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STRONG EFFICIENT EDGE DOMINATION NUMBER OF SOME SUBDIVISION GRAPHS

M.Annapoopathi[1], N.Meena[2]

[1]Reg.No:17231072092002, Research Scholar, Department of Mathematics,

P.G. & Research Department of Mathematics, The M.D.T. Hindu College,Tirunelveli.

(Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli – 627 012, Tamil Nadu, India)

Assistant Professor, National Engineering College,K.R.Nagar, Kovilpatti,Tamil Nadu, India.annapoopathi.nec@gmail.com

[2]Assistant Professor, P.G. & Research Department of Mathematics,

The M.D.T. Hindu College,Tirunelveli.(Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli – 627 012, Tamil Nadu India

meenamdt@gmail.com

Abstract : Let $G = (V, E)$ be a simple graph. A subset S of E(G) is a strong (weak) efficient edge dominating set of G if $|N_s[e] \cap S| = 1$ for all $e \in E(G)$($|N_w[e] \cap S| = 1$ for all $e \in E(G)$) where $N_s(e) = \{f / f \in E(G)$ and $\deg f \geq \deg e\}$ ($N_w(e) = \{f / f \in E(G)$ and $\deg f \leq \deg e\}$) and $N_s[e] = N_s(e) \cup \{e\}$($N_w[e] = N_w(e) \cup \{e\}$). The minimum cardinality of a strong efficient edge dominating set of G (weak efficient edge dominating set of G) is called a strong efficient edge domination number of G and is denoted by $\gamma'_{se}(G)$ ($\gamma'_{we}(G)$). In this paper, the strong efficient edge domination number of some subdivision graphs is studied.

Keywords: Domination, edge domination, strong edge domination, efficient edge domination, strong efficient edge domination.

AMS subject classification (2010): 05C69

1. Introduction

Throughout this paper, only finite, undirected and simple graphs are considered. Two volumes on domination have been published by T. W. Haynes, S. T. Hedetniemi and P. J. Slater [8, 9]. Edge dominating sets were studied by S. L. Mitchell and S. T. Hedetniemi [11]. A set F of edges in a graph G is called an edge dominating set of G if every edge in E – S is adjacent to at least one edge in F. The edge domination number γ′(G) of a graph G is the minimum cardinality of an edge dominating set of

G. The degree of an edge was introduced by V. R. Kulli [7]. The concept of efficient domination was introduced by D.W. Bange et al [3, 4]. The concept of strong domination graphs was introduced by E. Sampath Kumar and L. Pushpalatha [12] and efficient edge domination were studied by C. L. Lu et al [10] G. Santhosh [13] and D. M. Cardoso et al [5]. The strong efficient edge domination was introduced by M. Annapoopathi and N. Meena [1,2]. For all graph theoretic terminologies and notations, Harary [5] is referred to. The strong (weak) domination number $\gamma'_s(G)$($\gamma'_w(G)$) of G is the minimum cardinality of a strong (weak) dominating set of G and $\Gamma_s(G)$ is the maximum cardinality of a minimal strong dominating set of G. A subset D of E (G) is called an efficient edge dominating set if every edge in E (G) is dominated by exactly one edge in D. The cardinality of the minimum efficient edge dominating set is called the efficient edge domination number of G. In this paper, the strong efficient edge domination numbers of some subdivision related graphs are studied.

Definition 1.1 *[1]*

Let $G = (V, E)$ be a simple graph. A subset S of E(G) is a strong (weak) efficient edge dominating set of G if $|N_s[e] \cap S| = 1$ for all $e \in E(G)[|N_w[e] \cap S| = 1$ for all $e \in E(G)]$ where $N_s(e) = \left\{\frac{f}{f} \in E(G) \text{ and } \deg f \geq \deg e\right\}(N_w(e) = \left\{\frac{f}{f} \in E(G) \text{ and } \deg f \leq \deg e\right\})$ and $N_s[e] = N_s(e) \cup \{e\}(N_w[e] = N_w(e) \cup \{e\})$. The minimum cardinality of a strong efficient edge dominating set of G (weak efficient edge dominating set of G) is called a strong(weak) efficient edge domination number of G and is denoted by $\gamma'_{se}(G)$ $(\gamma'_{we}(G))$.

Note: *[1]* $\gamma'_{se}(C_{3n}) = n, \ \forall n \in N$.

Definition 1.2

If G1 and G2 are graphs and G1 has n vertices then the corona of G1 and G2 denoted by $G_1 \circ G_2$, is the graph obtained by taking one copy of G1 with an edge to every vertex in the i- th copy of G2.

Definition 1.3

The subdivision of an edge e = uv of a graph G is the replacement of the edge e by a path (u, v, w). If every edge of G is subdivided exactly once, then the resulting graph is called the subdivision graph S (G).

2. Strong efficient edge domination number of subdivision Graphs

Theorem 2.1

Let $G = S\left(P_n \circ K_1\right)$ then $\gamma'_{se}(G) = n, n \geq 2$.

Proof: Let $G = S\left(P_n \circ K_1\right)$. Let V (G) = {ui, vi1, vi / 1 ≤ i ≤ n}∪ { xi / 1 ≤ i ≤ n - 1}, E(G) = {ei = ui xi / 1 ≤ i ≤ n- 1}∪ { fi = xiui+1,1 ≤ i ≤ n - 1}∪ { gi = uivi / 1 ≤ i ≤ n }∪ {hi = vi vi1, 1 ≤ i ≤ n}. Deg u1 = Deg un = 2, deg vi = 2, 1 ≤ i ≤ n, deg xi = 2, 1 ≤ i ≤ n – 1, deg vi1 = 1, 1 ≤ i ≤ n, deg ui = 3, 2 ≤ i ≤ n – 1, deg e1 = deg fn - 1 = deg g1 = deg gn = 2, deg hi = 1, 1 ≤ i ≤ n and the remaining edges are of degree 3. S = {g1, g2, g3, g4, g5 ,…, gn } is the unique strong efficient edge dominating set of G and │S│= n, n ≥ 2. Hence $\gamma'_{se}(G) = n, n \geq 2$.

Theorem 2.2

Let $G = S\left(C_n \circ K_1\right)$ then $\gamma'_{se}(G) = n, n \geq 3$.

Proof: Let $G = S\left(C_n \circ K_1\right)$. Let V (G) = {ui, vi1, vi, xi / 1 ≤ i ≤ n}, E(G) = {ei = ui xi / 1 ≤ i ≤ n}∪ { fi = xiui+1,1 ≤ i ≤ n - 1}∪ { fn = xnu1 }∪ { gi = uivi / 1 ≤ i ≤ n }∪ {hi = vi vi1, 1 ≤ i ≤ n}. Deg ui = 3, 1 ≤ i ≤ n, deg xi = deg vi = 2, 1 ≤ i ≤ n, deg vi1 = 1, 1 ≤ i ≤ n, deg ei = deg fi = deg gi = 3, deg hi = 1, 1 ≤ i ≤ n. S = {gi / 1 ≤ i ≤ n} is the unique strong efficient edge dominating set of G and │S│= n, n ≥ 3. Hence $\gamma'_{se}(G) = n, n \geq 3$.

Theorem 2.3

Let $G = S\left(W_n \circ K_1\right)$ then $\gamma'_{se}(G) = n, n \geq 4$.

