ADV MATH SCI JOURNAL

Advances in Mathematics: Scientific Journal 8 (2019), no.3, 580–585
(Special issue on ICRAPAM)
ISSN 1857-8365 printed version
ISSN 1857-8438 electronic version

VERTEX LABELING OF SPIDER GRAPH

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Abstract. A (p, q) connected graph is an even vertex graceful labeling if there exists an injective map f: E(G) → {1, 3, …, 2q} so that induced map f+: V(G) → {0, 2, 4, …,(2k-2)} defined by f+(x)≡ Σf(x, y) (mod 2k), Where the summation runs through the edges xy and here k = max { p, q } makes the distinct lables to all the vertices of G(p,q). Even vertex graceful labeling for some graphs is discussed by A. Solairaju and P. Muruganantham.

Keywords: Spider graph, Graceful Graph, Even vertex graceful Graph.

1. Introduction

Graph Theory is one of the part of discrete mathematics, notable by geometric move towards to the study of objects. The standard aim of the theory is a graph and its generation. Any problem or object under the consideration is noted in the type of nodes (Vertices, points, elements) and edges (lines, link, connections). The vertex set and edge set are represented by V(G) and E(G) in that order. The Ringel – Kotzig [1] conjectured that all trees are graceful and this has been the focus of many papers.

A. Solairaju and P.Muruganantham [2] proved that even-edge gracefulness of ladder. (even vertex graceful). They found the connected graphs Pn o nC3 and Pn o nC7 are both even vertex graceful, where n is any positive integer. They also obtained [3] that Even vertex gracefulness of path merging circuits, where n is any even positive integer, and $m_1 + n_1 m_2 + n_1 n_2$ edges.These concepts were studied by R. Frucht and F. Harary [5]. In this paper, we investigate the detour domination number of Corona product of some standard graphs.

***Definition 1.1* (Spider graph)**

A Spider graph is a a cyclic graph with at most one vertex of degree more than two, called the center of Spider (if no vertex of degree more than two, then any vertex can be the center). A leg of a Spider is a path from the center to a vertex of degree one. Thus, a star with *k* legs is a Spider of *k* legs, each of length 1.

Definition 1.2

A labeling of a graph G(V,E) is graceful, if there exists an injective map f: V(G) → {0,1,2,…,q} such that its induced map f^+:E(G) →{1,2,3,…,q} is defined by $f^+(uv) = |f(u) - f(v)|$ for every edge uv in G where f^+ is injective map. A graph G is called graceful if it admits a graceful labeling.

Definition 1.3

A (p, q) graph is an even-vertex graceful, if there exists an injective map f: E(G) → {1, 2, …, 2q} such that induced map f^+: V(G) → {2,4,6,…,2q} defined by $f^+(x) \equiv \Sigma f(xy) \pmod{2k}$ is injective , where k = max {p, q}. Hence the graph G is an even-vertex graceful.

Definition 1.4

A (p, q) connected graph is edge-odd graceful graph if there exists an injective map f: E(G) → {1, 3, …, 2q-1} so that induced map f+: V(G) → {0, 1, 2, …,(2k-1)} defined by f+(x)≡ f(x, y) (mod 2k), where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd. Hence the graph G is edge- odd graceful.

2. Spider graphs with edge odd graceful labeling

Definition 2.1 **Spider graph with respect to C_3**

A spider graph $nC_3 * 3P_{n+2}$ is a connected graph identified from n copies of a circuit C_3 of length of 3, and 3 copies of a path P_{n+2} as follows:

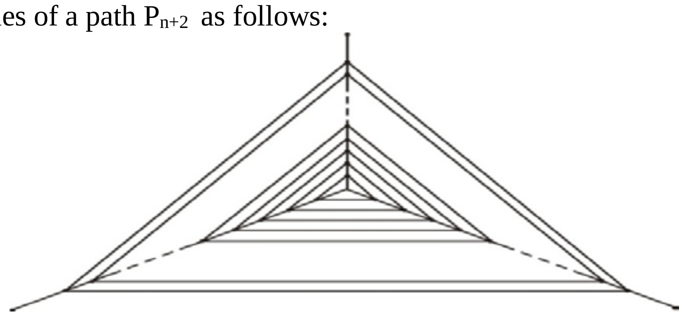


Figure 2.1 – A spider graph $nC_3 * P_{n+2}$

Definition 2.2

A spider graph $nC_3 * 3P_{n+1}$ copies of circuit with three vertices and three copies of path with (n+1) verticesis a connected graph whose vertex set is { v_0, v_1, v_2, v_3, …, v_{3n+3} }, and edge set is: {v_0v_1, v_0v_2, v_0v_3 } ∪ { v_iv_{i+1} ; i ≡ 1, 2 (mod 3), v_iv_{i+2} ; i ≡ 0 (mod 3) } ∪ { v_iv_{i+3} ; i = 1 to 3n }.
Here p = 3n+4, q = 6n+3.

