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ON INTUITIONISTIC SEMI GROUP *Q*- FUZZY IDEALS IN SEMI RING

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Abstract: We introduce the notions of anti *Q*- fuzzy right ideals, anti *Q*- fuzzy right *K*- ideals and intuitionistic *Q*- fuzzy right *K* - ideals of semi rings. We investigate some of their properties and connections with right *K* - ideals, *Q*- fuzzy right *K* - ideals, anti *Q*- fuzzy right *K* - ideals. We characterize regular semi ring through intuitionistic *Q*- fuzzy right ideals. The semi regularity of semi rings is characterized by intuitionistic *Q*- fuzzy right *K* - ideals.

Keywords: *Q* - fuzzy right ideal, anti *Q*- fuzzy right ideal, intuitionistic *Q*- fuzzy right ideal, *Q*- fuzzy right *k*- ideal, anti *Q*- fuzzy right *k*- ideal, intuitionistic *Q*- fuzzy right *k*- ideal.

1. Introduction

As a generalization of rings, semi rings have been found useful for solving problems in different areas of applied mathematics and information sciences, since the structure of a semi rings provides an algebraic framework for modelling and studying the key factors in these applied areas. They play an important role in studying optimization theory, graph theory, theory of discrete event dynamical systems, matrices, determinants, generalized fuzzy computation, automate theory, formal language theory, coding theory, analysis of computer programs, and so on. Semi ring, ordered semi rings and semi rings appear in a natural manner in some applications to the theory of automata (see [5], [8]) and formal languages (see [1],[6],[7]).

Atanassov introduced inituitionistic fuzzy sets which constitute a generalization of the notion of fuzzy sets [3], [4]. The degree of membership of an element in a given set, while intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership. M.Akarm and W.A.Dudek introduced the notion of intuitionistic fuzzy left K-ideals in semi ring [2]. In this paper, we apply the concept of intuitionistic *Q*-fuzzy set to semi rings. We introduced the notion of anti Q- fuzzy right ideals, anti *Q*-fuzzy right *K* - ideals and intuitionistic *Q*-fuzzy right *K* - ideals in semi ring. We investigate their properties and connections with right *K* - ideal, *Q* fuzzy right *k*- ideals, anti *Q*- fuzzy right *k*- ideal. We describe various methods of constructions of intuitionistic *Q*-fuzzy right ideal. The semi regularity of semi rings is characterized by intuitionistic *Q*-fuzzy right *K*- ideal.

2. Preliminaries

In this section, we refer to some elementary aspects of the theory of semi rings and fuzzy algebraic systems that are necessary for this paper.

Definition 2.1

A non empty set R together with two binary operations " + " and "." is said to be a semi ring if

(i) $(R,+)$ is a commutative semigroup,

(ii) $(R,.)$ is a semigroup

(iii) $a(b + c) = ab + ac$ and $(a + b)c = ac + bc$ for all $a,b,c \in R$.

Definition 2.2

A nonempty set S together with two associative binary operations called addition and multiplication (denoted by + and . respectively) is called a semi ring if $(S,+)$ is a commutative semigroup, $(S,.)$ is a semigroup and multiplication distributes over addition both from the left and the right, i.e., $a(b + c) = ab + ac$ and $(a + b)c = ac + bc$ for all $a,b,c,\in S$

Definition 2.3.

A semi ring R has a *zero* if there exits an element $0 \in R$ such that $0x = x0 = 0$ and $0 + x = x + 0 = x$ for all $x \in R$.

Definition 2.4

A nonempty subset S of R is said to be a sub semi ring of R if S is closed under the operation of addition and multiplication in R.

Definition 2.5

A sub semi ring I of a semi ring R is called an *right*(*left*)*ideal* of R if for all $r \in R$, $x \in I$, $xr \in I$, $(rx \in I)$.

Definition 2.6

A sub semi ring I of a semi ring R is called an ideal of R if it is both left and right ideal.

Definition 2.7

A right ideal B of a semi ring R is called a right k-ideal of R if $x + y, y \in B$ implies $x \in B$.

