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BIPOLAR VALUED MULTI FUZZY TRANSLATION IN SUBFIELD OF A FIELD

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Abstract: In this paper the conception of bipolar valued multi fuzzy translations are discussed and using this translation some properties in bipolar valued multi fuzzy subfield of a field are investigated. We also converse about bipolar valued multi fuzzy subfield of a field under homomorphisms with translation concepts.

Keywords: bipolar valued multi fuzzy subset, bipolar valued multi fuzzy subfield, bipolar valued multi fuzzy translation.

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1. Introduction

Continuously evolving science and technology has facing more complicated problems in handling varies types of uncertainty in the field of science, engineering, economics, medicine, environmental science and social sciences. Solving these uncertainties by traditional methods will be very complicated and unsolvable for most of the problems. We have considered this situation and solving the problems using some mathematical modelling like fuzzy set theory [13], interval mathematics [3], rough set theory [5], operative tools for handling with vagueness and uncertainty and probability theory are well-known, each of them has its own inherent limitations; one major fault shared by these mathematical methodologies may be due to the inadequacy of parametrization tools.

Fuzzy set is dealing with objects whose boundary is vague. Fuzzy set theory has several types of fuzzy set extensions (Ex. interval fuzzy sets, intuitionistic fuzzy sets, vague sets etc.), bipolar fuzzy set is another an extension of fuzzy set whose membership degree range is different from the above extensions. Lee and Zhang [4,14,15] imitated an extension of fuzzy set named bipolar valued fuzzy sets. bipolar- valued fuzzy sets has two kinds of representations of the notion. In case of Bipolar Valued fuzzy sets membership degree range is enlarged from the interval [0,1] to [-1,0]. Sabu Sebastian and T.V.Ramakrishnan [6,7] defined the multi-fuzzy sets. V.K.Shanthi and G.Shyamala [8],given the Notes on Bipolar-valued multi fuzzy subgroups of a group. C.Yamini et.al.[9,10] introduced the concept of Bipolar valued multi fuzzy subfield of a field and Notes on Bipolar valued multi fuzzy subfield of a field under homomorphism.

Anitha.M.S et al. [1] defined a homomorphism and antihomomorphism of bipolar-valued fuzzy subgroups of a group. B.Yasodara and K.E.Sathappan [11, 12] defined the bipolar valued multi fuzzy subsemirings of a semiring under homomorpisms. In2009 Jun et al.,[2] discussed about bipolar fuzzy translation in BCK/BCI-algebras.In this paper we study the notion of bipolar valued multi fuzzy translations with bipolar valued multi fuzzy subfield of a field and homomorphisms

2. Preliminaries

Definition 2.1 [11]

A bipolar valued fuzzy set (BVFS) A in X is defined as an object having the form $A = \{ < x, A^+(x), A^-(x)>/ x\in X\}$, where $A^+: X\to [0, 1]$ and $A^-: X\to [-1, 0]$. The positive membership degree $A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar-valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A.

Example 2.2: A = { < a, 0.9, –0.6>, < b, 0.8, –0.7 >, < c, 0.7, –0.5>} is a bipolar valued fuzzy subset of X= {a, b, c}.

Definition 2.3 [11]

A bipolar valued multi fuzzy set (BVMFS) A in X of order n is defined as an object of the form $A = \{ < x, A_1^+(x), A_2^+(x), \ldots, A_n^+(x), A_1^-(x), A_2^-(x), \ldots, A_n^-(x) >/ x\in X\}$, where $A_i^+: X\to [0, 1]$ and $\mathbf{A_i}^-: X\to [-1, 0]$, i = 1,2, 3, … n. The positive membership degrees $A_i^+(x)$ denote the satisfaction degree of an element x to the property corresponding to a bipolar valued multi fuzzy set A and the negative membership degrees $\mathbf{A_i}^-(x)$ denote the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar-valued multi fuzzy set A.

Note: In this paper, the bipolar valued multi fuzzy subfield of a field A means $A = < A^+, A^-> = <A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^->$.

Example 2.4: A = { < a, 0.5, 0,6, 0.3, –0.3, –0.6, –0.5>, < b, 0.1, 0.4, 0.7, –0.7, –0.3, –0.6>, < c, 0.5, 0.3, 0.8, –0.4, –0.5, –0.3 >} is a bipolar-valued multi fuzzy subset of order 3 in X = { a,b,c }.

Definition 2.5 *[9]*

Let F be a field. A bipolar valued multi fuzzy subset A of F is said to be a bipolar valued multi fuzzy subfield of F if the following conditions are satisfied, for all i,

(i) $A_i^+(x-y) \geq \min\{ A_i^+(x), A_i^+(y) \}$ for all x, y in F.
(ii) $A_i^-(x-y) \leq \max\{ A_i^-(x), A_i^-(y) \}$ for all x, y in F.
(iii) $A_i^+(xy^{-1}) \geq \min\{ A_i^+(x), A_i^+(y) \}$ for all x, y≠ 0 in F.
(iv) $A_i^-(xy^{-1}) \leq \max\{ A_i^-(x), A_i^-(y) \}$ for all x, y≠ 0 in F.

Example2.6: Let $F = Z_3 = \{ 0, 1, 2 \}$ be a field with respect to the ordinary addition and multiplication. Then A = { < 0, 0.5, 0.8, 0.6, – 0.6, – 0.5,– 0.7 > , < 1, 0.4, 0.7, 0.5, – 0.5, – 0.4,– 0.6 > ,< 2, 0.4, 0.7, 0.5, – 0.5, – 0.4, – 0.6 >} is a bipolar valued multi fuzzy subfield of order 3 in F.

