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A CONJUGATE OF M-N FUZZY SOFT SUBGROUPS

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Abstract. In this paper, we have discussed the concept of a conjugate of M-N fuzzy soft subgroup of a group and define the M-N fuzzy soft middle co-sets. Also its some elementary properties are discussed. The aim of the paper is to investigate conjugate of M –N fuzzy soft subgroup of a group from a general point of view.

Key words: Fuzzy subgroup; M-N fuzzy subgroup; M-N fuzzy soft subgroup; fuzzy soft middle co-sets.

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1. Introduction

There are various types of uncertainties in the real world, but few classical mathematical tools may not be suitable to model these uncertainties. Many intricate problems in economics, social science, engineering, medical science and many other fields involve undefined data. These problems which one comes face to face with in life cannot be solved using classical mathematic methods. In classical mathematics, a mathematical model of an object is devised and the concept of the exact solution of this model is not yet determined. Since, the classical mathematical model is too complex, the exact solution cannot be found. There are several well-renowned theories available to describe uncertainty. For instance, Rosenfeld [10] introduced the concept of fuzzy subgroup in 1971 and the theory of fuzzy sets was inspired by Zadeh [14] in addition to this, Molodtsov [7] have introduced the concept of soft sets in 1999. Furthermore, Majiet. al., [6] as well introduction the concept of fuzzy soft sets in 2001 and Jacobson [3] introduced the concept of M-group M-subgroup.
Sarala and Suganya [11] unravelled some properties of fuzzy soft groups in 2014. In addition, Vasantha Kandasamy and Smarandache [13] have introduced the Fuzzy Algebra during 2003. An introduction to the new definition of Soft sets and soft groups depending on inclusion relation and intersection of sets were exposed by Akta and Cagman [1]. In 1981, Das [2] studied the Fuzzy groups

and level subgroups. Moreover, Maij, Biswas and Ray [6] were introduced the fuzzy soft set in 2001. In [9] the notion of a conjugate fuzzy subgroup of a fuzzy group was introduced and studied. ShobhaShukla [12] studied the conjugate fuzzy subgroup in 2013. Mourad Oqla Massa'deh [9] studied M- fuzzy co-sets, M –conjugate of M –fuzzy subgroups.

In the present manuscript, we have discussed the concept of conjugate of M-N fuzzy soft group based on the concept of M - N fuzzy soft group [5 and 9]. In section 2, we presented the basic definition, notations on conjugate of M-N fuzzy soft group and required results on fuzzy soft group. In section 3, we define the conjugate of M-N fuzzy soft subgroups, define the M-N fuzzy soft middle co-sets and related results are discussed.

2. Preliminaries

In this section, some basic definitions and results needed are given. For the sake of convenience we set out the former concepts which will be used in this paper.

Definition 2.1

Let G be any non-empty set. A mapping μ : G → [0, 1] is called fuzzy set in G.

Definition 2.2

Let x be a non-empty set. A fuzzy subset μ of X is a function μ : X → [0, 1]

Definition 2.3

Let G be a group. A fuzzy subset μ of G is called a fuzzy subgroup if for x, y $\in G$

(1) μ (x y) ≥ min {μ (x), μ (y)}

(2) μ (x^{-1}) =μ(x)

Definition 2.4

Let μ be an M- N fuzzy subgroup of a set G. For t ∈ [0, 1], the level subset of μ is the set μ_t = {x ∈ G / μ (mx ≥ t, μ(xn) ≥ t, m ∈ M, n ∈ N}. This is called a M – N level subset of μ.

Definition 2.5

A pair (μ, A) is called a soft set over U, where μ is a mapping given by μ: A → $P(U)$

Definition 2.6

Let (μ, A) be a soft set over G. Then (μ, A) is called a soft group over G if μ (a) is a group G for all a ∈ A.

Definition 2.7

A pair (μ, A) is called a fuzzy soft set over U, where μ: A → I^U is a mapping I = [0, 1], μ (a) is a fuzzy subset of U for all a∈ A.

Definition 2.8 *[5]*

Let (μ, A) be a fuzzy soft set over G. Then (μ, A) is a called a fuzzy soft group if μ (a) is a fuzzy subgroup G for all a∈ A.