Definition 2.8 *[9] [10]*

A mapping $\mu : M \longrightarrow [0,1]$, where M, are an arbitrary non-empty set, is called a fuzzy set of M.

Definition 2.9 *[11]*

A mapping $\mu : M \times Q \longrightarrow [0,1]$, where M,Q are an arbitrary non-empty semi ring and semigroup respectively , is called a Q-fuzzy set of M.

Definition 2.10 *[9] [10]*

An upper level set of a fuzzy set μ denote by $U(\mu;t)$ is defined as $U(\mu;t) = \{x \in M | \mu(x) \geq t\}$ and a lower level set of a fuzzy set μ denote by $L(\mu;t)$ is defined as $L(\mu;t) = \{x \in M | \mu(x,q) \leq t\}$, for all $t \in [0,1]$.

Definition 2.11 *[11]*

An upper level set of a Q-fuzzy set μ denote by $U(\mu;t)$ is defined as $U(\mu;t) = \{x \in M | \mu(x,q) \geq t \forall q \in Q\}$ and a lower level set of a Q-fuzzy set μ denote by $L(\mu;t)$ is defined as

$$L(\mu;t) = \{x \in M | \mu(x,q) \leq t \forall q \in Q\}, \text{ for all } t \in [0,1].$$

As an important generalization of the notion of fuzzy sets in M, Atanassov [3],[4] introduced the concept of An intuitionistic fuzzy set (IFS for short) defined on a non-empty set M as objects of the form $A = \{< x, \mu A(x), \lambda A(x) > | x \in M\}$, where the functions $\mu : M \longrightarrow [0,1]$ and $\lambda : M \longrightarrow [0,1]$ denote the degree of membership (namely $\mu A(x)$) and the degree of non-membership (namely $\lambda A(x)$) for each element $x \in M$ to the set A, respectively, and $0 \leq \mu A(x) + \lambda A(x) \leq 1$ for each $x \in M$.

Obviously, every fuzzy set μ we can have intuitionistic fuzzy set : $A = \{< x, \mu_A(x), (1 - \mu_A)(x) > | x \in M\}$.

Definition 2.12 *[11]*

An intuitionistic Q-fuzzy set (IQFS for short) defined on a non-empty set M as objects of the form $A = \{< \mu A(x,q), \lambda A(x,q) > | x \in M, q \in Q\}$, where the functions $\mu A : M \times Q \longrightarrow [0,1]$ and $\lambda A : M \times Q \longrightarrow [0,1]$ denote the degree of membership (namely $\mu A(x,q)$) and the degree of non-membership (namely $\lambda A(x,q)$) for each element $x \in M, q \in Q$ to the set A, respectively, and $0 \leq \mu A(x,q) + \lambda A(x,q) \leq 1$ for each $x \in M, q \in Q$.
Obviously, every Q fuzzy set μ, we can have intuitionistic Q-fuzzy set : $A = \{< \mu A(x,q), (1 - \mu A)(x,q) > | x \in M, q \in Q\}$.

Remark. For the sake of simplicity, we shall use the symbol $A = (\mu_A, \lambda_A)$ for the intuitionistic Q-fuzzy set : $A = \{< \mu_A(x,q), \lambda_A(x,q) > | x \in M, q \in Q\}$. Obviously for an IQFS $A = (\mu_A, \lambda_A)$ in M, the IQFS A is a Q fuzzy set. Hence the notion of intuitionistic Q fuzzy set theory is a generalization of fuzzy set theory.

Let $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ be intuitionistic Q - fuzzy sets in a set M . We define

- $A \subseteq B \Leftrightarrow (\forall x \in M, q \in Q)$ $\quad (\mu A(x,q) \leq \mu B(x,q), \lambda A(x,q) \geq \lambda B(x,q))$.
- $A = B \Leftrightarrow A \subseteq B$ and $A \supseteq B$.
- $A \cap B = (\mu A \wedge \mu B, \lambda A \vee \lambda B)$.
- $A \cup B = (\mu A \vee \mu B, \lambda A \wedge \lambda B)$.
- $0 = (0,1)$and $1 = (1,0)$.