***Definition 2.7** [1]*

Let F and $\mathbf{F'}$ be any two subfields. Then the function g : F→ $\mathbf{F'}$ is said to be an antihomomorphism if g(x+y) = g(y) + g(x) and g(xy) = g(y)g(x) for all x and y in F.

Definition 2.8

Let A = $\langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^-\rangle$ be a bipolar valued multi fuzzy subset of X and α_i in [0, 1– sup { $A_i^+(x)$}:x ϵ X], β_i in [–1– inf { $A_i^-(x)$}, 0 :x ϵ X]. Then:

$$\mathbb{T} = \langle \mathbb{T}_i^+, \mathbb{T}_i^-\rangle = \mathbb{T}^{\boldsymbol{A}}_{((\boldsymbol{\alpha_1},\boldsymbol{\alpha_2},\ldots,\boldsymbol{\alpha_n}),(\boldsymbol{\beta_1},\boldsymbol{\beta_2},\ldots,\boldsymbol{\beta_n}))}$$

is called a bipolar valued multi fuzzy translation of A if

$\mathbb{T}_i^+(x) = \mathbb{T}^{+\boldsymbol{A}}_{\boldsymbol{\alpha_i}}(x) = A_i^+(x) + \alpha_i \ \forall x \epsilon X$, $\mathbb{T}_i^-(x) = \mathbb{T}^{-\boldsymbol{A}}_{\boldsymbol{\beta_i}}(x) = A_i^-(x) + \beta_i \ \forall x \epsilon X$, for all i.

Example2.9: Consider the set X = { 0, 1, 2, 3, 4 }. Let A = { (0, 0.5, 0.6, 0.3, –0.1, –0.5, –0.7), (1, 0.4, 0.5, 0.8, –0.3, –0.5, –0.6), (2, 0.6, 0.4, 0.8, –0.05, –0.4, –0.5), (3, 0.45, 0.6, 0.9, –0.2, –0.4, –0.7), (4, 0.2, 0.4, 0.5, –0.5, –0.6, –0.7) } be a bipolar valued multi fuzzy subset of X and $\alpha_1 = 0.4$, $\alpha_2 = 0.3$, $\alpha_3 = 0.4$, $\beta_1 = –0.1$, $\beta_2 = –0.2$, $\beta_3 = –0.1$. Then the bipolar valued multi fuzzy translation of A is $\mathbb{T} = \mathbb{T}^{\mathbf{A}}$((0.1, 0.1, 0.1), (–0.3, –0.2, –0.1) = { (0, 0.6, 0.7, 0.4, –0.4, –0.7, –0.8), (1, 0.5, 0.6, 0.9, –0.6, –0.7, –0.7), (2, 0.7, 0.5, 0.9, –0.35, –0.6, –0.6), (3, 0.55, 0.7, 1.0, –0.5, –0.6, –0.8), (4, 0.3, 0.5, 0.6, –0.8, –0.8, –0.8) }.

3. Properties of bipolar valued multi fuzzy translations:

Theorem 3.1

Let A = $\langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^-\rangle$ be a bipolar valued multi fuzzy subset of X. Then bipolar valued multi fuzzy translation $\mathbb{T} = \langle \mathbb{T}_i^+, \mathbb{T}_i^-\rangle$ of A is a bipolar valued multi fuzzy subfield of a field of X if and only if A = $\langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^-\rangle$ is a bipolar valued multi fuzzy subfield of a field of X.

Proof: Let A = $\langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^-\rangle$ be a bipolar valued multi fuzzy subfield of a field of X and x,y ϵ X and for all i. Then:

$\mathbb{T}^{+A}_{\alpha_i}(x-y) = A_i^+(x-y) + \alpha_i \geq \min\{ A_i^+(x), A_i^+(y) \}) + \alpha_i = \min \{ A_i^+(x) + \alpha_i, A_i^+(y) + \alpha_i \}$

$= \min \{ \mathbb{T}^{+A}_{\alpha_i}(x), \mathbb{T}^{+A}_{\alpha_i}(y) \}$.

$\mathbb{T}^{-A}_{\beta_i}(x-y) = A_i^-(x-y) + \beta_i \leq \max\{ A_i^-(x), A_i^-(y) \} + \beta_i = \max\{ A_i^-(x) + \beta_i, A_i^-(y) + \beta_i$

$= \max \{ \mathbb{T}^{-A}_{\beta_i}(x), \mathbb{T}^{-A}_{\beta_i}(y) \}$ for all i.

Also for all x, y≠ 0 in X and for all i,

$\mathbb{T}^{+A}_{\alpha_i}(xy^{-1}) = A_i^+(xy^{-1}) + \alpha_i \geq \min\{ A_i^+(x), A_i^+(y) \}) + \alpha_i = \min\{ A_i^+(x) + \alpha_i, A_i^+(y) + \alpha_i \}$

$= \min\{ \mathbb{T}^{+A}_{\alpha_i}(x), \mathbb{T}^{+A}_{\alpha_i}(y) \}$.