Proof: Let $G = S\left(W_n \circ K_1\right)$. Let V (G) = {ui, xi, vi1, vi, yi, u, v, w / 1 ≤ i ≤ n - 1}, E(G) = {ei = ui xi / 1 ≤ i ≤ n - 1} ∪ { fi = xiui+1,1 ≤ i ≤ n - 2} ∪ { fn -1= xnu1 }∪ { gi = uivi / 1 ≤ i ≤ n-1 }∪ {hi = vi vi1, 1 ≤ i ≤ n - 1} ∪ { ai = uiyi / 1 ≤ i ≤ n-1 } ∪ {bi = u yi, 1 ≤ i ≤ n - 1}∪ {e = uv, f = vw }. Deg ui = 4, 1 ≤ i ≤ n - 1, deg xi = 2, 1 ≤ i ≤ n - 1, deg vi = 2, deg vi1 = 1, deg yi = 2, 1 ≤ i ≤ n-1, deg u = n, deg v = 2, deg w = 1, deg ei = deg fi = deg gi = 4, deg hi = 1, 1 ≤ i ≤ n – 1, deg ai = 2, deg bi = n, 1 ≤ i ≤ n - 1, deg e = n, deg f = 1. S = {e, gi / 1 ≤ i ≤ n - 1} is the unique strong efficient edge dominating set of G and │S│= n, n ≥ 4. Hence $\gamma'_{se}(G) = n, n \geq 4$.

Theorem 2.4

Let $G = S\left(K_{1,n} \circ K_1\right)$ then $\gamma'_{se}(G) = n+1, n \geq 1$.

Proof: Let $G = S\left(K_{1,n} \circ K_1\right)$. Let V (G) = {u, v, w, ui, xi, vi1, vi / 1 ≤ i ≤ n}, E(G) = {e = uv, f = vw } ∪ {ei = u xi , fi = xiui , gi = uivi, hi = vi vi1, / 1 ≤ i ≤ n}. Deg u = n + 1, deg v = 2, deg w = 1, deg xi = deg ui = deg vi = 2, deg vi1 = 1, 1 ≤ i ≤ n, deg fi = deg gi = 2, deg hi = 1, 1 ≤ i ≤ n, deg ei = n + 1, 1 ≤ i ≤ n, deg e = n + 1. S = {e, gi / 1 ≤ i ≤ n} is the unique strong efficient edge dominating set of G and │S│= n + 1, n ≥ 1. Hence $\gamma'_{se}(G) = n+1, n \geq 1$.

Theorem 2.5:

Let $G = S\left(D_{r,s} \circ K_1\right)$ then $\gamma'_{se}(G) = r + s + 2, \;\; r,\, s \geq 1$.

Proof: Let $G = S\left(D_{r,s} \circ K_1\right)$. Let V (G) = { u, v, w, x1, x2, y1, y2, x1i, u1i, v1i, v1i(1) / 1 ≤ i ≤ r} ∪ { x2ji, u2j, v2j, v2j(1) / 1 ≤ j ≤ s}, E(G) = { e1 = u x1, e2 = x1x2, e = uw, f = wv, f1 = v y1, f2 = y1y2 } ∪ {e1i = u x1i , f1i = x1iu1i, g1i = u1iv1i, h1i = v1i v1i(i), / 1 ≤ i ≤ r} ∪ {e2j = u x2j , f2j = x2ju1j, g2j = u2jv2j, h2j = v2j v2j(j), / 1 ≤ j ≤ s}. Deg u = r + 2, deg v = s + 2 , deg w = 2, deg x1 = deg y1 = 2, deg x2 = deg y2 = 1, deg x1i = deg x2j = deg u1i = deg u2j = deg v1i = deg v2j = 2, deg v1i(1) = deg v2j(1) = 1, 1 ≤ i ≤ r, 1 ≤ j ≤ s, deg e1 = deg e = r + 2, deg f1 = deg f = s + 2, deg e2 = deg f2 = 1, deg e1i = r + 2, deg e2j = s + 2, deg f1i = deg f2j = 2, deg g1i = deg g2j = 2, deg h1i = deg h2j = 1, 1 ≤ i ≤ r, 1 ≤ j ≤ s. S = { e1, f1, g1i, g2j / 1 ≤ i ≤ r, 1 ≤ j ≤ s } is the unique strong efficient edge dominating set of G and │S│= r + s + 2, r, s ≥ 1. Hence $\gamma'_{se}(G) = r + s + 2, \;\; r,\, s \geq 1$.

Theorem 2.6: Let $G = S\left(K_n \circ K_1\right)$ then $\gamma'_{se}(G) = n, \; n \geq 2$.

Proof: Let $G = S\left(K_n \circ K_1\right)$. Let V (G) = {ui, vi, v1(i) / 1 ≤ i ≤ n} ∪ { v1i / 1 ≤ i ≤ n – 1, v2i / 1 ≤ i ≤ n – 2, v3i / 1 ≤ i ≤ n – 3, v4i / 1 ≤ i ≤ n – 4, …, v(n -1) 1 }, E(G) = {fi = vi v1(i), ei = uivi / 1 ≤ i ≤ n } ∪ {e1i = u1 v1i / 1 ≤ i ≤ n – 1} ∪ {e2i = u2 v2i / 1 ≤ i ≤ n – 2} ∪ {e3i = u3 v3i / 1 ≤ i ≤ n – 3} ∪ {e4i = u4 v4i / 1 ≤ i ≤ n – 4} ∪ … ∪ {e(n-1)1 = u(n-1) v(n-1)1 } ∪ {f1i = ui+1 v1i / 1 ≤ i ≤ n – 1} ∪ {f2i = ui+2 v2i / 1 ≤ i ≤ n – 2} ∪ {f3i = ui+3 v3i / 1 ≤ i ≤ n – 3} ∪ {f4i = ui+4 v4i / 1 ≤ i ≤ n – 4} ∪ … ∪ {f(n-1)1 = un v(n-1)1 }. Deg v1(i) = 1, deg vi = 2, deg ui = n, 1 ≤ i ≤ n and the remaining vertices are of degree 2, deg fi = 1, deg ei = n, 1 ≤ i ≤ n and the remaining edges are of degree n. S = {ei / 1 ≤ i ≤ n } is the unique strong efficient edge dominating set of G and │S│= n, n ≥ 1. Hence $\gamma'_{se}(G) = n, \; n \geq 2$.