3. Intuitionistic *Q*- Fuzzy Right Ideals

Definition 3.1

A Q-fuzzy set μ of a semi ring R is said to be a Q- fuzzy right (left) ideal if

(i) $\mu(x + y,q) \geq \mu(x,q) \wedge \mu(y,q)$

(ii) $\mu(xy,q) \geq \mu(x,q)$ $\quad (\mu(xy,q) \geq \mu(y,q))$

(iii) $\mu(x,q_1 \cdot q_2) \geq \mu(x,q_1)$ $\quad (\mu(x,q_1 \cdot q_2) \geq \mu(x,q_2))$ for all $x,y \in R; q,q_1,q_2 \in Q$

Example 3.2 Let $S = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \Big/ a,b,c \in [0,1] \right\}$ be the semi ring $Q = (Z_6, \odot_6$ and) be the semigroup. Let $A = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$ be a subset of S and $I = [\bar{0}, \bar{3}]$ be a subset of Z_6. Let the Q-fuzzy set μ defined by

$$\mu(x,q) = \begin{cases} 0.7, & if \quad x \in A, q \in I. \\ 0.3, & otherwise. \end{cases}$$

then μ is a Q- fuzzy right ideal of S.

Definition 3.3

A Q-fuzzy set μ of a semi ring R is said to be a anti Q- fuzzy right (left) ideal if

(i) $\mu(x + y,q) \leq \mu(x,q) \vee \mu(y,q)$

(ii) $\mu(xy,q) \leq \mu(x,q)$ $(\mu(xy,q) \leq \mu(y,q))$

(iii) $\mu(x,q_1 \cdot q_2) \leq \mu(x,q_1)$ $(\mu(x,q_1 \cdot q_2) \leq \mu(x,q_2))$ forall $x,y \in R; q,q_1,q_2 \in Q$.

Example 3.4

Let $S = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \Big/ a,b,c \in [0,1] \right\}$ be the semi ring and $Q = (Z_6, \odot_6)$ be the semigroup. Let $A = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$ be a subset of S and $I = [\bar{0}, \bar{3}]$ be a subset of Z_6. Let the Q-fuzzy set μ defined by

$$\mu(x,q) = \begin{cases} 0.3, & if \quad x \in A, q \in I. \\ 0.7, & otherwise. \end{cases}$$

then μ is a anti Q- fuzzy right ideal of S.

Definition 3.5

An intuitionistic Q- fuzzy set $A = \{< x,q,\mu_A(x,q),\lambda_A(x,q) > | x \in R, q \in Q\}$ is called an intuitionistic Q- fuzzy right (left) ideal of R if

(i) $\mu_A(x + y,q) \geq \mu_A(x,q) \wedge \mu_A(y,q)$

(ii) $\mu_A(xy,q) \geq \mu_A(x,q)$ $(\mu_A(xy,q) \geq \mu_A(y,q))$

(iii) $\mu_A(x,q_1 \cdot q_2) \geq \mu_A(x,q_1)$ $(\mu_A(x,q_1 \cdot q_2) \geq \mu_A(x,q_2))$

(iv) $\lambda_A(x + y,q) \leq \lambda_A(x,q) \vee \lambda_A(y,q)$

(v) $\lambda_A(xy,q) \leq \lambda_A(x,q)$ $(\lambda_A(xy,q) \leq \lambda_A(y,q))$

(vi) $\lambda_A(x,q_1 \cdot q_2) \leq \lambda_A(x,q_1)$ $(\lambda_A(x,q_1 \cdot q_2) \leq \lambda_A(x,q_2))$ forall $x,y \in R; q,q_1,q_2 \in Q$.

Definition 3.6

An intuitionistic Q-fuzzy set $A = (\mu_A,\lambda_A)$ is called an intuitionistic Q-fuzzy ideal of R if it is both intuitionistic Q-fuzzy right and left ideal.

Definition 3.7.

Let $A = (\mu_A,\lambda_A)$ be an intuitionistic Q-fuzzy set in R and let $s,t \in [0,1]$.