Also, $\mathbb{T}^{-A}_{\beta_i}(xy^{-1}) = A_i^-(xy^{-1}) + \beta_i \leq \max\{ A_i^-(x), A_i^-(y) \} + \beta_i = \max\{A_i^-(x) + \beta_i, A_i^-(y) + \beta_i\}$

$= \max \{ \mathbb{T}^{-A}_{\beta_i}(x), \mathbb{T}^{-A}_{\beta_i}(y)\}$.

Hence $\mathbb{T} = \langle \mathbb{T}_i^+, \mathbb{T}_i^-\rangle$ is a bipolar valued multi fuzzy subfield of a field of X.

Conversely, Let the bipolar valued multi fuzzy translation $\mathbb{T} = \langle \mathbb{T}_i^+, \mathbb{T}_i^-\rangle$ is a bipolar valued multi fuzzy subfield of a field of X. Then for any x, y ϵ X and for all i, we have:

$A_i^+(x-y) + \alpha_i = \mathbb{T}^{+A}_{\alpha_i}(x-y) \geq \min\{ \mathbb{T}^{+A}_{\alpha_i}(x), \mathbb{T}^{+A}_{\alpha_i}(y) \} = \min\{ A_i^+(x) + \alpha_i, A_i^+(y) + \alpha_i \}$

$= \min\{ A_i^+(x), A_i^+(y) \}) + \alpha_I$,

and

$A_i^-(x-y) + \beta_i = \mathbb{T}^{-A}_{\beta_i}(x-y) \leq \max \{ \mathbb{T}^{-A}_{\beta_i}(x), \mathbb{T}^{-A}_{\beta_i}(y) \} = \max \{ A_i^-(x) + \beta_i, A_i^-(y) + \beta_i \}$

$= \max \{ A_i^-(x), A_i^-(y) \} + \beta_i$. Also for all x, y≠ 0 in X and for all i,

$A_i^+(xy^{-1}) + \alpha_i = \mathbb{T}_{\alpha_i}^{+A}(xy^{-1}) \geq \min\{\mathbb{T}_{\alpha_i}^{+A}(x), \mathbb{T}_{\alpha_i}^{+A}(y)\} = \min\{A_i^+(x) + \alpha_i, A_i^+(y) + \alpha_i\}$
$= \min\{A_i^+(x), A_i^+(y)\}) + \alpha_i$,

and we have:

$A_i^-(xy^{-1}) + \beta_i = \mathbb{T}_{\beta_i}^{-A}(xy^{-1}) \leq \max\{\mathbb{T}_{\beta_i}^{-A}(x), \mathbb{T}_{\beta_i}^{-A}(y)\} = \max\{A_i^-(x) + \beta_i, A_i^-(y) + \beta_i\}$
$= \max\{A_i^-(x), A_i^-(y)\} + \beta_i$.

Hence $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ is a bipolar valued multi fuzzy subfield of a field of X.

Theorem 3.2:

If $\mathcal{M}$ and $\mathcal{N}$ are two bipolar valued multi fuzzy translations of bipolar valued multi fuzzy subfield $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ of a field F, then their intersection $\mathcal{M} \cap \mathcal{N}$ is also a bipolar valued multi fuzzy translation of A**.**

Proof: Let x be in F and $(\alpha,\beta) = ((\alpha_1, \alpha_2, \ldots\alpha_n), (\beta_1, \beta_2, \ldots\beta_n))$, $(\gamma,\delta) = ((\gamma_1, \gamma_2, \ldots\gamma_n), (\delta_1, \delta_2, \ldots\delta_n))$. Let $\mathcal{M} = \mathbb{T}_{(\alpha,\beta)}^{A} = \{\langle x, A_1^+(x)+\alpha_1, A_2^+(x)+\alpha_2, \ldots, A_n^+(x)+\alpha_n, A_1^-(x)+\beta_1, A_2^-(x)+\beta_2, \ldots, A_n^-(x)+\beta_n \rangle / x \in R\}$ and $\mathcal{N} = \mathbb{T}_{(\gamma,\delta)}^{A} = \{\langle x, A_1^+(x)+\gamma_1, A_2^+(x)+\gamma_2, \ldots, A_n^+(x)+\gamma_n, A_1^-(x)+\delta_1, A_2^-(x)+\delta_2, \ldots, A_n^-(x) + \delta_n \rangle / x \in R\}$ be two bipolar valued multi fuzzy translations of bipolar valued multi fuzzy subfield A of F.

Let $C = \mathcal{M} \cap \mathcal{N}$ and $C = \{\langle x, C_1^+(x), C_2^+(x), \ldots, C_n^+(x), C_1^-(x), C_2^-(x), \ldots, C_n^-(x) \rangle / x \in F\}$, where $C_i^+(x) = \min\{A_i^+(x)+\alpha_i, A_i^+(x)+\gamma_i\}$ and $C_i^-(x) = \max\{A_i^-(x)+\beta_i, A_i^-(x)+\delta_i\}$ for all i.

Case (i): $\alpha \leq \gamma$ and $\beta \leq \delta$. Now:

$C_i^+(x) = \min\{\mathcal{M}_i^+(x), \mathcal{N}_i^+(x)\} = \min\{A_i^+(x) + \alpha_i, A_i^+(x) + \gamma_i\} = A_i^+(x) + \alpha_i = \mathcal{M}_i^+(x)$ for all x in F and for all i. And,

$C_i^-(x) = \max\{\mathcal{M}_i^-(x), \mathcal{N}_i^-(x)\} = \max\{A_i^-(x) + \beta_i, A_i^-(x) + \delta_i\} = A_i^-(x) + \delta_i = \mathcal{N}_i^-(x)$ for all x in F and for all i. Therefore $C = \mathbb{T}_{(\alpha,\delta)}^{A}$ is a bipolar valued multi fuzzy translation of bipolar valued multi fuzzy subfield A of F.