Theorem 2.7:

Let $G = S(K_{m,n})$ then $\gamma'_{se}(G) = m + (n-1),\ m,\ n \geq 1, m \neq n.$

Proof: Let $G = S(K_{m,n})$, $m,\ n \geq 1, m \neq n$. Without loss of generality, m ≤ n. Let V (G) = {ui, vj, u1(i), v1(j), xij, xi, yj / 1 ≤ i ≤ m, 1 ≤ j ≤ n}, E(G) = {eij = ui xij / 1 ≤ i ≤ m, 1 ≤ j ≤ n } ∪ {fij = vj xij / 1 ≤ i ≤ m, 1 ≤ j ≤ n }. Deg ui = m, deg vj = n, 1 ≤ i ≤ m, 1 ≤ j ≤ n , deg xij = 2, deg eij = m, deg fij = n, 1 ≤ i ≤ m, 1 ≤ j ≤ n. S11 = {e11, e21, e31, ….., em1, f12, f13, f14, ….., f1n }, S12 = {e12, e22, e32, ….., em2, f11, f13, f14, ….., f1n }, S13 = {e13, e23, e33, ….., em3, f11, f12, f14, ….., f1n },…, S1n = {e1n, e2n, e3n, ….., emn, f11, f12, f13, ….., f1(n-1) }, are some strong efficient edge dominating set of $S(K_{m,n})$ and ❘S1j❘= m + (n – 1), 1 ≤ j ≤ n and m, n ≥ 1, $m \neq n$. Hence $\gamma'_{se}(G) \leq m + (n-1),\ m,\ n \geq 1,\ m \neq n$. Suppose T is any strong efficient edge dominating set of $S(K_{m,n})$ with ❘T❘< m + (n – 1). Since deg eij = m = $\Delta'(S(K_{m,n}))$ and any of eij is not adjacent with ekj $i \neq k, 1 \leq i, j \leq n$, at least one of e1j, e2j, e3j, e4j,….., enj, 1 ≤ j ≤ n belong to T. Without loss of generality, let e11, e21, e31, …, em1 belong to T. Also f11 is strongly dominated by e11. To strongly dominate the other edges, at least one of f2j, f3j, f4j,….., fnj, 1 ≤ j ≤ n belong to T and ❘T❘ ≥ m + (n – 1), a contradiction. Hence $\gamma'_{se}(G) = m + (n-1),\ m,\ n \geq 1,\ m \neq n.$

Theorem 2.8

Let $G = S(K_{m,n} \circ K_1)$ then $\gamma'_{se}(G) = m + n,\ m, n \geq 1.$

Proof: Let $G = S(K_{m,n} \circ K_1)$, m, n>1. Let V (G) = {ui, vj, ui1, vj1, xij, xi, yj / 1 ≤ i ≤ m, 1 ≤ j ≤ n}, E(G) = {fi = xi ui1/ 1 ≤ i ≤ m } ∪ {ei = ui xi / 1 ≤ i ≤ m } ∪ {gj = vj yj / 1 ≤ j ≤ n } ∪ {hj = yj vj1/ 1 ≤ j ≤ n } ∪ {eij = ui xij, fij = vj xij / 1 ≤ i ≤ m, 1 ≤ j ≤ n }. Deg ui = n + 1, 1 ≤ i ≤ m, deg vj = m+1, 1 ≤ j ≤ n, deg ui1 = 1, 1 ≤ i ≤ m, deg vj1= 1, 1 ≤ j ≤ n, deg xij = deg xi = deg yj = 2, 1 ≤ i ≤ m, 1 ≤ j ≤ n, deg fi = 1, deg ei = n + 1, deg gj = m + 1, deg hj = 1, 1 ≤ i ≤ m, 1 ≤ j ≤ n, deg eij = n +1, deg fij = m + 1, 1 ≤ i ≤ m, 1 ≤ j ≤ n. S = {ei, gj / 1 ≤ i ≤ m, 1 ≤ j ≤ n} is the unique strong efficient edge dominating set of G and ❘S❘= m + n, m, n ≥ 1. Hence $\gamma'_{se}(G) = m + n,\ m, n \geq 1.$

Theorem 2.9

Let $G = S(P_n \circ K_2)$ then $\gamma'_{se}(G) = 2n, n \geq 2.$

Proof: Let $G = S(P_n \circ K_2)$. Let V (G) = {ui, vi1, vi2, wi1, wi2, wi3 / 1 ≤ i ≤ n}∪ {xi / 1 ≤ i ≤ n - 1}, E(G) = {ei = ui xi, fi = xiui+1,1 ≤ i ≤ n - 1}∪ { gi1 = uiwi1, gi2 = uiwi2, hi1 = wi1vi1, hi2 = wi2vi2, li1 = vi1wi3, li2 = wi3vi2 / 1 ≤ i ≤ n }. Deg u1 = Deg un = 3, deg ui = 4, 2 ≤ i ≤ n – 1 and the remaining vertices have degree 2, deg e1 = deg fn - 1 = deg g11 = deg g12 = deg g1n = deg g2n = 3, deg gi1 = deg gi2 = 4, 2 ≤ i ≤ n – 1, deg ei = 4, 2 ≤ i ≤ n – 1, deg fi = 4, 1 ≤ i ≤ n – 2 and the remaining edges have degree 2. $S_1 = \{g_{i1}, l_{i2}, 1 \leq i \leq n\}, S_2 = \{g_{i2}, l_{i1}, 1 \leq i \leq n\}$ are some strong efficient edge dominating sets of G and ❘S1❘= ❘S2❘= 2n. Therefore $\gamma'_{se}(G) \leq 2n, n \geq 2$. There are n copies of C_6 in G. $\gamma'_{se}(C_6) = 2$. Therefore $\gamma'_{se}(G) \geq 2n,\ n \geq 2$. Hence $\gamma'_{se}(G) = 2n, n \geq 2.$

Theorem 2.10

Let $G = S(C_n \circ K_2)$ then $\gamma'_{se}(G) = 2n, n \geq 3.$

Proof: Let $G = S(C_n \circ K_2)$. Let V (G) = {ui, vi1, vi2, wi1, wi2, wi3, xi / 1 ≤ i ≤ n }, E(G) = {ei = ui xi/ 1 ≤ i ≤ n, fi = xiui+1,1 ≤ i ≤ n – 1, fn = xnu1}∪ { gi1 = uiwi1, gi2 = uiwi2, hi1 = wi1vi1, hi2 = wi2vi2, li1 = vi1wi3, li2 = wi3vi2 / 1 ≤ i ≤ n }. Deg ui = 4, 1 ≤ i ≤ n, and the remaining vertices have degree 2, deg ei = deg fi = deg gi1 = deg gi2 = 4, 1 ≤ i ≤ n and the remaining edges have degree 2.

$S_1 = \left\{ g_{i1}, l_{i2}, 1 \le i \le n \right\}, S_2 = \left\{ g_{i2}, l_{i1}, 1 \le i \le n \right\}$ are some strong efficient edge dominating sets of G and │S1│= │S2│= 2n. Therefore $\gamma'_{se}(G) \le 2n, n \ge 2.$ There are n copies of C_6 in G. $\gamma'_{se}(C_6) = 2$. Therefore $\gamma'_{se}(G) \ge 2n,\ n \ge 2$. Hence $\gamma'_{se}(G) = 2n, n \ge 2.$