Then the set $R_A^{(s,t)} = \{x \in R | \mu_A(x,q) \geq s, \lambda_A(x,q) \leq t, q \in Q\}$ is called a (s,t)- level set of $A = (\mu_A,\lambda_A)$. The set $\{(s,t) \in Im(\mu_A) \times Im(\lambda_A) s + t \leq 1\}$ is called image of $A = (\mu_A,\lambda_A)$.

Theorem 3.8.

A Q- fuzzy set μ is a Q - fuzzy right (left) ideal of a semi ring R if and only if $1-\mu$ is an anti Q- fuzzy right (left) ideal of a semi ring R.

Proof: Let μ be a Q-fuzzy right ideal in a semi ring R. Let $x,y \in R, q \in Q$. Then:

$$\mu(x+y,q) \geq \mu(x,q) \wedge \mu(y,q)$$
$$-\mu(x+y,q) \leq -[\mu(x,q) \wedge \mu(y,q)]$$
$$-\mu(x+y,q) \leq -\mu(x,q) \vee -\mu(y,q)$$
$$1-\mu(x+y,q) \leq 1-\mu(x,q) \vee 1-\mu(y,q) .$$

Therefore, $1-\mu(x+y,q) \leq 1-\mu(x,q) \vee 1-\mu(y,q)$.

Similarly, we can prove

$$1-\mu(xy,q) \leq 1-\mu(x,q) \text{ and } 1-\mu(x,q_1 \cdot q_2) \leq 1-\mu(x,q_1).$$

Hence $1-\mu$ is an anti Q- fuzzy right (left) ideal of a semi ring R.

Conversely, we can prove that μ is a Q - fuzzy right (left) ideal of a semi ring R in similar manner.

Definition 3.9

An upper level set of a Q-fuzzy set μ with respect to the semi ring R, denote by $U(\mu;t)$ is defined as $U(\mu;t) = \bigcap_{q\in Q}\{Uq(\mu,t)\}$ where $U_q(\mu,t) = \{x \in M | \mu(x,q) \geq t\}$ and an upper level set of a Q-fuzzy set μ with respect to the semigroup Q, denote by $U'(\mu;t)$ is defined as $U'(\mu;t) = \bigcap_{x\in R}\left\{U'_x(\mu,t)\right\}$ where $U'_x(\mu,t) = \{q \in Q | \mu(x,q) \geq t\}$.

We have $U(\mu;t) = \{x \in R | \mu(x,q) \geq t \forall q \in Q\}$ and $U(\mu;t) = \{q \in Q | \mu(x,q) \geq t \forall x \in R\}$.

Similarly, we defined a lower level set of a Q-fuzzy set μ as

$L(\mu;t) = \{x \in R | \mu(x,q) \leq t \forall q \in Q\}$ and $L'(\mu;t) = \{q \in Q | \mu(x,q) \leq t \forall x \in R\}$.

Definition 3.10

An upper level set of a Q-fuzzy set μ with respect to the semi ring R, denote by $U(\mu;t)$ is defined as $U(\mu;t) = \{x \in R | \mu(x,q) \geq t \forall q \in Q\}$ and an upper level$_0$ set of a Q-fuzzy set μ with respect to the semigroup Q, denote by $U'(\mu;t)$ is defined as $U'(\mu;t) = \{q \in Q | \mu(x,q) \geq t \ \forall x \in R\}$.

Similarly, we defined a lower level set of a Q-fuzzy set μ as:

$L(\mu;t) = \{x \in R | \mu(x,q) \leq t \ \forall q \in Q\}$ and $L'(\mu;t) = \{q \in Q | \mu(x,q) \leq t \ \forall x \in R\}$.

Lemma 3.11

A Q- fuzzy set μ in a semi ring R is a Q- fuzzy right (left) ideal if and only if $U(\mu;t)$ and $U'(\mu;t)$ are a right (left) ideal of a semi ring R and semigroup Q respectively for all $t \in [0,1], q \in Q$ whenever nonempty.

Proof: Let μ be a Q- fuzzy right (left) ideal of a semi ring R. Let $t \in [0,1]$ such that $x,y \in U(\mu;t)$ and $q \in Q$,implies $\mu(x,q) \geq t$ and $\mu(y,q) \geq t$. Then $\mu(x,q) \wedge \mu(y,q) \geq t$. Thus $\mu(x+y,q) \geq \mu(x,q) \wedge \mu(y,q) \geq t$ implies $\mu(x+y,q) \geq t$. Therefore, $x+y \in U_q(\mu;t)$.