Case (ii): $\alpha \geq \gamma$ and $\beta \geq \delta$. Now:

$C_i^+(x) = \min\{\mathcal{M}_i^+(x), \mathcal{N}_i^+(x)\} = \min\{A_i^+(x) + \alpha_i, A_i^+(x) + \gamma_i\} = A_i^+(x) + \gamma_i = \mathcal{N}_i^+(x)$ for all x in F and for all i. And,

$C_i^-(x) = \max\{\mathcal{M}_i^-(x), \mathcal{N}_i^-(x)\} = \max\{A_i^-(x) + \beta_i, A_i^-(x) + \delta_i\} = A_i^-(x) + \beta_i = \mathcal{M}_i^-(x)$ for all x in F and for all i.

Therefore $C = \mathbb{T}_{(\gamma,\beta)}^{A}$ is a bipolar valued multi fuzzy translation of bipolar valued multi fuzzy subfield A of F.

Case (iii): $\alpha \leq \gamma$ and $\beta \geq \delta$. Clearly, $C = \mathbb{T}_{(\alpha,\beta)}^{A}$ is a bipolar valued multi fuzzy translation of bipolar valued multi fuzzy subfield A of F.

Case (iv): $\alpha \geq \gamma$ and $\beta \leq \delta$. Clearly, $C = \mathbb{T}_{(\gamma,\delta)}^{A}$ is a bipolar valued multi fuzzy translation of bipolar valued multi fuzzy subfield A of F.

In other cases are true, so in all the cases, the intersection of any two bipolar valued multi fuzzy translations of bipolar valued multi fuzzy subfield A of F is a bipolar valued multi fuzzy translation of A.

Theorem 3.3:

The intersection of a family of bipolar valued multi fuzzy translations of bipolar valued multi fuzzy subfield $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ of a field F is a bipolar valued multi fuzzy translation of A.

Proof: It is trivial.

Theorem 3.4

Union of any two bipolar valued multi fuzzy translations of bipolar valued multi fuzzy subfield $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ of a field F is a bipolar valued multi fuzzy translation of A.

Proof: Let x be in F and $(\alpha,\beta) = ((\alpha_1, \alpha_2, \ldots\alpha_n), (\beta_1, \beta_2, \ldots\beta_n))$, $(\gamma,\delta) = ((\gamma_1, \gamma_2, \ldots\gamma_n), (\delta_1, \delta_2, \ldots\delta_n))$. Let $\mathcal{M} = \mathbb{T}^A_{(\alpha,\beta)} = \{ \langle x, A_1^+(x)+\alpha_1, A_2^+(x)+\alpha_2, \ldots, A_n^+(x)+\alpha_n, A_1^-(x)+\beta_1, A_2^-(x)+\beta_2, \ldots, A_n^-(x)+\beta_n \rangle / x\in F \}$ and $\mathcal{N} = \mathbb{T}^A_{(\gamma,\delta)} = \{ \langle x, A_1^+(x)+\gamma_1, A_2^+(x)+\gamma_2, \ldots, A_n^+(x)+\gamma_n, A_1^-(x)+\delta_1, A_2^-(x)+\delta_2, \ldots, A_n^-(x) + \delta_n \rangle / x\in F \}$ be two bipolar valued multi fuzzy translations of bipolar valued multi fuzzy subfield A of F.

Let $C = \mathcal{M} \cup \mathcal{N}$ and $C = \{ \langle x, C_1^+(x), C_2^+(x), \ldots, C_n^+(x), C_1^-(x), C_2^-(x), \ldots, C_n^-(x) \rangle / x\in F \}$, where $C_i^+(x) = \max \{ A_i^+(x)+\alpha_i, A_i^+(x)+\gamma_i \}$ and $C_i^-(x) = \min \{ A_i^-(x)+\beta_i, A_i^-(x)+ \delta_i \}$ for all i.

Case (i): $\alpha \leq \gamma$ and $\beta \leq \delta$. Now:

$C_i^+(x) = \max \{ \mathcal{M}_i^+(x), \mathcal{N}_i^+(x) \} = \max \{ A_i^+(x) + \alpha_i, A_i^+(x)+ \gamma_i \} = A_i^+(x) + \gamma_i = \mathcal{N}_i^+(x)$ for all x in F and for all i . And,

$C_i^-(x) = \min \{ \mathcal{M}_i^-(x), \mathcal{N}_i^-(x) \} = \min \{ A_i^-(x) + \beta_i, A_i^-(x) + \delta_i \} = A_i^-(x) + \beta_i = \mathcal{M}_i^-(x)$ for all x in F and for all i. Therefore $C = \mathbb{T}^A_{(\gamma,\beta)}$ is a bipolar valued multi fuzzy translation of bipolar valued multi fuzzy subfield A of F.

