n\}_n$ in X has a convergent subsequence $\{x_{n_i}\}_{i\in\mathbb{N}}$ (say). Let $\{x_{n_i}\}_{i\in\mathbb{N}}$ converges to y. We also assume that y, $f(y) \notin \{x_n: n \in \mathbb{N}\}$ (if not, choose a subsequence with such a property). According to the above assumptions we may now write for all $i \in \mathbb{N}$ and $t > 0$

$$\mu(x_{n_i}, y, y, t) \le \psi(\mu(f(x_{n_i}), f(y), f(y), t)) < \mu(f(x_{n_i}), f(y), f(y), t)$$
$$\nu(x_{n_i}, y, y, t) \ge \phi(\nu(f(x_{n_i}), f(y), f(y), t)) > \nu(f(x_{n_i}), f(y), f(y), t)$$

Since ψ and ϕ are continuous for all x, y, z∈X. From lemma (4.2) we obtain

$$\lim_{i\to\infty}\mu(x_{n_i}, y, y, t) \le \lim_{i\to\infty}\mu(f(x_{n_i}), f(y), f(y), t)$$
$$\Rightarrow 1 \le \lim_{i\to\infty}\mu(f(x_{n_i}), f(y), f(y), t)$$
$$\Rightarrow \lim_{i\to\infty}\mu(f(x_{n_i}), f(y), f(y), t) = 1 \text{ and}$$
$$\lim_{i\to\infty}\nu(x_{n_i}, y, y, t) \ge \lim_{i\to\infty}\nu(f(x_{n_i}), f(y), f(y), t)$$
$$\Rightarrow 0 \ge \lim_{i\to\infty}\nu(f(x_{n_i}), f(y), f(y), t)$$
$$\Rightarrow \lim_{i\to\infty}\nu(f(x_{n_i}), f(y), f(y), t) = 0.$$

i.e., $$f(x_{n_i}) \to f(y) \tag{4.1}$$

Similarly, we obtain

$$f^2(x_{n_i}) \to f^2(y) \tag{4.2}$$

Now, we see that

$\mu(x_{n_1}, f(x_{n_1}), f(x_{n_1}), t) \le \psi(\mu(f(x_{n_1}), f^2(x_{n_1}), f^2(x_{n_1}), t)) < \mu(f(x_{n_1}), f^2(x_{n_1}), f^2(x_{n_1}), t) < \cdots <$
$\mu(x_{n_i}, f(x_{n_i}), f(x_{n_i}), t) < \mu(f(x_{n_i}), f^2(x_{n_i}), f^2(x_{n_i}), t) < \cdots < 1$ and

$\nu(x_{n_1}, f(x_{n_1}), f(x_{n_1}), t) \ge \phi(\nu(f(x_{n_1}), f^2(x_{n_1}), f^2(x_{n_1}), t)) > \nu(f(x_{n_i}), f2(x_{n_i}), t) > \cdots >$
$\nu(x_{n_i}, f(x_{n_i}), f(x_{n_i}), t) > \nu(f(x_{n_i}), f^2(x_{n_i}), f^2(x_{n_i}), t) > \cdots > 0.$

Thus $\{\mu(x_{n_i}, f(x_{n_i}), f(x_{n_i}), t)\}_{i\in\mathbb{N}}$ and $\{\mu(f(x_{n_i}), f^2(x_{n_i}), f^2(x_{n_i}), t)\}_{i\in\mathbb{N}}$ converges to a common limit..

Also, $\{\nu(x_{n_i}, f(x_{n_i}), f(x_{n_i}), t)\}_{i\in\mathbb{N}}$ and $\{\nu(f(x_{n_i}), f^2(x_{n_i}), f^2(x_{n_i}), t)\}_{i\in\mathbb{N}}$ converges to a common limit.

So, by (4.1), (4.2) and lemma (4.2) we get

$$\mu(y, f(y), f(y), t) = \mu(\lim_{i\to\infty}x_{n_i}, f(\lim_{i\to\infty}x_{n_i}), f(\lim_{i\to\infty}x_{n_i}), t)$$
$$= \lim_{i\to\infty}\mu(x_{n_i}, f(x_{n_i}), f(x_{n_i}), t)$$
$$= \lim_{i\to\infty}\mu(f(x_{n_i}), f^2(x_{n_i}), f^2(x_{n_i}), t)$$
$$= \mu(f(\lim_{i\to\infty}x_{n_i}), f^2(\lim_{i\to\infty}x_{n_i}), f^2(\lim_{i\to\infty}x_{n_i}), t)$$
$$= \mu(f(y), f^2(y), f^2(y), t) \text{ and}$$
$$\nu(y, f(y), f(y), t) = \nu(\lim_{i\to\infty}x_{n_i}, f(\lim_{i\to\infty}x_{n_i}), f(\lim_{i\to\infty}x_{n_i}), t)$$
$$= \lim_{i\to\infty}\nu(x_{n_i}, f(x_{n_i}), f(x_{n_i}), t)$$
$$= \lim_{i\to\infty}\nu(f(x_{n_i}), f^2(x_{n_i}), f^2(x_{n_i}), t)$$
$$= \nu(f(\lim_{i\to\infty}x_{n_i}), f^2(\lim_{i\to\infty}x_{n_i}), f^2(\lim_{i\to\infty}x_{n_i}), t)$$
$$= \nu(f(y), f^2(y), f^2(y), t) \text{ for all } t > 0.$$

Suppose $f(y) \ne y$, then we have $\mu(y, f(y), f(y), t) \le \psi(\mu(f(y), f^2(y), f^2(y), t)) < \mu(f(y), f^2(y), f^2(y), t)$
And $\nu(y, f(y), f(y), t) \ge \phi(\nu(f(y), f^2(y), f^2(y), t)) > \nu(f(y), f^2(y), f^2(y), t)$, $t > 0$, a contradiction.
Hence $y = f(y)$ is a fixed point.
If x, y, zare fixed points of f then $\mu(f(x), f(y), f(z), t) = \mu(x, y, z, t) \le \psi(\mu(f(x), f(y), f(z), t))$ and
$\nu(f(x), f(y), f(z), t) = \nu(x, y, z, t) \ge \phi(\nu(f(x), f(y), f(z), t))$, $\forall\, t > 0$.
If $x \ne y \ne z$ then $\mu(x, y, z, s) < 1$ and $\nu(x, y, z, s) > 0$ for some $s > 0$ i.e., $0 < \mu(x, y, z, s) < 1$ and $0 < \nu(x, y, z, s) < 1$ hold, implying $\mu(f(x), f(y), f(z), s) \le \psi(\mu(f(x), f(y), f(z), s)) < \mu(f(x), f(y), f(z), s)$ and $\nu(f(x), f(y), f(z), s) \ge \phi(\nu(f(x), f(y), f(z), s)) > \nu(f(x), f(y), f(z), s)$, which are contradictions.
Thus $x = y = z$. This completes the proof.

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