Similarly, if $x \in U(\mu;t)$,then $xy \in U(\mu;t)$. If $q \in Q$ and $l \in U'(\mu;t)$ implies $\mu(x,l) \geq t$, then $\mu(x,lq) \geq \mu(x,l) \geq t$. Therefore, $lq \in U'(\mu;t)$.

Hence $U(\mu;t)$ and $U'(\mu;t)$ are a right (left) ideal of a semi ring R and a semigroup Q respectively for all $t \in [0,1], q \in Q$.

Conversely, if there exists $x,y \in R$, $q \in Q$ and $\mu(x,q) \wedge \mu(y,q) = t$ implies $\mu(x,q) \geq t, \mu(y,q) \geq t$ then $x,y \in U(\mu,t)$. Since $U(\mu,t)$ is an right (left) ideal of R then $x+y \in U(\mu,t)$ implies $\mu(x+y,q) \geq t = \mu(x,t) \wedge \mu(y,q)$. Therefore, $\mu(x+y,q) \geq \mu(x,q) \wedge \mu(y,q)$. Similarly, $\mu(x \cdot y,q) \geq \mu(x,q)$.

Let $\mu(x,q_1) = t$ and $q_1 \in U'(\mu,t), q_2 \in Q$ then $q_1 \cdot q_2 \in U'(\mu,t)$ implies $\mu(x,q_1 \cdot q_2) \geq t$.

So $\mu(x,q_1 \cdot q_2) \geq t = \mu(x,q_1)$. Therefore, $\mu(x,q_1 \cdot q_2) \geq \mu(x,q_1)$.

Hence μ is a Q- fuzzy right (left) ideal in a semi ring R.

Lemma 3.12

A Q- fuzzy set λ of a semi ring R is an anti Q- fuzzy right (left) ideal if and only if $L(\lambda;t)$ and $L'(\lambda;t)$ are a right (left) ideal of a semi ring R and semigroup Q respectively for all $t \in [0,1], q \in Q$ whenever nonempty.

Proof: Let λ be an anti Q- fuzzy right (left) ideal of a semi ring R. Let $t \in [0,1]$ such that $x,y \in L(\lambda;t)$ and $q \in Q$ implies $\lambda(x,q) \le t$ and $\lambda(y,q) \le t$. Then $\lambda(x,q) \vee \lambda(y,q) \le t$. ,
$\lambda(x+y,q) \le \lambda(x,q) \vee \lambda(y,q) \le t$ implies $\lambda(x+y,q) \le t$.
Therefore, $x + y \in L(\lambda;t)$.
Similarly if $x \in L(\lambda;t)$,then $xy \in L(\lambda;t)$. If $q \in Q$ and $l \in L'(\lambda;t)$ implies $\lambda(x,l) \le t$, then
$\lambda(x,lq) \le \lambda(x,l) \le t$. Therefore $lq \in L'(\lambda;t)$.
Hence $L(\lambda;t)$ and $L'(\lambda;t)$ are a right (left) ideal of a semi ring R and semigroup Q respectively for all $t \in [0,1], q \in Q$.

Conversely, If there exists $x,y \in R$, $q \in Q$ and $\lambda(x,q) \vee \lambda(y,q) = t$ implies $\lambda(x,q) \le t, \lambda(y,q) \le t$ then $x,y \in L(\lambda,t)$. Since $L(\lambda,t)$ is an right(left) ideal of R then $x + y \in L(\lambda,t)$ implies $\lambda(x+y,q) \le t = \lambda(x,t) \vee \lambda(y,q)$. Therefore, $\lambda(x+y,q) \le \lambda(x,q) \vee \lambda(y,q)$.
Similarly, $\lambda(x \cdot y,q) \le \lambda(x,q)$.Let $\lambda(x,q_1) = t$ and $q_1 \in L'(\lambda,t), q_2 \in Q$ then $q_1q_2 \in L'(\lambda,t)$ implies $\lambda(x,q_1 \cdot q_2) \le t$. So $\lambda(x,q_1 \cdot q_2) \le t = \lambda(x,q_1)$. Therefore $\lambda(x,q_1 \cdot q_2) \le \lambda(x,q_1)$.
Hence λ is an anti Q- fuzzy right (left) ideal in a semi ring R.