Theorem 2.11

Let $G = S\left(W_n \circ K_2\right)$ then $\gamma'_{se}(G) = 2n, n \ge 4.$

Proof: Let $G = S\left(W_n \circ K_2\right),\ n \ge 4.$ Let V (G) = {u, ui, xi, yi / 1 ≤ i ≤ n - 1 }∪ { vi1, vi2, wi1, wi2, wi3 / 1 ≤ i ≤ n }, E(G) = {ei = ui xi/ 1 ≤ i ≤ n - 1}∪ { fi = xiui+1,1 ≤ i ≤ n – 2, fn - 1 = xn - 1u1}∪ { ni = uyi, mi = yi ui, 1 ≤ i ≤ n - 1 }∪ { gi1 = uiwi1, gi2 = uiwi2 / 1 ≤ i ≤ n – 1, gn1 = uwn1, gn2 = uwn2 }∪ { hi1 = wi1vi1, hi2 = wi2vi2, hn1 = wn1vn1, hn2 = wn2vn2} }∪ { li1 = vi1wi3, li2 = wi3vi2, ln1 = vn1wn3, ln2 = vn2wn3}. Deg u = n+1, deg ui = 5, 1 ≤ i ≤ n - 1, and the remaining vertices have degree 2, deg ei = deg fi = deg gi1 = deg gi2 = deg mi = 5, 1 ≤ i ≤ n – 1, deg gn1 = deg gn2 = deg ni = n + 1 and the remaining edges have degree 2. $S_1 = \left\{ g_{i1}, l_{i2}, 1 \le i \le n-1, g_{n1}, l_{n2} \right\}$, $S_2 = \left\{ g_{i2}, l_{i1}, 1 \le i \le n-1, g_{n2}, l_{n1} \right\}$ are some strong efficient edge dominating sets of G and │S1│= │S2│= 2n. Therefore $\gamma'_{se}(G) \le 2n, n \ge 4.$ There are n copies of C_6 in G. $\gamma'_{se}(C_6) = 2$. Therefore $\gamma'_{se}(G) \ge 2n,\ n \ge 4$. Hence $\gamma'_{se}(G) = 2n, n \ge 4.$

Theorem 2.12

Let $G = S\left(K_{1,n} \circ K_2\right)$ then $\gamma'_{se}(G) = 2(n+1), n \ge 1.$

Proof: Let $G = S\left(K_{1,n} \circ K_2\right),\ n \ge 1.$ Let V (G) = {u, ui, xi /1 ≤ i ≤ n }∪ {vi1, vi2, wi1, wi2, wi3 /1 ≤ i ≤ n + 1 }, E(G) = {ei = ui xi/ 1 ≤ i ≤ n}∪ { fi = uxi,1 ≤ i ≤ n}∪ { gi1 = uiwi1, gi2 = uiwi2 / 1 ≤ i ≤ n, g(n+1)1 = uw(n+1)1, g(n+1)2 = uw(n+1)2 }∪ { hi1 = wi1vi1, hi2 = wi2vi2, li1 = vi1wi3, li2 = wi3vi2 / 1 ≤ i ≤ n+1 }. Deg u = n+2, deg ui = 3, 1 ≤ i ≤ n, and the remaining vertices have degree 2, deg ei = 3, 1 ≤ i ≤ n, deg fi = n+2, 1 ≤ i ≤ n, deg gi1 = deg gi2 = 3, 1 ≤ i ≤ n, deg g(n+1)1 = deg g(n+1)2 = n + 2 and the remaining edges have degree 2. $S_1 = \left\{ g_{i1}, l_{i2}, 1 \le i \le n, g_{(n+1)1}, l_{(n+1)2} \right\}$, $S_2 = \left\{ g_{i2}, l_{i1}, 1 \le i \le n\ , g_{(n+1)2}, l_{(n+1)1} \right\}$ are some strong efficient edge dominating sets of G and │S1│= │S2│= 2(n+1). Therefore $\gamma'_{se}(G) \le 2(n+1), n \ge 1.$ There are n+1 copies of C_6 in G. $\gamma'_{se}(C_6) = 2$. Therefore $\gamma'_{se}(G) \ge 2(n+1),\ n \ge 1$. Hence $\gamma'_{se}(G) = 2(n+1), n \ge 1.$

Theorem 2.13

Let $G = S\left(D_{r,s} \circ K_2\right)$ then $\gamma'_{se}(G) = 2(r+s+2),\ r, s \ge 1.$

Proof: Let $G = S\left(D_{r,s} \circ K_2\right),\ r, s \ge 1.$ Let V (G) = {u, v, w, ui, vj, xi, yj / 1 ≤ i ≤ r, 1 ≤ j ≤ s } ∪ {w11(i), w12(i), w13(i), 1 ≤ i ≤ r + 1, w21(j), w22(j), w23(j) /1 ≤ j ≤ s + 1 } }∪ { v11(i), v12(i), 1 ≤ i ≤ r + 1, v21(j), v22(j) / 1 ≤ j ≤ s + 1 }, E(G) = {e = uw, f = vw, f1i = uxi, e1i = ui xi / 1 ≤ i ≤ r}∪ { f2j = vyj, e2j = yj vj, 1 ≤ j ≤ s}∪ {g11(i) = uiw11(i), g12(i) = uiw12(i), g21(j) = vjw21(j), g22(j) = vjw22(j)/ 1 ≤ i ≤ r, 1 ≤ j ≤ s, g11(r+1) = uw11(r+1), g12(r+1) = uw12(r+1), g21(s+1) = vw21(s+1), g22(s+1) = vw22(s+1)}∪ { h11(i) = w11(i) v11(i), h12(i) = w12(i) v12(i), l11(i) = v11(i) w13(i), l12(i) = v12(i) w13(i) , 1 ≤ i ≤ r + 1, h21(j) = w21(j) v21(j), h22(j) = w22(j) v22(j), l21(j) = v21(j) w23(j), l22(j) = v22(j) w23(j) , 1 ≤ j ≤ s + 1}. Deg u = r + 3, deg v = s + 3, deg w = 2, deg ui = deg vj = 3, 1 ≤ i ≤ r, 1 ≤ j ≤ s and the remaining vertices have degree 2, deg e = deg g11(r+1) = g12(r+1) = deg f1i = r + 3, 1 ≤ i ≤ r, deg f = deg g21(s+1) = g22(s+1) = deg f2j = s + 3, 1 ≤ j ≤ s, deg g11(i) = g12(i) = deg g21(j) = g22(j) = 3, 1 ≤ i ≤ r, 1 ≤ j ≤ s and the remaining edges have degree 2.

$$S_1 = \left\{ g_{11}{}^{(i)}, g_{21}{}^{(j)}, l_{12}{}^{(i)}, l_{22}{}^{(j)}, 1 \le i \le r, 1 \le j \le s, g_{11}{}^{(r+1)}, g_{21}{}^{(s+1)} \right\},$$
$$S_2 = \left\{ g_{12}{}^{(i)}, g_{22}{}^{(j)}, l_{11}{}^{(i)}, l_{21}{}^{(j)}, 1 \le i \le r, 1 \le j \le s, g_{12}{}^{(r+1)}, g_{22}{}^{(s+1)} \right\}$$

are some strong efficient edge dominating sets of G and ∣S1∣=∣S2∣=2(r+s+2). Therefore $\gamma'_{se}(G) \le 2(r+s+2), r,s \ge 1$. There are r + s + 2 copies of C_6 in G. $\gamma'_{se}(C_6) = 2$. Therefore $\gamma'_{se}(G) \ge 2(r+s+2),\ r,s \ge 1$. Hence $\gamma'_{se}(G) = 2(r+s+2), r,s \ge 1$.

Theorem 2.14

Let $G = S\left(K_n \circ K_2\right)$ then $\gamma'_{se}(G) = 2n, n \ge 2$.