Definition 2.9 *[4]*

Let (μ, A) and (λ, B) be two fuzzy soft set over U. Then (μ, A) is called a fuzzy soft subset of (λ, B) denoted by (μ, A) ⊆ (λ, B) if

(1) A ⊆ B

(2) μ (a) is a fuzzy subset of λ (a) for each a∈ A.

Definition 2.10

A fuzzy set μ is called a fuzzy soft subgroup of a group G, if for x, y ∈ G

(1) μ (xy) $\geq$ min{μ (x), μ (y)}

(2) μ (x^{-1}) $\geq$ μ (x)

Definition 2.11 *[8]*

Let M, N be left and right operator sets of group G respectively if (m x) n = m (x n) for all x ∈ G, m ∈ M, n∈ N. Then G is said be an M – N group.

Definition 2.12 *[4]*

Let G be an M – N group and (μ, A) be a fuzzy soft subgroup of G if

(1) μ {m(x y) n} $\geq$ min { μ (x), μ (y)}

(2) μ {(m x^{-1}) n} $\geq\mu$ (x) hold for any x, y ∈ G, m ∈ M, n ∈ N, then (μ, A) is said be an M – N fuzzy soft subgroup of G. Here μ: A → P (G)

Definition 2.13 *[5]*

Let G be an M – N group and (μ, A) be a fuzzy soft subgroup of G if

(1) μ (m x)≥ μ (x)

(2) μ (x n) ≥ μ (x) hold for any x ∈G, m ∈M, and n ∈N, then (μ, A) is said be an M – N fuzzy soft subgroup of G.

Definition 2.14 *[12]*

Let μ and λ be two fuzzy subgroup of G, then μ and λ are said to be conjugate fuzzy subgroup of G if for some g ∈ G, μ (x) = λ (g^{-1} xg) for every x∈G.

3. Conjugate of M-N fuzzy soft subgroup

In this section, we shall define the conjugate of M-N fuzzy soft subgroup, discussed the Conjugate of M-N fuzzy soft group based on the concept of fuzzy soft group [5and 9], and give some elementary properties are discussed.

Definition 3.1

Let μ and λ be two M - N fuzzy soft subgroup of G, then μ and λ are said to be conjugate of M - N fuzzy soft subgroup of G if for some g ∈ G,

(1) μ (mx) = λ (g^{-1} x g) for every x ∈ G, m ∈ M

(2) μ (xn) = λ (g^{-1} x g) for every x ∈ G, n ∈ N

Definition 3.2

Let G be an M – N group and (μ, A) be a conjugate of fuzzy soft subgroup of G if

(1) μ (m x) ≥ μ (x) = λ (g^{-1} x g)

(2) μ (y n) ≥ μ (y) = λ (g^{-1} y g) hold for any x, y ∈ G, m ∈ M, and n ∈ N, then (μ, A) is said be a conjugate of M – N fuzzy soft subgroup of G.

Theorem 3.3

Let μ and λ be any M - N fuzzy soft subgroup of the group G, then μ and λ are conjugate of M - N fuzzy soft subgroup of G iff μ = λ.

Proof: Given that μ and λ are conjugate of M - N fuzzy soft subgroup of the group G. We have to prove that μ = λ. Since μ and λ are conjugate of M - N fuzzy soft subgroup of the group G, by the definition there exists g ∈ G, such that, μ (mx) = λ (g^{-1} x g) for every x ∈ G, m ∈ M, μ (xn) = λ (g^{-1} x g) for every x ∈ G, n ∈ N. Let mx = gmx for allg, x ∈ G, m ∈ M, then:

μ (gmx) = λ (g^{-1} gx g)

μ (gmx) = λ (x g)

μ (gx) = λ (x g), since μ (mx) ≥ μ (x).

And, let xn = ɡxn for allɡ, x ∈ G, n ∈ N, then:

μ (ɡxn) = λ ($ɡ^{-1}$ ɡx ɡ)

μ (ɡxn) = λ (x ɡ)

μ (ɡx) = λ (x ɡ), since μ (xn) ≥ μ (x).

For some ɡ = e ∈ G,m ∈ M, we have μ (mex) = λ (mxe)

μ (mx) = λ (mx), since μ (xm) ≥ μ (x)

μ (x) = λ (x), μ = λ.

Similarly we can prove that μ (exn) = λ (xen) , hence μ = λ.