Theorem 3.13

An intuitionistic Q- fuzzy set $A = \{< \mu_A(x,q), \lambda_A(x,q) > | x \in R, q \in Q\}inR$ is an intuitionistic Q-fuzzy right ideal in R if and only if $U(\mu_A;t)$ and $U'(\mu_A;t)$ are a right(left) ideal of a semi ring R and semigroup Q and $L(\lambda_A;t)$ and $L'(\lambda_A;t)$ are a right (left) ideal of a semi ring R and semigroup Q for all t $\in [0,1], q \in Q$ whenever nonempty.
Proof: The proof follows from Lemma3.11 and Lemma 3.12

Theorem 3.14

An intuitionistic Q- fuzzy set $A = (\mu_A, \lambda_A)$ in R is an intuitionistic Q- fuzzy right ideal in R if and only if $R_A^{(s,t)}$ is a right ideal for all s,t $\in [0,1]$ whenever nonempty.
Proof: Let $A = (\mu_A, \lambda_A)$ be an intuitionistic Q- fuzzy right ideal in R. Clearly $R_A^{(s,t)} = U(\mu_A;s) \cap L(\lambda_A,t)$.
Then by lemma 3.11 and lemma 3.12 ,$R_A^{(s,t)}$ is a right ideal for all $s,t \in [0,1]$ whenever nonempty.
Conversely, let $x,y \in R, q \in Q$ and $\mu_A(x,q) \wedge \mu_A(y,q) = s$; $\lambda_A(x,q) \vee \lambda_A(y,q) = t$. Then,
$x,y \in R_A^{(s,t)}$ implies $x + y \in R_A^{(s,t)}$. Thus,
$\mu_A(x+y,q) \ge \mu_A(x,q) \wedge \mu_A(y,q)$ and $\lambda_A(x+y,q) \le \lambda_A(x,q) \wedge \lambda_A(y,q)$.
Let $\mu_A(x,q) = s; \lambda_A(x,q) = t$then $x \in R_A^{(s,t)}$ implies $xy \in R_A^{(s,t)}$.
Therefore, $\mu_A(xy,q) \ge \mu_A(x,q); \lambda_A(xy,q) \le \lambda_A(x,q)$.
Similarly, $\mu_A(x,q_1 \cdot q_2) \ge \mu_A(x,q_1); \lambda_A(x,q_1 \cdot q_2) \le \lambda_A(x,q_1)$ for a ll $q_1,q_2 \in Q$.
Hence $A = (\mu_A, \lambda_A)$is an intuitionistic Q- fuzzy right ideal in R.

Corollary 3.15

An intuitionistic Q- fuzzy set $A = \{< x,q,\mu_A(x,q), \lambda_A(x,q) > | x \in R, q \in Q\}$in R is an intuitionistic Q- fuzzy right ideal in R if and only if μ_A is a Q-fuzzy right ideal in a semi ring R and λ_A is an anti Q-fuzzy right ideal in a semi ring R.
Proof: The proof follows from Theorem 3.14, Lemma 3.11 and Lemma 3.12

Corollary 3.16

An intuitionistic Q- fuzzy set $A = \{< x,q,\mu_A(x,q),(1-\mu_A)(x,q) > | x \in R, q \in Q\}$in R is an intuitionistic Q- fuzzy right ideal in R if and only if μ_A is a Q-fuzzy right ideal in a semi ring R
Proof: The proof follows from Theorem 3.14 and corollary 3.15

Corollary 3.17

An intuitionistic Q- fuzzy set $A = \{< x,q,(1-\lambda_A)(x,q), \lambda_A(x,q) > | x \in R, q \in Q\}$in R is an intuitionistic Q- fuzzy right ideal in R if and only if λ_A is an anti Q-fuzzy right ideal in a semi ring R
Proof: The proof follows from Theorem 3.14 and corollary 3.16

4. Intuitionistic *Q*- Fuzzy Right k-Ideals

Definition 4.1

A right (left) ideal I is called right(left) k-ideal of a semi ring R if $x + y$, $y \in I$ implies $x \in I$.