Proof: Let $G = S\left(K_n \circ K_2\right),\ n \ge 2$. Let V (G) = {ui, vi1, vi2, wi1, wi2, wi3 / 1 ≤ i ≤ n }∪ { v1i / 1 ≤ i ≤ n – 1, v2i / 1 ≤ i ≤ n – 2, v3i / 1 ≤ i ≤ n – 3, v4i / 1 ≤ i ≤ n – 4, …, v(n -1) 1 }, E(G) = {e1i = u1 v1i / 1 ≤ i ≤ n – 1} ∪ {e2i = u2 v2i / 1 ≤ i ≤ n – 2} ∪ {e3i = u3 v3i / 1 ≤ i ≤ n – 3} ∪ {e4i = u4 v4i / 1 ≤ i ≤ n – 4} ∪ … ∪ {e(n-1)1 = u(n-1) v(n-1)1 } ∪ {f1i = ui+1 v1i / 1 ≤ i ≤ n – 1} ∪ {f2i = ui+2 v2i / 1 ≤ i ≤ n – 2} ∪ {f3i = ui+3 v3i / 1 ≤ i ≤ n – 3} ∪ {f4i = ui+4 v4i / 1 ≤ i ≤ n – 4} ∪ … ∪ {f(n-1)1 = un v(n-1)1 }∪ { gi1 = uiwi1, gi2 = uiwi2, hi1 = wi1vi1, hi2 = wi2vi2, li1 = vi1wi3, li2 = wi3vi2 / 1 ≤ i ≤ n }. Deg ui = n+1, 1 ≤ i ≤ n and the remaining vertices have degree 2, deg li1 = deg li2 = deg hi1 = deg hi2 = 2, 1 ≤ i ≤ n and the remaining edges have degree n + 1. $S_1 = \left\{g_{i1}, l_{i2}, 1 \le i \le n\right\}$, $S_2 = \left\{g_{i2}, l_{i1}, 1 \le i \le n\right\}$ are some strong efficient edge dominating sets of G and ∣S1∣= ∣S2∣= 2n. Therefore $\gamma'_{se}(G) \le 2n, n \ge 2$. There are n copies of C_6 in G. $\gamma'_{se}(C_6) = 2$. Therefore $\gamma'_{se}(G) \ge 2n,\ n \ge 2$. Hence $\gamma'_{se}(G) = 2n, n \ge 2$.

Theorem 2.15

Let $G = S\left(K_{m,n} \circ K_2\right)$ then $\gamma'_{se}(G) = 2(m+n),\ m,n \ge 1$.

Proof: Let $G = S\left(K_{m,n} \circ K_2\right),\ m,n \ge 1$. Let V (G) = {ui, vj / 1 ≤ i ≤ m, 1 ≤ j ≤ n } ∪ {v11(i), v12(i), 1 ≤ i ≤ m, v21(j), v22(j) / 1 ≤ j ≤ n, w11(i), w12(i), w13(i), w21(j), w22(j), w23(j) , xij / 1 ≤ i ≤ m, 1 ≤ j ≤ n }, E(G) = { eij = ui xij , fij = xij vj / 1 ≤ i ≤ m, 1 ≤ j ≤ n }∪ {g11(i) = uiw11(i), g12(i) = uiw12(i), g21(j) = vjw21(j), g22(j) = vjw22(j), h11(i) = w11(i) v11(i), h12(i) = w12(i) v12(i), h21(j) = w21(j) v21(j), h22(j) = w22(j) v22(j), l11(i) = v11(i) w13(i), l12(i) = v12(i) w13(i) , l21(j) = v21(j) w23(j), l22(j) = v22(j) w23(j) , 1 ≤ i ≤ m, 1 ≤ j ≤ n }. Deg ui = n+2, deg vj = m + 2, 1 ≤ i ≤ m, 1 ≤ j ≤ n and the remaining vertices have degree 2, deg eij = deg g11(i) = g12(i) = n + 2, 1 ≤ i ≤ m, 1 ≤ j ≤ n, deg fij = deg g21(j) = g22(j) = m + 2, 1 ≤ i ≤ m, 1 ≤ j ≤ n and the remaining edges have degree 2.

$$S_1 = \left\{ g_{11}{}^{(i)}, g_{21}{}^{(j)}, l_{12}{}^{(i)}, l_{22}{}^{(j)}, 1 \le i \le m, 1 \le j \le n \right\},$$
$$S_2 = \left\{ g_{12}{}^{(i)}, g_{22}{}^{(j)}, l_{11}{}^{(i)}, l_{21}{}^{(j)}, 1 \le i \le m, 1 \le j \le n \right\}$$

are some strong efficient edge dominating sets of G and ∣S1∣= ∣S2∣= 2(m + n). Therefore $\gamma'_{se}(G) \le 2(m+n), m,n \ge 1$. There are m + n copies of C_6 in G. $\gamma'_{se}(C_6) = 2$. Therefore $\gamma'_{se}(G) \ge 2(m+n),\ m,n \ge 1$. Hence $\gamma'_{se}(G) = 2(m+n), m,n \ge 1$.

Theorem 2.16

Let $G = S\left(P_n \circ K_3\right)$ then $\gamma'_{se}(G) = 3n, n \ge 2$.

Proof: Let $G = S\left(P_n \circ K_3\right),\ n \ge 2$. Let V (G) = {ui, vik, wik, yik / 1 ≤ i ≤ n, 1 ≤ k ≤ 3 }∪ {xi / 1 ≤ i ≤ n - 1}, E(G) = {ei = ui xi, fi = xiui+1,1 ≤ i ≤ n - 1}∪{ eik = ui yik / 1 ≤ i ≤ n, 1 ≤ k ≤ 3 }∪ { fi1 = yi1vi3, fi2 = yi2vi1, fi3 = yi3vi2 / 1 ≤ i ≤ n }∪ { gi1 = vi1wi1, gi2 = wi1vi3, hi1 = vi1wi2, hi2 = wi2vi2, li1 = vi3wi3, li2 = wi3vi2 / 1 ≤ i ≤ n }. Deg u1 = deg un = 4, deg ui = 5, 2 ≤ i ≤ n – 1, deg vik = 3, 1 ≤ i ≤ n, 1 ≤ k ≤ 3 and the remaining vertices have degree 2, deg e1 = deg en - 1 = 4, deg ei = 5, 2 ≤ i ≤ n - 2,deg fi = 5, 1 ≤ i ≤ n – 2, deg fn - 1 = 4, deg e1k = deg enk = 4, 1 ≤ k ≤ 3, deg eik = 5, 2 ≤ i ≤ n - 1, 1 ≤ k ≤ 3 and the remaining edges have degree 2. $S_1 = \{e_{i1},\ g_{i1}, l_{i2}, 1 \le i \le n\}$, $S_2 = \{e_{i2}, g_{i2}, h_{i2}, 1 \le i \le n\}$, $S_3 = \{e_{i3}, h_{i1}, l_{i1}, 1 \le i \le n\}$ are some strong efficient edge dominating sets of G and │S1│=│S2│= │S3│= 3n. Therefore $\gamma'_{se}(G) \le 3n,\ n \ge 2.$ Any strong efficient edge dominating set S contains either the edge e1 or en-1 or both. Suppose the edge $e_j,\ j \ne 1, n-1$ belongs to S. The edge ej strongly efficiently dominate $f_j, f_{j-1}, e_{j1}, e_{j2}, e_{j3}$. Then f_{j+1} belongs to S or e_{j+2} belongs to S. Also one of the edges $e_{(j+1)1}, e_{(j+1)2}, e_{(j+1)3}$ belongs to S. Then f_j and $e_{(j+1)}$ are strongly dominated by two edges, a contradiction. Hence the edges $e_j, f_j, 2 \le j \le n-2$ do not belong to S. Therefore any one of the edges $e_{1j}, e_{2j}, ..., e_{nj}, 1 \le j \le 3$ must belong to S. Without loss of generality, let it be $e_{11}, e_{21}, ..., e_{n1}$ belong to S. Also there are n copies of C6 exists in G and $\gamma'_{se}(C_6) = 2$. Therefore $\gamma'_{se}(G) \ge 3n,\ n \ge 2$. Hence $\gamma'_{se}(G) = 3n, n \ge 2.$