Case (ii): $\alpha \geq \gamma$ and $\beta \geq \delta$. Now,

$C_i^+(x) = \max \{ \mathcal{M}_i^+(x), \mathcal{N}_i^+(x) \} = \max \{ A_i^+(x) + \alpha_i, A_i^+(x)+ \gamma_i \} = A_i^+(x) + \alpha_i = \mathcal{M}_i^+(x)$ for all x in F and for all i. And,

$C_i^-(x) = \min \{ \mathcal{M}_i^-(x), \mathcal{N}_i^-(x) \} = \min\{ A_i^-(x) + \beta_i, A_i^-(x) + \delta_i \} = A_i^-(x) + \delta_i = \mathcal{N}_i^-(x)$ for all x in F and for all i. Therefore, $C = \mathbb{T}^A_{(\alpha,\delta)}$ is a bipolar valued multi fuzzy translation of bipolar valued multi fuzzy subfield A of F.

Case (iii): $\alpha \leq \gamma$ and $\beta \geq \delta$. Clearly, $C = \mathbb{T}^A_{(\gamma,\delta)}$ is a bipolar valued multi fuzzy translation of bipolar valued multi fuzzy subfield A of F.

Case (iv): $\alpha \geq \gamma$ and $\beta \leq \delta$. Clearly, $C = \mathbb{T}^A_{(\alpha,\beta)}$ is a bipolar valued multi fuzzy translation of bipolar valued multi fuzzy subfield A of F.

In other cases are true, so in all the cases, union of any two bipolar valued multi fuzzy translations of bipolar valued multi fuzzy subfield A of field F is a bipolar valued multi fuzzy translation of A.

Theorem 3.5

The union of a family of bipolar valued multi fuzzy translations of bipolar valued multi fuzzy subfield $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ of a field F is a bipolar valued multi fuzzy translation of A.

Proof: It is trivial.

Theorem 3.6

Let (F, +, .) and (F$^{\prime}$, +, .) be any two fields and g be a homomorphism. Then the homomorphic image of a bipolar valued multi fuzzy translation of a bipolar valued multi fuzzy subfield $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ of F is also a bipolar valued multi fuzzy subfield of F$^{\prime}$.

Proof: Let $U = \langle U_1^+, U_2^+, \ldots, U_n^+, U_1^-, U_2^-, \ldots, U_n^- \rangle = g\,(\mathbb{T}^A_{(\alpha,\beta)})$, where $\mathbb{T}^A_{(\alpha,\beta)}$ is a bipolar valued multi fuzzy translation of a bipolar valued multi fuzzy subfield A of F. We have to prove that U is a bipolar valued multi fuzzy subfield of F$^{\prime}$. For all g(x) and g(y) in F$^{\prime}$, we have:

$U_i^+[g(x)-g(y)] = U_i^+[g(x-y)] \geq \mathbb{T}^{+A}{}_{\alpha_i}(x-y) = A_i^+(x-y) + \alpha_i \geq \min\{ A_i^+(x), A_i^+(y) \} + \alpha_i$

$= \min \{ A_i^+(x) +\alpha_i, A_i^+(y) +\alpha_i \} = \min \{ \mathbb{T}^{+A}_{\alpha_i}(x), \mathbb{T}^{+A}_{\alpha_i}(y) \}$

which implies that:

$U_i^+[\, g(x) - g(y) \,] \geq \min \{ U_i^+(g(x)), U_i^+(g(y)) \}$ for all g(x) and g(y) in F^I and for all i.

Also, $U_i^-[\, g(x)-g(y) \,] = U_i^-[\, g(x-y) \,] \leq \mathbb{T}_{\beta_i}^{-A} (x-y) = A_i^-(x-y) + \beta_i \leq \max \{ A_i^-(x), A_i^-(y) \} + \beta_i$

$= \max \{ A_i^-(x)+ \beta_i, A_i^-(y)+ \beta_i \} = \max \{\mathbb{T}_{\beta_i}^{-A} (x), \mathbb{T}_{\beta_i}^{-A} (y) \}$

which implies that $U_i^-[\, g(x) - g(y) \,] \leq \max \{ U_i^-(g(x)), U_i^-(g(y)) \}$ for all g(x) and g(y)in F^I and for all i. And,

$U_i^+[\, g(x)g(y)^{-1}] = U_i^+[\, g(xy^{-1}) \,] \geq \mathbb{T}^{+A}{}_{\alpha_i} (xy^{-1}) = A_i^+(xy^{-1}) + \alpha_i \geq \min\{ A_i^+(x), A_i^+(y) \} + \alpha_i$

$= \min \{ A_i^+(x) +\alpha_i, A_i^+(y) +\alpha_i \} = \min \{ \mathbb{T}_{\alpha_i}^{+A} (x), \mathbb{T}_{\alpha_i}^{+A} (y) \}$

which implies that $U_i^+[\, g(x)g(y)^{-1} \,] \geq \min \{ U_i^+(g(x)), U_i^+(g(y)) \}$ for all g(x) and g(y) in F^I and forall i. And,

$U_i^-[\, g(x)g(y)^{-1} \,] = U_i^-[\, g(xy)^{-1} \,] \leq \mathbb{T}_{\beta_i}^{-A} (xy^{-1}) = A_i^-(xy^{-1}) + \beta_i \leq \max \{ A_i^-(x), A_i^-(y) \} + \beta_i$

$= \max \{ A_i^-(x)+ \beta_i, A_i^-(y)+ \beta_i\} = \max \{\mathbb{T}_{\beta_i}^{-A} (x), \mathbb{T}_{\beta_i}^{-A} (y) \}$

which implies that $U_i^-[\, g(x)g(y)^{-1}] \leq \max \{ U_i^-(g(x)), U_i^-(g(y)) \}$ for all g(x) and g(y)in F^I and for all i. Therefore, U is a bipolar valued multi fuzzy subfield of F^I.