Definition 4.2

A Q- fuzzy right(left) ideal μ of a semi ring R is called Q- fuzzy right(left) k ideal if for all $x,y \in R, q \in Q, \mu(x,q) \geq \mu(x + y,q) \wedge \mu(y,q)$.

Definition 4.3

An anti Q- fuzzy right(left) ideal λ of a semi ring R is called anti Q- fuzzy right(left) k- ideal if for all $x,y \in R, q \in Q, \lambda(x,q) \leq \lambda(x + y,q) \vee \lambda(y,q)$.

Definition 4.4

An intuitionistic Q- fuzzy right (left)ideal $A = \{< \mu_A(x,q), \lambda_A(x,q) > | x \in R, q \in Q\}$ in R is said to be an intuitionistic Q- fuzzy right(left)k-ideal if

(i) $\mu_A(x,q) \geq \mu_A(x + y,q) \wedge \mu_A(y,q)$

(ii) $\lambda_A(x,q) \leq \lambda_A(x + y,q) \vee \lambda_A(y,q)$ for all $x,y \in R, q \in Q$.

Theorem 4.5

A Q- fuzzy set μ is a Q - fuzzy right (left) k-ideal of a semi ring R if and only if $1 - \mu$ is an anti Q- fuzzy right (left) k-ideal of a semi ring R.

Proof: Let μ be a Q-fuzzy right k-ideals in a semi ring R. Let $x,y \in R$. Then $\mu(x,q) \geq \mu(x + y,q) \wedge \mu(y,q)$ implies $-\mu(x,q) \leq -(\mu(x+y,q) \wedge \mu(y,q))$. Thus, $-\mu(x,q) \leq -\mu(x+y,q) \vee -\mu(y,q)$. Therefore, $1-\mu(x,q) \leq 1-\mu(x+y,q) \vee 1-\mu(y,q)$. Hence $1-\mu(x,q) \leq 1-\mu(x+y,q) \vee 1-\mu(y,q)$. By Theorem 3.6, $1 - \mu$ is an anti Q-fuzzy right (left) k-ideal of a semi ring R. By similar argument, we can prove the converse part.

Lemma 4.6

A Q- fuzzy set μ of a semi ring R is a Q- fuzzy right k- ideal if and only if $U_q(\mu,t)$ is a right k-ideal in a semi ring R for all $t \in [0,1]$ whenever nonempty.

Proof: Let μ be a Q-fuzzy right k-ideal in a semi ring R. Let $x,y \in R$ such that $x+y$, $y \in U(\mu,t)$ then $\mu(x + y,q) \geq t$ and $\mu(y,q) \geq t$ implies $\mu(x,q) \geq \mu(x + y,q) \wedge \mu(y,q) \geq t$. Hence $x \in U(\mu,t)$.Therefore, by Lemma 3.7 $U(\mu,t)$is a right k- ideal in a semi ring R.

Conversely, let $\mu(x + y,q) \wedge \mu(y,q) = t$ implies $\mu(x + y,q) \geq t$ and $\mu(y,q) \geq t$. Thus $x + y$, y $\in U(\mu,t)$ and since $U(\mu,t)$ is a right k- ideal in R. $\mu(x,q) \geq \mu(x + y,q) \wedge \mu(y,q) = t$ implies $x \in U(\mu,q)$. Therefore, by Lemma 3.7, μ be a Q- fuzzy right k-ideal in a semi ring R.

Lemma 4.7

A Q- fuzzy set λ of a semi ring R is an anti Q- fuzzy right (left) k-ideal if and only if $L(\lambda;t)$ is a right (left)k- ideal in a semi ring R for all $t \in [0,1]$,q $\in Q$ whenever nonempty.