Theorem 2.17

Let $G = S(C_n \circ K_3)$ then $\gamma'_{se}(G) = 3n, n \ge 3.$

Proof: Let $G = S(C_n \circ K_3),\ n \ge 3.$ Let V (G) = {ui, xi, vik, wik, yik / 1 ≤ i ≤ n, 1 ≤ k ≤ 3 }, E(G) = {ei = ui xi, 1 ≤ i ≤ n} ∪ {fi = xiui+1,1 ≤ i ≤ n – 1, fn = xnu1}∪{ eik = ui yik / 1 ≤ i ≤ n, 1 ≤ k ≤ 3 }∪ { fi1 = yi1vi3, fi2 = yi2vi1, fi3 = yi3vi2 / 1 ≤ i ≤ n }∪ { gi1 = vi1wi1, gi2 = wi1vi3, hi1 = vi1wi2, hi2 = wi2vi2, li1 = vi3wi3, li2 = wi3vi2 / 1 ≤ i ≤ n }. Deg ui = 5, deg vik = 3, 1 ≤ i ≤ n, 1 ≤ k ≤ 3 and the remaining vertices have degree 2, deg ei = deg fi = deg eik = 5, 1 ≤ i ≤ n, 1 ≤ k ≤ 3 and the remaining edges have degree 2. $S_1 = \{e_{i1},\ g_{i1}, l_{i2}, 1 \le i \le n\}$, $S_2 = \{e_{i2}, g_{i2}, h_{i2}, 1 \le i \le n\}$, $S_3 = \{e_{i3}, h_{i1}, l_{i1}, 1 \le i \le n\}$ are some strong efficient edge dominating sets of G and │S1│= S2│= │S3│= 3n. Therefore $\gamma'_{se}(G) \le 3n,\ n \ge 3.$ Any strong efficient edge dominating set S contains either the edge e1 or en-1 or both. Suppose the edge $e_j,\ j \ne 1, n-1$ belongs to S. The edge ej strongly efficiently dominate $f_j, f_{j-1}, e_{j1}, e_{j2}, e_{j3}$. Then f_{j+1} belongs to S or e_{j+2} belongs to S. Also one of the edges $e_{(j+1)1}, e_{(j+1)2}, e_{(j+1)3}$ belongs to S. Then f_j and $e_{(j+1)}$ are strongly dominated by two edges, a contradiction. Hence the edges $e_j, f_j, 2 \le j \le n-2$ do not belong to S. Therefore any one of the edges $e_{1j}, e_{2j}, ..., e_{nj}, 1 \le j \le 3$ must belong to S. Without loss of generality, let it be $e_{11}, e_{21}, ..., e_{n1}$ belong to S. Also there are n copies of C6 exists in G and $\gamma'_{se}(C_6) = 2$. Therefore $\gamma'_{se}(G) \ge 3n,\ n \ge 3$. Hence $\gamma'_{se}(G) = 3n, n \ge 3.$

Theorem 2.18

Let $G = S(W_n \circ K_3)$ then $\gamma'_{se}(G) = 3n, n \ge 4.$

Proof: Let $G = S(W_n \circ K_3),\ n \ge 4.$ Let V (G) = {u, ui, xi, yi, vik, wik, yik / 1 ≤ i ≤ n - 1, 1 ≤ k ≤ 3 }, E(G) = {ei = ui xi, 1 ≤ i ≤ n - 1} ∪ {fi = xiui+1,1 ≤ i ≤ n – 2, fn-1 = xn-1u1}∪{ eik = ui yik / 1 ≤ i ≤ n - 1, 1 ≤ k ≤ 3, enk = u ynk }∪ { fi1 = yi1vi3, fi2 = yi2vi1, fi3 = yi3vi2 / 1 ≤ i ≤ n }∪ { gi1 = vi1wi1, gi2 = wi1vi3, hi1 = vi1wi2, hi2 = wi2vi2, li1 = vi3wi3, li2 = wi3vi2 / 1 ≤ i ≤ n }. Deg u = n+2, deg ui = 5, 1 ≤ i ≤ n – 1, deg vik = 3, 1 ≤ i ≤ n, 1 ≤ k ≤ 3 and the remaining vertices have degree 2, deg ei = deg fi = 5, 1 ≤ i ≤ n-1, deg gi = deg eik = 5, 1 ≤ i ≤ n - 1, 1 ≤ k ≤ 3, deg hi = n+2, 1 ≤ i ≤ n-1, deg enk = n+2, 1 ≤ k ≤ 3 and the remaining edges have degree 3. $S_1 = \{e_{i1},\ g_{i1}, l_{i2}, 1 \le i \le n\}$, $S_2 = \{e_{i2}, g_{i2}, h_{i2}, 1 \le i \le n\}$, $S_3 = \{e_{i3}, h_{i1}, l_{i1}, 1 \le i \le n\}$ are some strong efficient edge dominating sets of G and │S1│=│S2│= │S3│= 3n. Therefore $\gamma'_{se}(G) \le 3n,\ n \ge 4.$ Any strong efficient edge dominating set S contains either the edge e1 or en-1 or both. Suppose the edge $e_j, j \neq 1, n-1$ belongs to S. The edge ej strongly efficiently dominate $f_j, f_{j-1}, e_{j1}, e_{j2}, e_{j3}$. Then f_{j+1} belongs to S or e_{j+2} belongs to S. Also one of the edges $e_{(j+1)1}, e_{(j+1)2}, e_{(j+1)3}$ belongs to S. Then f_j and $e_{(j+1)}$ are strongly dominated by two edges, a contradiction. Hence the edges $e_j, f_j, 2 \leq j \leq n-2$ do not belong to S. Therefore any one of the edges $e_{1j}, e_{2j}, ..., e_{nj}, 1 \leq j \leq 3$ must belong to S. Without loss of generality, let it be $e_{11}, e_{21}, ..., e_{n1}$ belong to S. Also there are n copies of C6 exists in G and $\gamma_{se}^{'}(C_6) = 2$. Therefore $\gamma_{se}^{'}(G) \geq 3n,\ n \geq 4$. Hence $\gamma_{se}^{'}(G) = 3n, n \geq 4.$

Theorem 2.19

Let $G = S\left(K_{1,n} \circ K_3\right)$ then $\gamma_{se}^{'}(G) = 3(n+1), n \geq 1.$

