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FUZZY QUASI- IDEALS IN NEAR- SUBTRACTION SEMIGROUPS

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Abstract: Dheena discussed and derived some properties of near subtraction semigroups. The concept of fuzzy set was first initiated by Zadeh. In this paper, we introduced the notation of fuzzy quasi- ideals in near- subtraction semigroups.

Keywords: Fuzzy sub-near subtraction semigroups, fuzzy two sided X-Subalgebra, fuzzy two-sided ideal, fuzzy bi-ideal and fuzzy quasi- ideal..

1. Introduction

B.M.Schein [12] considered systems of the form (X; o ;/), where X is a set of functions closed under the composition “o” of functions (and hence (X; o) is a function semigroup) and the set theoretic subtraction “/” (and hence (X;/) is a subtraction algebra in the sense of [3]).Y.B.Jun et al [5] introduced the notation of ideals in subtraction algebras and discussed the characterization of ideals. In [5], Y.B.Jun and H.S.Kim established the ideal generated by a set,and discussed related results.The concept of fuzzy set was first initiated by Zadeh [14]. Narayanan et al.[10] defined the concept of generalized fuzzy ideals of near-rings. Mahalakshmi et al. [5] studied the notation of bi-ideals in near-subtraction semigroups. Manikandan [7] studied fuzzy fuzzy bi-ideals in near-rings. Recently, Narayanan and Manikandan [11] studied Interval-valued fuzzy idealsgenerated by an interval-valued fuzzy subset in semi-groups and investigated some of itsproperties.Thillaigovindan and Chinnadurai [13] studied on interval-valued fuzzy quasi-ideals ofsemi-groups and investigated some of its properties. [10] studied interval-valuedfuzzy Quasi-ideals in a semigroups and investigated some of its properties. Murugadas etal. [8] studied interval-valued Q-fuzzy ideals generated by an interval-valued Q-fuzzysubset in ordered semi-groups and investigated some of its properties.

2.Preliminaries

Definition 2.1

A nonempty set X together with two binary operation – and .is called **near subtraction algebra** if it satisfying the following:

(i) x-(y-x) = x
(ii) x-(x-y) = y-(y-x)
(iii) (x-y)-z = (x-z)-y

Definition 2.2

A nonempty set X together with two binary operation – and . is said to be **subtraction semigroup** if it satisfying the following:

(i) (x,-) is a subtraction algebra.
(ii) (x,.) is a semigroup.
(iii) x(y-z) = xy-xz and (x-y)z = xz-yz $\forall$x,y,z$\in$X

Definition 2.3

A near- subtraction semigroup X is called **zero-symmetric,** if x0 = 0, for all x in X.

Definition 2.4

A non empty subset S of a subtraction algebra X is said to be a **subalgebra** of X, if x-y ϵ S. For all x,y ϵ S.

Note: Let X be a near- subtraction semigroup. Given two subsets A and B of X,

A$\circ$B = {ab/a$\in$A, b$\in$B}. Also we define another operation "$*$ "

A$*$B = {ab–a(a'-b) / a',a,$\in$A, b$\in$B}.

Definition 2.5

A function A from a non-empty set X to the unit interval.[0,1] is called a **fuzzy subset of X**.

Definition 2.6

A subalgebra B of X is called **bi-ideal** if BXB$\cap BX * B \subseteq B$. In case of zero symmetric, BXB$\subseteq B$.

Notation: Let A and B be two fuzzy subsets of a semigroup X. We define the relation $\subseteq$between A and B , the union, intersection and product of A and B, respectively as follows:

1. A$\subseteq$B if A(x)≤B (x), for all x∈X,
2. (A$\cup$B) (x) = max{A(x), B(x)}, for all x∈X,
3. (A $\cap$B) (x) = min{A(x), B (x)}, for all x$\in$X.
4. (A$\circ$B) (x) = $\begin{cases} \sup\limits_{x=yz} \min\{A(y), B(z)\}, & \text{if } x=yz \text{ for all } x,y\epsilon X \\ 0, & \text{Otherwise} \end{cases}$
5. (A$*$B) (x) = $\begin{cases} \inf\limits_{x=yz} \max\{A(y), B(z)\}, & \text{if } x=yz \text{ for all } x,y\epsilon X \\ 0, & \text{Otherwise} \end{cases}$

Definition 2.7

A Fuzzy subalgebraA of X is called **fuzzy bi-ideal** of X,if (AXA)$\cap$(AX$*$ A)$\subseteq$A.In case of zero symmetric if AX A$\subseteq$ A.

Definition 2.8

Let (X,-,.) be a near subtraction semigroup.A non-emty subset I of X is called

(i). A **Left ideal** if I is a subalgebra of (X,-) and $xi\text{-}x(y-i)\in I$ for every $x,y\in X,\ i\in I$

(ii). A **right ideal** if I is a subalgebra of (X,-) and $IX\subseteq I$.