Proof: Let λ be a Q-fuzzy right k-ideal in a semi ring R. Let $x,y \in R$ such that $x+y$, y $\in L(\lambda,t)$ then $\lambda(x+y,q) \leq t$ and $\lambda(y,q) \leq t$ implies $\lambda(x,q) \leq \lambda(x+y,q) \vee \lambda(y,q) \leq t$. Hence $x \in L(\lambda,t)$. Thus $L(\lambda,t)$ is a right k- ideal in a semi ring R .

Conversely, let $\lambda(x + y,q) \vee \lambda(y,q) = t$ implies $\lambda(x + y,q) \leq t$ and $\lambda(y,q) \leq t$. Thus $x + y$, $y \in L(\lambda,t)$ and since $L(\lambda,t)$ is a right k- ideal in R. $\lambda(x,q) \leq \lambda(x + y,q) \vee \lambda(y,q) = t$. Thus λ be a Q- fuzzy right k-ideal in a semi ring R.

Theorem 4.8

An intuitionistic Q- fuzzy set $A = \{< \mu_A(x,q), \lambda_A(x,q) > | x \in R, q \in Q\}$in R is an intuitionistic Q-fuzzy right k - ideal in R if and only if $U(\mu_A;t)$, $L(\lambda_A;t)$ is a right (left) k-ideals in a semi ring R for all $t \in [0,1], q \in Q$ whenever nonempty.

Proof: The proof follows from Lemma 4.6 and Lemma 4.7

Theorem 4.9

An intuitionistic Q- fuzzy set $A = (\mu_A, \lambda_A)$ in R is an intuitionistic Q- fuzzy right k-ideal in R if and only if $R_A^{(s,t)}$ is a right k-ideal for all $s,t \in [0,1]$ whenever nonempty.

Proof: Let $A = (\mu_A, \lambda_A)$ be an intuitionistic Q- fuzzy right k-ideal in R. Clearly, $R_A^{(s,t)} = U(\mu_A; s) \cap L(\lambda_A, t)$. Then by Lemma 4.6 and Lemma 4.7, $R_A^{(s,t)}$ is a right k - ideal for all s,t $\in [0,1]$ whenever nonempty.

Conversely, $A = (\mu_A, \lambda_A)$ in R is an intuitionistic Q- fuzzy right k-ideal in R. Let $x,y \in R, q \in Q$ and $\mu_A(x + y,q) \wedge \mu_A(y,q) = s; \lambda_A(x + y,q) \vee \lambda_A(y,q) = t$. Then $x + y, y \in R_A^{(s,t)}$ implies $x \in R_A^{(s,t)}$. Thus $\mu_A(x,q) \geq \mu_A(x+y,q) \wedge \mu_A(y,q)$ and $\lambda_A(x,q) \leq \lambda_A(x+y,q) \vee \lambda_A(y,q)$.

Hence $A = (\mu_A, \lambda_A)$ is an intuitionistic Q- fuzzy right k-ideal in R.

Corollary 4.10

An intuitionistic Q - fuzzy set $A = \{< \mu_A(x), \lambda_A(x) > | x \in R, q \in Q\}$ in R is an intuitionistic Q- fuzzy right k - ideal in R if and only μ_A is a Q-fuzzy right k- ideal in a semi ring R and λ_A is an anti Q- fuzzy right k - ideal in a semi ring R.

Proof: The proof follows from Theorem 4.8 and Lemma 4.6 and Lemma 4.7.

Corollary 4.11

An intuitionistic Q- fuzzy set $A = \{< x,q,\mu_A(x),(1-\mu_A)(x) > | x \in R, q \in Q\}$in R is an intuitionistic Q- fuzzy right k - ideal in R if and only μ_A is a Q- fuzzy right k- ideal in a semi ring R.

Proof: The proof follows from Theorem 4.5 and Corollary 4.10.

Corollary 4.12

An intuitionistic Q- fuzzy set $A = \{< x,q,(1-\lambda_A)(x),\lambda_A(x) > | x \in R,$ q $\in Q\}$in R is an intuitionistic Q- fuzzy right k - ideal in R if and only λ_A is an anti Q - fuzzy right k ideal in a semi ring R.

Proof: The proof follows from Theorem 4.5 and Corollary 4.10.

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