Conversely, we have to prove that, μ and λ are conjugate of M - N fuzzy soft subgroup of G. Let μ = λ, μ (mx) = λ (mx), μ (mx) = μ (x) = λ (x), since μ (xm) ≥ μ (x). By the definition, μ (mx) = λ (e^{-1} xe) . Similarly we can prove that μ (xn) = λ (e^{-1} xe) . Hence μ and λ are conjugate of M - N fuzzy soft subgroup of G.

Theorem 3.4

Let λ be an M - N fuzzy soft subgroup of a group G, and μ be a fuzzy soft subset of G. If μ and λ are conjugate of M - N fuzzy soft subgroup of the group G, then μ is an M - N fuzzy soft subgroup of a group G.

Proof: Let e be an identity element of the group G. If μ and λ are conjugate of M - N fuzzy soft subgroup of the group G, by the definition, since there exists an element ɡ ∈ G, such that:

λ (x) ≤ λ (mx) = μ ($ɡ^{-1}$ x ɡ) for all x ∈ G

⇒ λ (x) = μ ($ɡ^{-1}$ x ɡ) and

λ (x) ≤ λ (xn) = μ ($ɡ^{-1}$ x ɡ) for all x ∈ G, m ∈ M, and n ∈ N,

⇒ λ (x) = μ ($ɡ^{-1}$ x ɡ).

Also, μ (x) ≤ μ (mx) = μ (exe)

μ (x) = μ ($ɡ^{-1}$ ɡ x $ɡ^{-1}$ ɡ)

= λ (ɡx $ɡ^{-1}$) .

Therefore, μ (x) = λ (ɡx $ɡ^{-1}$) . Similarly we can prove that μ (x) ≤ μ (xn) = μ (ɡx $ɡ^{-1}$).

We have to prove that, μ is an M - N fuzzy soft subgroup of a group G. Since λ be an M - N fuzzy soft subgroup of a group G, now:

μ (xy) ≤ μ (mxyn) = μ (emxeyne)

μ (mxyn) = μ ($ɡ^{-1}$ ɡ mx $ɡ^{-1}$ ɡ y n $ɡ^{-1}$)

= λ(ɡm x $ɡ^{-1}$ɡ yn $ɡ^{-1}$)

≥ min { λ (ɡ mx $ɡ^{-1}$), λ(ɡ yn $ɡ^{-1}$) }, since μ (xm) ≥ μ (x)

μ (mxyn) ≥ min { λ (ɡx $ɡ^{-1}$), λ(ɡy $ɡ^{-1}$) }

μ (xy) ≥ min { μ (x),μ(y) }

Also, μ ($mx^{-1}n$) = λ (ɡmxy^{-1} $ɡ^{-1}$ ɡn y $ɡ^{-1}$)

≥ min { λ (ɡ mx $ɡ^{-1}$ ɡ $y^{-1}ɡ^{-1}$), λ(ɡ yn $ɡ^{-1}$) },

≥ min {min{ λ (ɡ mx $ɡ^{-1}$) λ (ɡ $y^{-1}ɡ^{-1}$)}, λ(ɡ yn $ɡ^{-1}$) }, since μ (xm) ≥ μ (x)

≥ min {min{ λ (ɡx $ɡ^{-1}$) λ (ɡ $y^{-1}ɡ^{-1}$)}, λ(ɡy$ɡ^{-1}$) },

≥ min {{ λ (ɡx $ɡ^{-1}$), λ(ɡy$ɡ^{-1}$) }, μ (y) = μ (y^{-1})

≥ min {λ (ɡx $ɡ^{-1}$), λ(ɡ$y^{-1}ɡ^{-1}$) }

= λ(ɡx $ɡ^{-1}$),

μ ($mx^{-1}n$) = λ(ɡx $ɡ^{-1}$).

Hence μ is an M - N fuzzy soft subgroup of a group G.

4. M-N Fuzzy Soft Middle Co-set

Definition 4.1

Let λ be an M- N fuzzy soft subgroup of a group G. then for any a, b ∈ G the M – N fuzzy soft middle coset aλb of the G is defined by (a λ b) (mxn) = λ (a^{-1} x b^{-1}) for all x∈G.

Theorem 4.2

If λ is an M – N fuzzy soft subgroup of a group G, then for any a ∈G, the M – N fuzzy soft middle coset $a\lambda a^{-1}$ of the group G is also a M – N fuzzy soft subgroup of the group G.