(iii). An **Ideal** of X if I is both a left and right.

Definition 2.9

A Fuzzy subset Aof X is called **fuzzy ideal** if it satisfying the following conditions:

(i) A(x-y)≥min{ A(x), A(y)}.

(ii)A(xi-x(y-i) ≥ A(i).

(iii)A(xy) ≥ A(x).,for every x,y∈X.

A fuzzy subset with (i) and (ii) is called a fuzzy left ideal of X,Whereas a fuzzy subset with (i) and (iii) is called a fuzzy right ideal of X.

Definition 2.10

A Fuzzy subset A of X is called **fuzzy X subalgebra** if it satisfying the following conditions:

(i) A is a fuzzy subalgebra of (X,-).

(ii)A(xy) ≥A(x).

(iii)A(xy) ≥A(y),for every x,y ∈X.

A fuzzy subset with (i) and (ii) is called a fuzzy right X-subalgebra of X ,Whereas a fuzzy subset with (i) and (iii) is called a fuzzy left X-subalgebra of X.

3.Fuzzy quasi- ideals in near subtraction semigroups

Defnition 3.1

A fuzzy subalgebra Aof X is called a **fuzzy quasi-ideal** of X if (A∘X) ∩(X∘A) ∩(A∗X)⊆A.In case of zero symmetric of X if (A∘X) ∩(X∘A)⊆A.

Example 3.2 Let X={0,a,b,c} be a near-subtraction semigroup with two binary operations '-'and '•' is defined as follows.

-	0	a	b	c
0	0	0	0	0
A	a	0	a	0
B	b	b	0	0
C	c	b	a	0

∘	0	A	b	c
0	0	0	0	0
a	a	A	b	a
b	0	0	b	b
c	a	A	c	c

Define a fuzzy subset A:X→(0,1) defined by A(0) = 0.2, A(a)= A(b) =0.7,and A(c)=0.3 Then (A∘X) ∩(X∘A)∩(A*X)(0)=0.2, (A∘X) ∩(X∘A)∩(A*X) (a)=0.7, (A∘X)∩(X∘A)∩(A*X) (b)=0.7,(A∘X)∩(X∘A)∩(A*X)(c)=0.3.

Theorem 3.3

Let X be a zero symmetric near subtraction semigroup and if fuzzy subalgebra A of X is a fuzzy quasi -ideal of X then (A∘X ∩(X∘A)⊆A.

Proof: Assume that A is a fuzzy quasi- ideal of X Then (A∘X ∩X∘A) ∩*(X∘A)*∩(A*X)⊆A.Since A is a fuzzy subalgebra of X.A(0)≥ A(x) for all xϵX.we have:

$$(A\circ X)(0) \geq (A\circ X)(x) \text{ for all } x\epsilon X$$

Since X is zero symmetric, (A∘X ∩X∘A) ⊆(A*X)⊆A. Therefore, A∘X ∩X∘A ⊆A .

Theorem 3.4

Every fuzzy quasi – ideal of A is a fuzzy bi-ideal of X.

Proof: Let A be a fuzzy quasi -ideal of X . Now,

$$A(ab) \geq (A \circ X)(ab) \cap (X \circ A)(ab)$$

$$= \sup_{ab=xy} \min\{A(x); X(y)\} \cap \sup_{ab=xy} \min\{X(x), A(y)\} \geq \{A(a) \cap A(b)\}.$$

Therefore, A(ab)≥min {A(a),A(b)}

$$A(axb) \geq (A \circ X)(axb) \cap (X \circ A)(axb)$$

$$= \sup_{axb=yz} \min\{A(y); X(z)\} \cap \sup_{axb=yz} \min\{X(y), A(z)\} \geq \min\{A(a), X(ab)\} \cap \min\{X(ax), A(b)\}$$

$$= A(a) \cap A(b).$$

Therefore A(axb)≥min {A(a),A(b)}.

Note: Every fuzzy quasi - ideal is a fuzzy bi- ideal .but Converse is not true.

Example 3.5

-	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	c	a	0

◦	0	a	B	c
0	0	0	0	0
a	0	b	B	c
b	0	b	0	b
c	0	c	B	a

Let X={0,a,b,c} be a near-subtraction semigroup with two binary operations ‘-’and ‘•’ is defined as follows.

Define a fuzzy subset A:X→(0,1) defined by A(0) = 0.2, A(a)= A(b) =0.7,and A(c)=0.3 Then $(A \circ X)(0) \cap (X \circ A)(0) \nleq A(0) \Rightarrow 0.7 \nleq 0.2$

Theorem 3.6

Let X be a left permutable near subtraction semigroup then every fuzzy quasi- ideal A of X is a fuzzy strong bi-ideal of X.