Proof: Let $G = S\left(K_{1,n} \circ K_3\right),\ n \geq 1.$ Let V (G) = {u, ui, xi / 1 ≤ i ≤ n }∪ {vik, wik, yik / 1 ≤ i ≤ n + 1, 1 ≤ k ≤ 3 }, E(G) = {ei = u xi, 1 ≤ i ≤ n} ∪ {fi = xiui,1 ≤ i ≤ n}∪{ eik = ui yik / 1 ≤ i ≤ n, 1 ≤ k ≤ 3, e(n+1)k = u y(n+1)k / 1 ≤ k ≤ 3}∪ { fi1 = yi1vi3, fi2 = yi2vi1, fi3 = yi3vi2 / 1 ≤ i ≤ n+1 }∪ {gi1 = vi1wi1, gi2 = wi1vi3, hi1 = vi1wi2, hi2 = wi2vi2, li1 = vi3wi3, li2 = wi3vi2 / 1 ≤ i ≤ n+1 }. Deg u = n+3, deg ui = 4, 1 ≤ i ≤ n, deg vik = 3, 1 ≤ i ≤ n+1, 1 ≤ k ≤ 3 and the remaining vertices have degree 2, deg ei = n+3, deg fi = 4, 1 ≤ i ≤ n, deg eik = 4, 1 ≤ i ≤ n + 1, 1 ≤ k ≤ 3 and the remaining edges have degree 3. $S_1 = \left\{e_{i1},\ g_{i1}, l_{i2}, 1 \leq i \leq n\right\}$, $S_2 = \left\{e_{i2}, g_{i2}, h_{i2}, 1 \leq i \leq n\right\}$, $S_3 = \left\{e_{i3}, h_{i1}, l_{i1}, 1 \leq i \leq n\right\}$ are some strong efficient edge dominating sets of G and │S1│=│S2│= │S3│= 3(n+1). Therefore $\gamma_{se}^{'}(G) \leq 3(n+1),\ n \geq 1.$ Any strong efficient edge dominating set S contain the edge $e_i, 1 \leq i \leq n$. Without loss of generality, let it be e1. The edge e1 strongly dominates $e_i,\ 2 \leq i \leq n,\ f_1, e_{(n+1)1}, e_{(n+1)2}, e_{(n+1)3}$. Also any one of e_{11}, e_{12}, e_{13} belongs to S. Suppose the edge e_{11} belongs to S. Then $\left|N_S\left[f_1\right] \cap S\right| = \left|\left\{e_1, e_{11}\right\}\right| = 2 > 1$, a contradiction. Suppose the edge e_{12} belongs to S. Then $\left|N_S\left[f_1\right] \cap S\right| = \left|\left\{e_1, e_{12}\right\}\right| = 2 > 1$, a contradiction. Suppose the edge e_{13} belongs to S. Then $\left|N_S\left[f_1\right] \cap S\right| = \left|\left\{e_1, e_{13}\right\}\right| = 2 > 1$, a contradiction. Therefore the edge e1 does not belong to S. Therefore any one of the edges $e_{1j}, e_{2j}, ..., e_{(n+1)j},\ 1 \leq j \leq 3$ must belong to S. Without loss of generality, let it be $e_{11}, e_{21}, ..., e_{n1}, e_{(n+1)1}$ belong to S. Also there are n copies of C6 exists in G and $\gamma_{se}^{'}(C_6) = 2$. Therefore $\gamma_{se}^{'}(G) \geq 3(n+1),\ n \geq 1.$ Hence $\gamma_{se}^{'}(G) = 3(n+1), n \geq 1.$

Theorem 2.20

Let $G = S\left(D_{r,s} \circ K_3\right)$ then $\gamma_{se}^{'}(G) = 3(r+s+2), r, s \geq 1.$

Proof: Let $G = S\left(D_{r,s} \circ K_3\right),\ r, s \geq 1.$ Let V (G) = {u, v, w, ui, xi / 1 ≤ i ≤ r }∪ {xik, uik, wik / 1 ≤ i ≤ r + 1, 1 ≤ k ≤ 3} ∪ {yj, vj / 1 ≤ j ≤ s }∪ {yjk, vjk, zjk / 1 ≤ j ≤ s + 1, 1 ≤ k ≤ 3 }, E(G) = {e = uw, f = wv, ei = u xi, fi = xiui, 1 ≤ i ≤ r} ∪{ eik = ui xik / 1 ≤ i ≤ r, 1 ≤ k ≤ 3, e(r+1)k = u x(r+1)k / 1 ≤ k ≤ 3}∪ { fi1 = xi1ui3, fi2 = xi2ui1, fi3 = xi3ui2 / 1 ≤ i ≤ r+1 }∪ {gi1 = ui1wi1, gi2 = wi1ui3, hi1 = ui1wi2, hi2 = wi2ui2, li1 = ui3wi3, li2 = wi3ui2 / 1 ≤ i ≤ r+1} ∪ {gj = v yj, hj = yjvj, 1 ≤ j ≤ s} ∪{ ajk = vj yjk / 1 ≤ j ≤ s, 1 ≤ k ≤ 3, a(s+1)k = v y(s+1)k / 1 ≤ k ≤ 3}∪ { bj1 = yj1vj3, bj2 = yj2vj1, bj3 = yj3vj2 / 1 ≤ j ≤ s+1 }∪ {cj1 = vj1zj1, cj2 = vj1zj3, dj1 = vj1zj2, dj2 = zj2vj2, mj1 = vj3zj3, mj2 = zj3vj2 / 1 ≤ j ≤ s+1}. Deg u = r+4, deg v = s+4, deg w = 2, deg ui = deg vj = 4, 1 ≤ i ≤ r, 1 ≤ j ≤ s, deg uik = deg vjk = 3, 1 ≤ i ≤ r, 1 ≤ j ≤ s, 1 ≤ k ≤ 3 and the remaining vertices have degree 2, deg e = r+4, deg f = s+4, deg ei = r+4, deg fi = 4, 1 ≤ i ≤ r, deg eik = 4, 1 ≤ i ≤ r, 1 ≤ k ≤ 3, deg e(r+1)k = r+4, 1 ≤ k ≤ 3, deg gj = s+4, deg hj = 4, 1 ≤ j ≤ s, deg ajk = 4, 1 ≤ j ≤ s, 1 ≤ k ≤ 3, deg a(s+1)k = s+4, 1 ≤ k ≤ 3 and the remaining edges have degree 3.

$$S_1=\left\{e_{i1},\, h_{i1},l_{i1},1\le i\le r+1,a_{j1},\, d_{j1},m_{j1},1\le j\le s+1\right\},$$
$$S_2=\left\{e_{i2},g_{i2},h_{i2},1\le i\le r+1,a_{j2},\, d_{j2},c_{j2},1\le j\le s+1\right\},$$
$$S_3=\left\{e_{i3},g_{i1},l_{i2},\,1\le i\le r+1,a_{j3},\, c_{j1},m_{j2},1\le j\le s+1\right\}$$
are some strong efficient edge dominating sets of G and │S1│= S2│= │S3│= 3(r+s+2). Therefore $\gamma'_{se}(G)\le 3(r+s+2),\ r,s\ge 1.$ Also no set with less than 3(r+s+2) edges is a strong efficient edge dominating set of G. Therefore $\gamma'_{se}(G)\ge 3(r+s+2),\ r,s\ge 1$. Hence $\gamma'_{se}(G)=3(r+s+2),r,s\ge 1.$