Theorem 3.7

Let (F, +, .) and (F^I, +, .) be any two fields and f be a homomorphism. Then the homomorphic pre-image of bipolar valued multi fuzzy translation of a bipolar valued multi fuzzy subfield $U = \langle U_1^+, U_2^+, \ldots, U_n^+, U_1^-, U_2^-, \ldots, U_n^- \rangle$ of F^I is a bipolar valued multi fuzzy subfield of F.

Proof: Let $\mathbb{T} = \mathbb{T}_{(\alpha,\beta)}^{U}$ = g(A), where $\mathbb{T}_{(\alpha,\beta)}^{U}$ is a bipolar valued multi fuzzy translation of bipolar valued multi fuzzy subfield $U = \langle U_1^+, U_2^+, \ldots, U_n^+, U_1^-, U_2^-, \ldots, U_n^- \rangle$ of F^I. We have to prove that $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ is a bipolar valued multi fuzzy subfield of F.

Let x and y in F. Then:

$A_i^+(x-y) = \mathbb{T}_{\alpha_i}^{+U} (g(x-y)) = \mathbb{T}_{\alpha_i}^{+U} (g(x)-g(y)) = U_i^+[\, g(x)-g(y) \,] + \alpha_i$

$\geq \min \{ U_i^+(g(x)), U_i^+(g(y)) \}+ \alpha_i = \min \{ U_i^+(g(x)) + \alpha_i, U_i^+(g(y)) + \alpha_i \}$

$= \min \{ \mathbb{T}_{\alpha_i}^{+U} (g(x)), \mathbb{T}_{\alpha_i}^{+U} (g(y)) \} = \min \{ A_i^+(x), A_i^+(y) \}$

which implies that $A_i^+(x-y) \geq \min \{ A_i^+(x), A_i^+(y) \}$ for all x and y in F and for all i. And,

$A_i^-(x-y) = \mathbb{T}_{\beta_i}^{-U} (g(x-y)) = \mathbb{T}_{\beta_i}^{-U} (g(x)-g(y)) = U_i^-[\, g(x)-g(y) \,] +\beta_i$

$\leq \max \{ U_i^-(g(x)), U_i^-(g(y)) \} + \beta_i = \max \{ U_i^-(g(x)) + \beta_i, U_i^-(g(y)) + \beta_i\}$

$= \max \{ \mathbb{T}_{\beta_i}^{-U} (g(x)), \mathbb{T}_{\beta_i}^{-U} (g(y)) \} = \max \{ A_i^-(x), A_i^-(y) \}$

which implies $A_i^-(x-y) \leq \max\{ A_i^-(x), A_i^-(y) \}$ for all x and y in F and for all i. Also,

$A_i^+(xy^{-1}) = \mathbb{T}_{\alpha_i}^{+U} (g(xy^{-1})) = \mathbb{T}_{\alpha_i}^{+U} (g(x)g(y)^{-1}) = U_i^+[\, g(x) g(y)^{-1}] + \alpha_i \geq \min \{ U_i^+(g(x)),$
$U_i^+(g(y)) \}+ \alpha_i = \min \{ U_i^+(g(x)) + \alpha_i, U_i^+(g(y)) + \alpha_i \} = \min \{ \mathbb{T}_{\alpha_i}^{+U} (g(x)), \mathbb{T}_{\alpha_i}^{+U} (g(y)) \}$

$= \min \{ A_i^+(x), A_i^+(y) \}$

which implies that $A_i^+(xy^{-1}) \geq \min \{ A_i^+(x), A_i^+(y) \}$ for all x and $y \neq 0$ in F and for all i. And,

$A_i^-(xy^{-1}) = \mathbb{T}_{\beta_i}^{-U} (g(xy^{-1})) = \mathbb{T}_{\beta_i}^{-U} (g(x)g(y)^{-1}) = U_i^-[\, g(x)g(y)^{-1} \,] + \beta_i$

$\leq \max \{ U_i^-(g(x)), U_i^-(g(y)) \} + \beta_i = \max \{ U_i^-(g(x)) + \beta_i, U_i^-(g(y)) + \beta_i\}$

$= \max \{ \mathbb{T}_{\beta_i}^{-U} (g(x)), \mathbb{T}_{\beta_i}^{-U} (g(y)) \} = \max \{ A_i^-(x), A_i^-(y) \}$

which implies $A_i^-(xy^{-1}) \leq \max\{ A_i^-(x), A_i^-(y) \}$ for all x and $y \neq 0$ in F and for all i.

Therefore, A is a bipolar valued multi fuzzy subfield of F.

Theorem 3.8

Let (F, +, .) and (F^I,+, .) be any two fields and g be anti-homomorphism. Then the anti-homomorphic image of a bipolar valued multi fuzzy translation of a bipolar valued multi fuzzy subfield $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ of F is also a bipolar valued multi fuzzy subfield of F^I.

Proof: Let $U = \langle U_1^+, U_2^+, \ldots, U_n^+, U_1^-, U_2^-, \ldots, U_n^- \rangle = g(\mathbb{T}_{(\alpha,\beta)}^{A})$, where $\mathbb{T}_{(\alpha,\beta)}^{A}$ is a bipolar valued multi fuzzy translation of a bipolar valued multi fuzzy subfield A of F. We have to prove that U is a bipolar valued multi fuzzy subfield of F^I.