Proof: Let A be a fuzzy quasi -ideal X. Then by the above theorem A is a fuzzy bi-ideal of X

$$A(axb) \geq (A \circ X)(axb) \cap (X \circ A)(axb)$$

$$= (A \circ X)(xab) \cap (X \circ A)(axb) \text{ (Since X is left permutable)}$$

$$= \sup_{xab=yz} \min\{A(y); X(z)\} \cap \sup_{axb=xy} \min\{X(x), A(y)\}$$

$$\geq \min\{A(x), X(ab)\} \cap \min\{X(ax), A(b)\} = A(x) \cap A(b) = \min\{A(x), A(b)\}$$

Theorem 3.7

Let X be regular near subtraction semigroup A be any fuzzy quasi- ideal of X Then A= (A◦X) ∩(X◦A).

Proof: Let X be regular near subtraction semigroup. Since X is regular,x∈X there exists a∈X Such that x=xax

$$(A \circ X)(x) = \sup_{x=xax=pq} \min\{A(p); X(q)\} \geq \min\{A(x), X(ax)\} = A(x)$$

$$(X \circ A)(x) = \sup_{x=xax=pq} \min\{X(p); A(q)\} \geq \min\{X(ax), A(x)\} = A(x)$$

$$(A \circ X)(x) \cap (X \circ A)(x) \geq A(x) \cap A(x) = A(x)$$

$(A \circ X) \cap (X \circ A) \supseteq A$ and also A is a fuzzy quasi -ideal of X. Then $(A \circ X) \cap (X \circ A) \subseteq A$.

Therefore, A= A◦X∩X◦A.

Theorem 3.8

Let A be a fuzzy near subtraction semigroup of X Then A is a fuzzy quasi-ideal of X if and only if upper level cut U(A;t) of X is a fuzzy quasi-ideal of X, for each t∈ [0,1].

Proof: Assume that A is a fuzzy quasi-ideal of X and U(A;t) is a non empty upper level subset of X. Let x,y∈ U(A;t);Then A(x)≥t and A(y)≥t. Now A(x-y)≥min { A(x), A(y)}≥t. This implies that: x-y∈ U(A; t).Hence U(A;t) is a subalgebra of X.

To prove U(A;t) is a fuzzy quasi-ideal of X . Let a∈U(A;t)◦X∩X◦U(A;t),

$\Rightarrow$a$\in$ U(A;t)$\circ$X and a$\in$ X$\circ$U(A;t) then a=xy and a=wz, where x,z$\in$X and y,w$\in$ U(A;t)
This implies that A(y)$\geq$t and A(w)$\geq$t,
A(a)$\geq$ (A$\circ$X)(a)$\cap$(X$\circ$A)(a) = min $\{\underset{a=pq=wz}{sup}\min\{A(p);X(q)\},\underset{a=p_1q_1=xy}{sup}\min\{X(p_1);A(q_1)\}$
$\geq$min {A(w),A(y)}$\geq$t.

Thus a$\in$U(A;t). Hence U(A;t) $\circ$X$\cap$X$\circ$ U(A;t) $\subseteq$ U(A;t). Thus U(A;t) is a fuzzy quasi-ideal of X.
Conversely , U(A;t) is a fuzzy quasi-ideal of X. Let a$\in$X such that, suppose A(a) $\not\supseteq$(A$\circ$X)(a)$\cap$(X$\circ$A)(a) and that A(a)$\leq t_1 \leq$(A$\circ$X)(a)$\cap$(X$\circ$A)(a) for some $t_1 \in (0,1)$. (A$\circ$X)(a) $\geq t_1$ and (X$\circ$A)(a) $\geq t_1$ but A(a)$< t_1$, $\Rightarrow$a$\notin$ U(A;t_1) which is contradiction . Therefore,

$$A(a)\supseteq (A\circ X)(a)\cap(X\circ A)(a).$$

Hence A is a fuzzy quasi-ideal of X.

Theorem 3.9

Suppose A is a fuzzy quasi- ideal of X then the set A_0 ={a$\in$A/A(a)>0} is a quasi- ideal of X.

Proof: To show that A_0 is a quasi- ideal of X . We need only to show that X$\circ$ $A_0 \cap A_0 \circ$X $\subseteq A_0$
Let a$\in$X$\circ$ $A_0 \cap A_0\circ$X implies that a$\in$X A_0 and a$\in A_0$ X . So a=rx and a=ys for r,s$\in$X and x,y$\in A_0$ then A(x)>0 and A(y)>0. Now, A(a)$\geq$ (A$\circ$X)(a)$\cap$(X$\circ$A)(a). Since:

(A$\circ$X)(a) = $\underset{a=pq}{sup}\min\{A(p),X(q)\}$$\geq$A(y) because a=ys
(X$\circ$A)(a) =$\underset{a=pq}{sup}\min\{X(p),A(q)\}$$\geq$A(x) because a=rx.