Theorem 2.21

Let $G=S\left(K_n\circ K_3\right)$ then $\gamma'_{se}(G)=3n,n\ge 2.$

Proof: Let $G=S\left(K_n\circ K_3\right),\ n\ge 2.$ Let V (G) = {ui / 1 ≤ i ≤ n }∪ { v1i / 1 ≤ i ≤ n – 1, v2i / 1 ≤ i ≤ n – 2, v3i / 1 ≤ i ≤ n – 3, v4i / 1 ≤ i ≤ n – 4, …, v(n -1)1} ∪ {yik, xik, wik / 1 ≤ i ≤ n, 1 ≤ k ≤ 3}, E(G) = {e1i = u1 v1i / 1 ≤ i ≤ n – 1} ∪ {e2i = u2 v2i / 1 ≤ i ≤ n – 2} ∪ {e3i = u3 v3i / 1 ≤ i ≤ n – 3} ∪ {e4i = u4 v4i / 1 ≤ i ≤ n – 4} ∪ … ∪ {e(n-1)1 = u(n-1) v(n-1)1 } ∪ {f1i = ui+1 v1i / 1 ≤ i ≤ n – 1} ∪ {f2i = ui+2 v2i / 1 ≤ i ≤ n – 2} ∪ {f3i = ui+3 v3i / 1 ≤ i ≤ n – 3} ∪ {f4i = ui+4 v4i / 1 ≤ i ≤ n – 4} ∪ … ∪ {f(n-1)1 = un v(n-1)1 }∪ {mik = ui yik / 1 ≤ i ≤ n, 1 ≤ k ≤ 3 } ∪ {ni1 = yi1 xi3, ni2 = yi2 xi1 , ni3 = yi1 xi2 / 1 ≤ i ≤ n} ∪ { gi1 = xi1wi1, gi2 = xi2wi2, hi1 = xi1wi2, hi2 = wi2xi3, li1 = xi2wi3, li2 = wi3xi3 / 1 ≤ i ≤ n }. Deg ui = n+2, deg xik= 3, 1 ≤ i ≤ n , 1 ≤ k ≤ 3 and the remaining vertices have degree 2, deg nik= 3, 1 ≤ i ≤ n , 1 ≤ k ≤ 3, deg gi1 = deg gi2 = deg hi1 = deg hi2 = deg li1 = deg li2 = 3, 1 ≤ i ≤ n and the remaining edges have degree n + 2. $S_1=\left\{m_{i1},g_{i2},h_{i1},\,1\le i\le n\right\},S_2=\left\{m_{i2},g_{i2},h_{i2},1\le i\le n\right\},S_3=\left\{m_{i3},g_{i1},h_{i2},1\le i\le n\right\}$ are some strong efficient edge dominating sets of G and │S1│=│S2│= │S3│= 3n. Therefore $\gamma'_{se}(G)\le 3n,n\ge 2.$ Also no set with less than 3n edges is a strong efficient edge dominating set of G. Therefore $\gamma'_{se}(G)\ge 3n,\ n\ge 2$. Hence $\gamma'_{se}(G)=3n,n\ge 2.$

Theorem 2.22

Let $G=S\left(K_{m,n}\circ K_3\right)$ then $\gamma'_{se}(G)=3(m+n),\ m,n\ge 1.$

Proof: Let $G=S\left(K_{m,n}\circ K_3\right),\ m,n\ge 1.$ Let V (G) = {ui, vj, xij / 1 ≤ i ≤ m, 1 ≤ j ≤ n} ∪ $\left\{x_{1k}^{(i)},y_{1k}^{(j)},u_{1k}^{(i)},v_{1k}^{(j)},w_{1k}^{(i)},z_{1k}^{(j)}/1\le k\le 3,1\le i\le m,1\le j\le n\right\}$, E(G) = { eij = ui xij , fij = xij vj / 1 ≤ i ≤ m, 1 ≤ j ≤ n }∪$\left\{e_{1k}^{(i)}=u_i x_{1k}^{(i)}/1\le i\le m,1\le k\le 3\right\}\cup\left\{f_{11}^{(i)}=x_{11}^{(i)}u_{13}^{(i)},\, f_{12}^{(i)}=x_{12}^{(i)}u_{11}^{(i)},\, f_{13}^{(i)}=x_{11}^{(i)}u_{12}^{(i)}\right\}\cup$ $\left\{g_{11}^{(i)}=u_{11}^{(i)}w_{11}^{(i)},g_{12}^{(i)}=w_{11}^{(i)}u_{13}^{(i)},h_{11}^{(i)}=u_{11}^{(i)}w_{12}^{(i)},g_{12}^{(i)}=w_{12}^{(i)}u_{12}^{(i)},l_{11}^{(i)}=u_{12}^{(i)}w_{13}^{(i)},l_{12}^{(i)}=w_{13}^{(i)}u_{13}^{(i)}/1\le i\le m\right\}\cup$ $\left\{a_{1k}^{(j)}=v_j y_{1k}^{(j)}/1\le i\le m,1\le k\le 3\right\}\cup\left\{b_{11}^{(j)}=y_{11}^{(j)}v_{13}^{(j)},b_{12}^{(j)}=y_{12}^{(j)}v_{11}^{(j)},b_{13}^{(j)}=y_{13}^{(j)}v_{12}^{(j)}\right\}\cup$ $\left\{c_{11}^{(j)}=v_{11}^{(j)}z_{12}^{(j)},c_{12}^{(j)}=z_{12}^{(j)}v_{12}^{(j)},d_{11}^{(ji)}=z_{11}^{(j)}b_{12}^{(j)},d_{12}^{(j)}=z_{11}^{(j)}v_{13}^{(j)},m_{11}^{(j)}=v_{12}^{(j)}z_{13}^{(j)},m_{12}^{(j)}=z_{13}^{(j)}v_{13}^{(j)}/1\le j\le n\right\}.$
Deg ui = n + 3, deg vj = m + 3, 1 ≤ i ≤ m, 1 ≤ j ≤ n, deg u1k(i) = deg v1k(j) = 3, 1 ≤ i ≤ m, 1 ≤ j ≤ n and the remaining vertices have degree 2. Deg eij = deg e1k(i) = n+3, deg fij = deg a1k(j) = m+3, 1 ≤ i ≤ m, 1 ≤ j ≤ n and the remaining edges have degree 2.
$$S_1=\left\{e_{11}^{(i)},g_{11}^{(i)},l_{11}^{(i)},a_{11}^{(j)},d_{11}^{(j)},m_{11}^{(j)},\,1\le i\le m,\,1\le j\le n\right\},$$
$$S_2=\left\{e_{12}^{(i)},g_{12}^{(i)},h_{12}^{(i)},a_{12}^{(j)},d_{12}^{(j)},c_{12}^{(j)},\,1\le i\le m,\,1\le j\le n\right\},$$
$$S_3=\left\{e_{13}^{(i)},h_{11}^{(i)},l_{12}^{(i)},a_{13}^{(j)},c_{11}^{(j)},m_{12}^{(j)},\,1\le i\le m,\,1\le j\le n\right\}$$
are some strong efficient edge dominating sets of G and │S1│= │S2│= │S3│= 3(m+n). Therefore $\gamma'_{se}(G)\le 3(m+n),\ m,n\ge 1.$ Also no set with less than 3(m+n) edges is a strong efficient edge dominating set of G. Therefore $\gamma'_{se}(G)\ge 3(m+n),\ m,n\ge 1$. Hence $\gamma'_{se}(G)=3(m+n),m,n\ge 1.$

3. Conclusion

In this paper, strong efficient edge domination numbers of some subdivision graphs are determined.

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