For all g(x) and g(y) in F^I, we have:

$U_i^+[\, g(x)-g(y)] = U_i^+[\, g(y-x) \,] \geq T^{+A}{}_{\alpha_i} (y-x) = A_i^+(y-x) + \alpha_i \geq \min\{ A_i^+(y), A_i^+(x) \} + \alpha_i$

$= \min \{ A_i^+(x) + \alpha_i, A_i^+(y) + \alpha_i \} = \min \{ \mathbb{T}_{\alpha_i}^{+A} (x), \mathbb{T}_{\alpha_i}^{+A} (y) \}$

which implies that $U_i^+[g(x) - g(y)] \geq \min \{ U_i^+(g(x)), U_i^+(g(y)) \}$ for all g(x) and g(y) in F^I and for all i, and

$U_i^-[g(x) - g(y)] = U_i^-[g(y-x)] \leq \mathbb{T}_{\beta_i}^{-A} (y-x) = A_i^-(y-x) + \beta_i \leq \max \{ A_i^-(y), A_i^-(x) \} + \beta_i$

$= \max \{ A_i^-(x) + \beta_i, A_i^-(y) + \beta_i \} = \max \{ \mathbb{T}_{\beta_i}^{-A} (x), \mathbb{T}_{\beta_i}^{-A} (y) \}$

which implies that $U_i^-[g(x) - g(y)] \leq \max \{ U_i^-(g(x)), U_i^-(g(y)) \}$ for all g(x) and g(y)in F^I and for all i. Also,

$U_i^+[g(x)g(y)^{-1}] = U_i^+[g(yx^{-1})] \geq \mathbb{T}^{+A}{}_{\alpha_i} (yx^{-1}) = A_i^+(yx^{-1}) + \alpha_i \geq \min\{ A_i^+(y), A_i^+(x) \} + \alpha_i$

$= \min \{ A_i^+(x) + \alpha_i, A_i^+(y) + \alpha_i \} = \min \{ \mathbb{T}_{\alpha_i}^{+A} (x), \mathbb{T}_{\alpha_i}^{+A} (y) \}$

which implies that $U_i^+[g(x)g(y)^{-1}] \geq \min \{ U_i^+(g(x)), U_i^+(g(y)) \}$ for all g(x) and g(y) in F^I and for all i, and

$U_i^-[g(x)g(y)^{-1}] = U_i^-[g(yx^{-1})] \leq \mathbb{T}_{\beta_i}^{-A}(yx^{-1}) = A_i^-(yx^{-1}) + \beta_i \leq \max\{ A_i^-(x), A_i^-(y) \} + \beta_i$

$= \max \{ A_i^-(x) + \beta_i, A_i^-(y) + \beta_i \} = \max\{ \mathbb{T}_{\beta_i}^{-A} (x), \mathbb{T}_{\beta_i}^{-A} (y)\}$

which implies that $U_i^-[g(x)g(y)^{-1}] \leq \max \{ U_i^-(g(x)), U_i^-(g(y)) \}$ for all g(x) and g(y)in F^I and for all i.

Therefore, U is a bipolar valued multi fuzzy subfield of F^I.

Theorem 3.9

Let (F, +, .) and (F^I, +, .) be any two fields and f be an anti-homomorphism. Then the anti-homomorphic pre-image of bipolar valued multi fuzzy translation of a bipolar valued multi fuzzy subfield $U = \langle U_1^+, U_2^+, \ldots, U_n^+, U_1^-, U_2^-, \ldots, U_n^- \rangle$ of F^I is a bipolar valued multi fuzzy subfield of F.

Proof: Let $\mathbb{T} = \langle \mathbb{T}_1^+, \mathbb{T}_2^+, \ldots, \mathbb{T}_n^+, \mathbb{T}_1^-, \mathbb{T}_2^-, \ldots, \mathbb{T}_n^- \rangle = \mathbb{T}_{(\alpha,\beta)}^{U} = g(A)$, where $\mathbb{T}_{(\alpha,\beta)}^{U}$ is a bipolar valued multi fuzzy translation of bipolar valued multi fuzzy subfield U of F^I. We have to prove that $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ is a bipolar valued multi fuzzy subfield of F.

Let x and y in F. Then:

$A_i^+(x-y) = \mathbb{T}_{\alpha_i}^{+U} (g(x-y)) = \mathbb{T}_{\alpha_i}^{+U} (g(y)-g(x)) = U_i^+[g(y)-g(x)] + \alpha_i$

$\geq \min \{ U_i^+(g(y)), U_i^+(g(x)) \} + \alpha_i = \min \{ U_i^+(g(x)) + \alpha_i, U_i^+(g(y)) + \alpha_i \}$

$= \min \{ \mathbb{T}_{\alpha_i}^{+U} (g(x)), \mathbb{T}_{\alpha_i}^{+U} (g(y)) \} = \min \{ A_i^+(x), A_i^+(y) \}$

which implies that $A_i^+(x-y) \geq \min \{ A_i^+(x), A_i^+(y) \}$ for all x and yin F and for all I, and

$A_i^-(x-y) = \mathbb{T}_{\beta_i}^{-U} (g (x-y)) = \mathbb{T}_{\beta_i}^{-U} (g(y)-g(x)) = U_i^-[g(y)-g(x)] + \beta_i$

$\leq \max \{ U_i^-(g(y)), U_i^-(g(x)) \} + \beta_i = \max \{ U_i^-(g(x)) + \beta_i, U_i^-(g(y)) + \beta_i\}$

$= \max \{ \mathbb{T}_{\beta_i}^{-U} (g(x)), \mathbb{T}_{\beta_i}^{-U} (g(y)) \} = \max \{ A_i^-(x), A_i^-(y) \}$

which implies $A_i^-(x-y) \leq \max\{ A_i^-(x), A_i^-(y) \}$ for all x and y in F and for all i.