Hence, (A$\circ$X)(a) $\geq$A(y) and (X$\circ$A)(a)$\geq$A(x) .
A(a) $\geq$ (A$\circ$X)(a)$\cap$(X$\circ$A)(a)$\geq$A(y)$\cap$ A(x)>0 because A(x)>0 and A(y)>0. Therefore,
$A_0 \circ$X$\cap$ X $\circ A_0 \subseteq A_0$, thus a$\in A_0$. Hence A_0 is a quasi-ideal of X.

Theorem 3.10

Suppose A is a non empty subsetof X, then A is a quasi-ideal of X if and only if K_A the characteristic function of A is a fuzzy quasi- ideal of X.

Proof: Suppose A is a quasi-ideal of X and K_A is the characteristic function of A . Let a$\in$X ,if a$\notin A$. Then a$\notin$XA or a$\notin AX$. Thus (X$\circ K_A$)(a) =0,($K_A \circ$X)(a)=0, and so ($K_A\circ$X)(a)$\cap$ (X$\circ$ K_A)(a) =0= K_A .
If a$\in K_A$, then (K_A)(a) =1 $\geq$ ($K_A \circ$X)(a)$\cap$ (X$\circ$ K_A)(a). Hence K_A is a fuzzy quasi-ideal of X .
Conversely, assume that K_A is a fuzzy quasi-ideal of X, Let a$\in$A$\circ$X$\cap$X$\circ$A. Then there exists y,z$\in$X and b,c$\in$A such that a=by and a=zc . Thus:

($K_A\circ$X)(a) = $\underset{a=pq}{sup}\min\{K_A(p),X(q)\}$ = min $\{K_A(a),X(a)\}$ = min $\{K_A(by),X(by)\}$
$\geq K_A(b)\cap X(y)$ =1$\cap$1 =1

So ($K_A\circ$X)(a) =1. Similarly (X$\circ K_A$)(a)=1. Since $K_A(a) \geq$($K_A\circ$X)(a)$\cap$ (X$\circ$ K_A)(a), thus $K_A(a) = 1$, which is implies that $a\in A$. Hence A$\circ$X$\cap$X$\circ$A $\subseteq$A.
Therefore A is a quasi-ideal of X.

Theorem 3.10

Let A and B be any two fuzzy quasi-ideal of X then A$\cap$B is also a fuzzy quasi-ideal of X.

Proof: (A$\cap$B)(x-y) = min {A(x-y),B(x-y)} $\geq$min {min{A(x),A(y)}, min{B(x),B(y)}}
=min {(A$\cap$B)(x), (A$\cap$B)(y)}.

Let x$\in$Xand choose a,b,c,y,z$\in$X and such that x=ab=zy-z(c-y) then:
(A$\circ$X)(x)$\cap$(X$\circ$A)(x)$\cap$(A*X)(x)= min $\{\underset{x=ab}{sup}\min\{A(a);X(b)\} \cap \underset{x=ab}{sup}\min\{X(a),A(b)\}\cap$
$\underset{x=zy-z(c-y)}{sup}\min\{A(c);X(z)\}$$\leq$A(x)

Thus min $\{\underset{x=ab}{sup}A(a), \underset{x=ab}{sup}A(b), \underset{x=zy-z(c-y)}{sup}A(c)\}$$\leq$A(x).
Similarly, min $\{\underset{x=ab}{sup}B(a), \underset{x=ab}{sup}B(b), \underset{x=zy-z(c-y)}{sup}B(c)\}$$\leq$ B(x)
(((A$\cap$B)$\circ$X)$\cap$(X$\circ$(A$\cap$B))$\cap$(A$\cap$B)*X))(x)=min{((A$\cap$B)$\circ$X)(x),(X$\circ$(A$\cap$B))(x),
(A$\cap$B)*X))(x)}=min$\{\underset{x=ab}{sup}(A\cap B)(a), \underset{x=ab}{sup}(A\cap B)(b), \underset{x=zy-z(c-y)}{sup}(A\cap B)(c)\}$
=min$\{\underset{x=ab}{\sup}\min\{A(a),B(a)\}, \underset{x=ab}{sup}\min\{A(b),B(b)\}, \underset{x=zy-z(c-y)}{sup}\min\{A(c),B(c)\}\}$

$\leq \min\{\min\{\sup_{x=ab} A(a), \sup_{x=ab} A(b), \sup_{x=zy-z(c-y)} A(c)\},$

$\min\{\sup_{x=ab} B(a), \sup_{x=ab} B(b), \sup_{x=zy-z(c-y)} B(c)\}\}$

$\leq \min\{A(x), B(x)\} = (A \cap B)(x).$

So $(A \cap B)$ is a fuzzy quasi-ideal of X.

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