Proof: Let λ be an M – N fuzzy soft subgroup of a group G and a ∈G. Let x, y ∈ G, m ∈ M and n ∈ N, then:

$$(a \lambda a^{-1}) (m xy^{-1}n) = \lambda (ma^{-1}xy^{-1}an) = \lambda (ma^{-1} xaa^{-1} y^{-1}an)= \lambda (m (a^{-1}xa) (a^{-1} y^{-1}a)n)$$
$$\geq \min \{\lambda(m(a^{-1}xa)), \lambda(a^{-1}y^{-1}a)n)\}, \text{since } A(mx) \geq A(x), A(yn) \geq A(y)$$
$$\geq \min \{\lambda((a^{-1}xa)), \lambda(a^{-1}y^{-1}a))\},$$

since λ is an M – N fuzzy soft subgroup of a group G. Therefore,

$$(a\lambda a^{-1}) (m xy^{-1}n) \geq \min \{\lambda((a^{-1}xa), \lambda(a^{-1}y^{-1}a))\}.$$

Hence $a\lambda a^{-1}$ is an M – N fuzzy soft subgroup of the group G.

Theorem 4.3

Let λ be any M – N fuzzy soft subgroup of a group G and$a\lambda a^{-1}$ be an M – N fuzzy soft middle co-set of G, then $o(a\lambda a^{-1}) = o(\lambda)$ for any a ∈ G.

Proof: Let λ be an M – N fuzzy soft subgroup of a group G and a ∈G . By the theorem 3.6 $a\lambda a^{-1}$ is an M – N fuzzy soft subgroup of the group G. Thus $(a \lambda a^{-1}) (m xn) = \lambda (ma^{-1} x an)$, for all x ∈ G,m ∈ M and n ∈ N. Therefore, λ and $a\lambda a^{-1}$ are conjugate of M - N fuzzy soft subgroup of G.

We know that the theorem if λ and μ are conjugate of M –N fuzzy soft subgroup of the group G, then $o(\lambda) = o(\mu)$. Hence $o(a\lambda a^{-1}) = o(\lambda)$ for any a ∈G.

Definition 4.4

Let λ and μ be an M – N fuzzy soft subgroup of the group G, and f be a positive fuzzy set, then for a∈G we define the M – N positive double fuzzy soft coset

$$(\lambda a\mu)^f \text{ by } (\lambda a \mu)^f = \min \{(a \lambda)^f , (a \mu)^f \}.$$

Theorem 4.5

The M – N positive double fuzzy soft co-set $(\lambda a \mu)^f$ is M – N fuzzy soft subgroup of the group G, when λ, μ are M – N fuzzy soft subgroup of G

Proof: Let x, y ∈ G, m ∈ M and n ∈ N. Now,

$$(\lambda a \mu)^f(mxy^{-1}n)= \min \{(a \lambda)^f (mxy^{-1}n), (a \mu)^f (mxy^{-1}n)\}$$
$$= \min \{f(a)\lambda(mxy^{-1}n), f(a) \mu(mxy^{-1}n)\}$$
$$\geq f(a) \min\{\min\{ \lambda (mx), \lambda (y^{-1}n), \min\{ \mu(mx), \mu(y^{-1}n)\}\}.$$

Since, $\lambda (mx) \geq \lambda (x), \lambda (x^{-1}) = \lambda (x), (xn) \geq \mu(x), \mu(x^{-1}) =\mu(x)$

$$\geq f(a) \min\{\min\{ \lambda (x), \lambda (y), \min\{ \mu(x), \mu(y)\}\}$$
$$\geq f(a) \min\{\min\{ \lambda(x), \mu(x)\}, \min\{ \lambda (y), \mu(y)\}\}$$
$$= \min \{ f(a) \min\{\lambda(x), \mu(x)\}, f (a) \min \{\lambda(y), \mu(y) \}$$
$$= \min \{(\lambda a \mu)^f (x), (\lambda a \mu)^f (y)\}.$$

Therefore, $(\lambda a \mu)^f(mxy^{-1}n) \geq \min \{(\lambda a \mu)^f (x), (\lambda a \mu)^f (y)\}$. Hence $(\lambda a \mu)^f$ is M – N fuzzy soft subgroup of the group G.

4. Conclusion

The main results in the present manuscript are based on the concept conjugate of M –N fuzzy soft group [5and 12]. We have also defined the M-N fuzzy soft middle co-set and its some elementary properties are discussed.

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