Theorem 2.3

A spider graph $nC_3 * 3P_{n+2}$ is an even vertex graceful.

Proof: Some arbitrary labeling of the given graph is taken as follows:

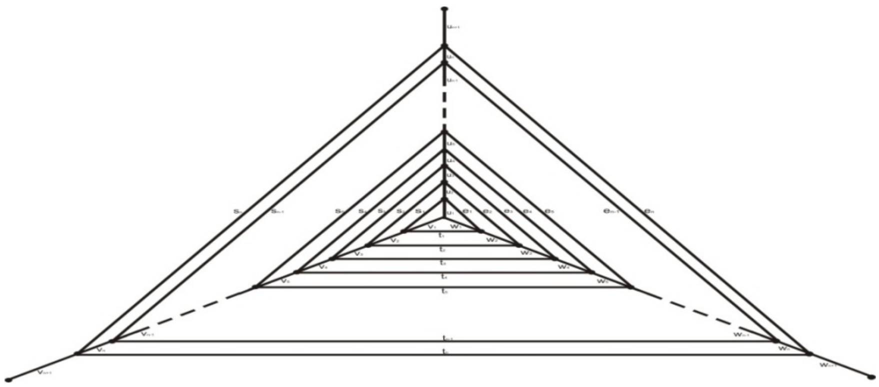


Figure 2.2 Edge labeling of a spider graph $nC_3 * 3P_{n+2}$

We define f: E(G) = {1, 2 ,.., 2q} by

$f(e_i) = 1+ 6i- 6$, i= 1 to n;
$f(s_i) = 5+ 6i- 6$, i= 1 to n;
$f(t_i) = 3+ 6i- 6$, i= 1 to n;

$f(u_i) =2+ 6i- 6$, i= 1 to n;
$f(v_i) = 6i$, i is odd i varies from 1 to n;
= 6i-2, i is even i varies from 1 to n;
$f(w_i) = 4+6(i- 1)=6i-2$,I is odd ; i=1to n;
= 6i, i is even i varies from 1 to n;

$n \equiv 0, 1 \pmod 4$

$f(u_{n+1}) = f(u_n) +6$;
$f(w_{n+1}) = f(u_{n+1}) +2$;
$f(v_{n+1}) = f(w_{n+1}) +2$;

$n \equiv 2, 3 \pmod 4$

$f(u_{n+1}) = f(u_n) +18$;
$f(w_{n+1}) = f(u_{n+1}) +2$;
$f(v_{n+1}) = f(w_{n+1}) +2$;

For all vertices V of $nC_3 * 3P_{n+2}$, define $f^+ (v) = (\sum_{vu \in E(G)} f (vu)) \pmod{2q}$,

The labeling f for all vertices and f^+ for all vertices satisfy the conditions of even vertex graceful labeling. Thus $nC_3 * 3P_{n+2}$ is an even vertex graceful.

Example 2.4 The spider graph $10C_4 * 3P_{12}$ is even vertex graceful.

Here n = 10.