Also, $A_i^+(xy^{-1}) = \mathbb{T}_{\alpha_i}^{+U} (g(xy^{-1})) = \mathbb{T}_{\alpha_i}^{+U} (g(y)g(x)^{-1}) = U_i^+[g(y)g(x)^{-1}] + \alpha_i$

$\geq \min \{ U_i^+(g(y)), U_i^+(g(x)) \} + \alpha_i = \min \{ U_i^+(g(x)) + \alpha_i, U_i^+(g(y)) + \alpha_i \}$

$= \min \{ \mathbb{T}_{\alpha_i}^{+U} (g(x)), \mathbb{T}_{\alpha_i}^{+U} (g(y)) \} = \min \{ A_i^+(x), A_i^+(y) \}$

which implies that $A_i^+(xy^{-1}) \geq \min \{ A_i^+(x), A_i^+(y) \}$ for all x and $y \neq 0$ in F and for all I, and we have:

$A_i^-(xy^{-1}) = \mathbb{T}_{\beta_i}^{-U} (g(xy^{-1})) = \mathbb{T}_{\beta_i}^{-U} (g(y)\, g(x)^{-1}) = V_i^-[g(y)g(x)^{-1}] + \beta_i$

$\leq \max \{ U_i^-(g(y)), U_i^-(g(x)) \} + \beta_i = \max \{ U_i^-(g(x)) + \beta_i, U_i^-(g(y)) + \beta_i\}$

$= \max \{ \mathbb{T}_{\beta_i}^{-U} (g(x)), \mathbb{T}_{\beta_i}^{-U} (g(y)) \} = \max \{ A_i^-(x), A_i^-(y) \}$

which implies $A_i^-(xy^{-1}) \leq \max\{ A_i^-(x), A_i^-(y) \}$ for all x and $y \neq 0$ in F and for all i. Therefore A is a bipolar valued multi fuzzy subfield of F.

References

[1] Anitha.M.S., Muruganantha Prasad & K.Arjunan, "Homomorphism and Antihomomorphism of Bipolar- valued fuzzy subgroups of a group", International journal of Mathematical Archive - 4(12), (2013), 274- 276.

[2] Jun, Y. B., Kim, H. S., Lee, K. J., "Bipolar Fuzzy Translations in BCK/BCI-algebras", Journal of the Chungcheong Mathematical Society, Volume 22:3, (2009).
[3] Jun, Y. B., Lee, K. J., and J. Zhan, "Soft p-ideals of soft BCI-algebras", Comput Math Appl 58 (2009), 2060–2068.
[4] Lee, K. M., "Bipolar-valued fuzzy sets and their operations", Proc. of Int. Conf. on Intelligent Technologies, Bangkok, Thailand, (2000), pp.307-312
[5] Pawlak, .Z., "Rough sets", International Journal of Computing andInformation Sciences 11 (1982), 341–356
[6] Sabu Sebastian and T.V.Ramakrishnan, "Multi-fuzzy sets", International Mathematical Forum, 6, (2010), no.50, 2471 – 2476.
[7] Sabu Sebastian and T.V.Ramakrishnan, "Multi-fuzzy subgroups", Int.J.Contemp.Math.Sciences, Vol. 6, (2011), no.8, 365 – 372.
[8] Shanthi, V.K., and Shyamala , G., "Notes on Bipolar-valued multi fuzzy subgroups of a group", International journal of Mathematical Archive-6(6),(2015),234-238.
[9] Yamini,.C.,Arjunan,.K., and Ananth,.B.,"Bipolar valued multi fuzzy subfield of a field", InternationalJournal of Management, Technology And Engineering, ISSN NO : 2249-7455,volume 8,issue XI, November (2018).
[10] Yamini,.C.,Arjunan,.K., and Ananth,.B., "Notes on Bipolar valued multi fuzzy subfield of a field under Homomorphisms",Journal of Applied Science and Computations,ISSN NO:1076-5131,VOLUME VI,Issue IV,April 2019.1162-1166.
[11] Yasodara .B and Sathappan .K.E, "Bipolar-valued multi fuzzy subsemirings of a semiring", International Journal of Mathematical Archive, 6(9), (2015), 75– 80.
[12] Yasodara .B and Sathappan .K.E, "Homomorphism and anti-homomorphism of bipolar-valued multifuzzy subsemirings of a semiring", Bulletin of Mathematics and Statistics Research, Vol.3 Issu.3,(2015), 229 – 233.
[13] Zadeh .L.A , "fuzzy sets", Inform. And Control, 8(1965), 338-353.
[14] Zhang .W.R, "Bipolar Fuzzy sets and Relations", a computational Frame work for cognitive modelingand multiple decision Analysis, proceedings of Fuzzy IEEE conferences,pp-305-309,(1994).
[15] Zhang. W.R. "Bipolar Fuzzy sets", proceedings of Fuzzy IEEE conferences, pp 835-840,(1998).