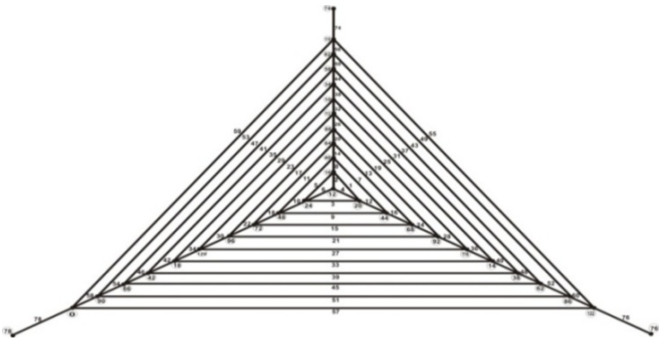


Figure 2.3 – A Spider graph $10C_3 * 3P_{12}$

***Example 2.5*:** The spider graph $13C_4 * 3P_{15}$is even vertex graceful.
Here n = 13

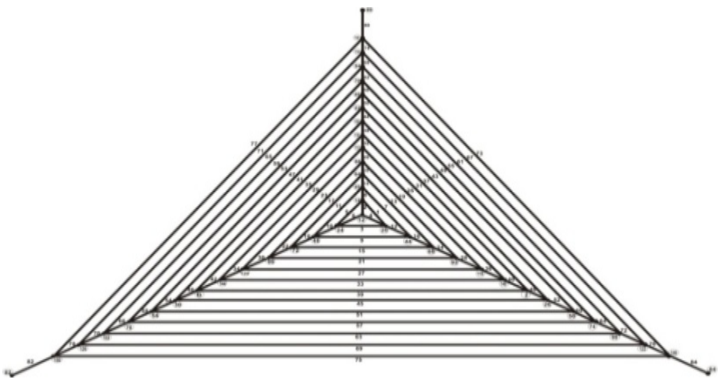


Figure 2.4 – A Spider graph $13C_3 * 3P_{15}$

3. Spider graphs with even vertex graceful labelling

Definition 3.1 **Spider graph with respect to C_5**

A spider graph $nC_5 * 3p_{n+1}$ is a connected graph whose vertex set is $\{v_0, v_1, v_2, v_3, \ldots, v_{5n+5}\}$, and edge set is:

$\{v_0v_1, v_0v_2, v_0v_3, v_0v_4, v_0v_5\} \cup \{v_iv_{i+1} ; i \equiv 1,2,3,4 \pmod 5 ; v_iv_{i-4} ; i \equiv 0 \pmod 5\} \cup \{v_iv_{i+5} ; i = 1 \text{ to } 5n\}$.

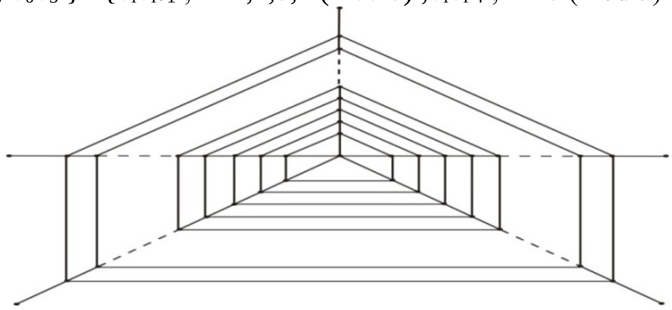


Figure 3.1 - Spider graph $nC_5 * 5P_{n+2}$

Theorem 3.2

Spider graph $nC_5 * 5P_{n+2}$ is even veretx graceful.

Proof: The edge labeling of the given graph $nC_5 * 5P_{n+2}$ is chosen as follows:

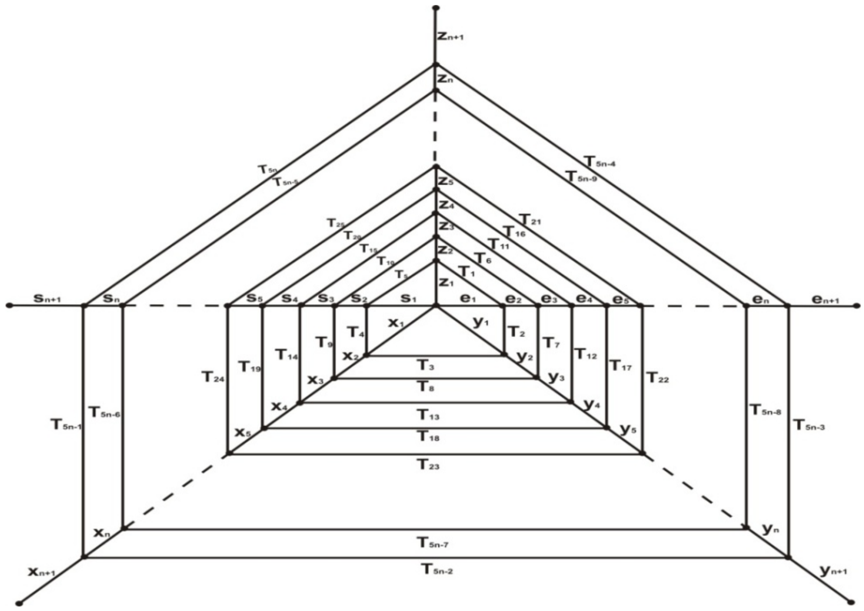


Figure 3.2 – Edge labeling of a Spider graph $nC_5 * 5P_{n+2}$

Definef: $E(nC_5 * 5P_{n+2}) \rightarrow \{1, 2, \ldots, 2q\}$ by

$f(t_i) = 2i-1, \qquad i = 1, 2, 3, \ldots, (5n);$

$f(e_1) = 4; f(e_2) = 20;$

$f(s_1) = 10; f(s_2) = 14;$

$f(x_1) = 8; f(x_2) = 16;$

$f(y_1) = 6; f(y_2) = 18;$

$f(z_1) = 2; f(z_2) = 12;$

$f(e_i) = f(e_1) + (i - 1)10,$

$f(s_i) = f(s_1) + (i - 1)10,$

$f(x_i) = f(x_1) + (i - 1)10,$

$f(y_i) = f(y_1) + (i - 1)10$, where i varies from 3 to (n+1), i is odd.

$f(e_i) = f(e_2) + (i - 2)10,$

$f(s_i) = f(s_2) + (i - 2)10,$

$f(x_i) = f(x_2) + (i - 2)10,$

$f(y_i) = f(y_2) + (i - 2)10$, where i varies from 4 to (n+1),i is even.

n is odd :

$f(z_i) = f(z_1) + (i-1)10$, i varies from 3 to n, i is odd.

$= f(z_2) + (i-2)10$, i varies from 4 to n, i is even.

$f(z_{n+1}) = f(z_n) +10 \quad$ if $n \equiv 1 \pmod 4$

$f(z_{n+1}) = f(z_n) +30 \quad$ if $n \equiv 3 \pmod 4$

n is even : $f(z_3) = 22$; $f(z_4) =32$

$f(z_5) = 72$; $f(z_6) =82$

$f(z_i) = f(z_5) + (i-5)10$, i varies from 7 to n, i is odd.

$= f(z_6) + (i-6)10$, i varies from 8 to n, i is even.

$f(z_{n+1}) = f(z_n) +30$, if $n \neq 10 \pmod{20}$ or $n \neq 0 \pmod{20}$:

$f(z_{n+1}) = f(z_n) + 70$, if $n \equiv 10 \pmod{20}$;

$f(z_{n+1}) = f(z_n) +40$, if $n \equiv 0 \pmod{20}$.

The induced labeling f_+ for all vertices of nC_5*5P_{n+2} is $f^+(v) = (\sum_{vu \in E(G)} f(vu)) \pmod{2q}$ where $v \in V(G)$. The Labeling for vertex and f^+ for all vertices satisfy the conditions of even vertex graceful labeling. The connected graph $nC_5 * 5P_{n+2}$ is an even vertex graceful.

Example 3.3 Spider graph $10C_5 * 5P_{12}$ is even vertex graceful.

Here n = 10.

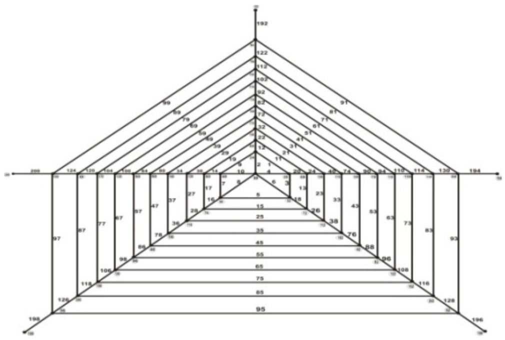
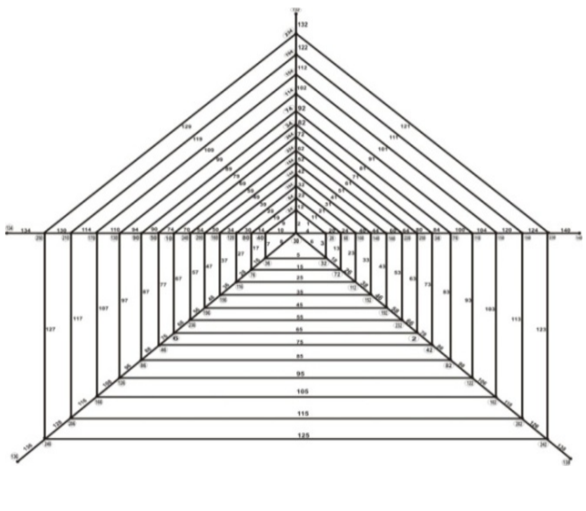


Figure 3.3 – Even vertex labeling of a Spider graph $10C_5 * 5P_{12}$

Example 3.4**:** Spider graph $13C_5 * 5P_{15}$ is even vertex graceful.



Here n = 13.

Figure 3.4 – Even vertex labeling of a Spider graph $13C_5 * 5P_{15}$

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