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PROPERTIES OF $\delta\omega$ -CLOSED SETS IN TOPOLOGICAL SPACES

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Abstract In this paper, we introduce a new class of sets called $\delta\omega$-closed sets in topological spaces. This class lies between the class of δ-closed sets and the class of δg-closed sets.

Keywords: $\delta\omega$-closed, δg-closed, δ-closed, ω-closed.

1. Introduction

In 1963 Levine [14] introduced the notion of semi-open sets. Velicko [17] introduced the notion of $\boldsymbol{\delta}$-closed sets and it is well known that the collection of all $\boldsymbol{\delta}$-closed sets of a topological space forms a topology and is denoted by $\tau\boldsymbol{\delta}$. Levine [13] also introduced the notion of g-closed sets and investigated its fundamental properties. This notion was shown to be productive and very useful.

After the advent of g-closed sets, Arya and Nour [3], Sheik John [15] and Dontchev [12] introduced gs-closed sets, $\boldsymbol{\omega}$-closed sets and gsp-closed sets respectively.

In this paper, we introduce a new class of sets called $\delta\omega$-closed sets in topological spaces. This class lies between the class of δ-closed sets and the class of δg-closed sets.

2. Preliminaries

Definition 2.1

A subset A of a space (X, τ) is called:

(i) semi-open set [8] if A $\subseteq$ cl(int(A));

(ii) preopen set [2] if A $\subseteq$ int(cl(A));

(iii) α-open set [1] if A $\subseteq$ int(cl(int(A)));

(iv) β-open set [1] (= semi-preopen) if A $\subseteq$ cl(int(cl(A)));

(v) regular open set [2] if A = int(cl(A)).

The complements of the above mentioned open sets are called their respective closed sets.

The preclosure [2] (resp. semi-closure [11], α-closure [1], semi-pre-closure [8]) of a subset A of X, denoted by pcl(A) (resp. scl(A), αcl(A), spcl(A)), is defined to be the intersection of all preclosed (resp. semi-closed, α -closed, semi-preclosed) sets of (X, τ) containing A. It is known that pcl(A) (resp. scl(A), αcl(A), spcl(A)) is a preclosed (resp. semi-closed, α -closed, semi-preclosed) set.

Definition 2.2 *[7]*

A point x of a space X is called a θ-adherent point of a subset A of X if cl(U) ∩ A ≠ ϕ, for every open set U containing x. The set of all θ-adherent points of A is called the θ-closure of A and is denoted by cl_θ(A). A subset A of a space X is called θ-closed if and only if A = cl_θ(A). The complement of a θ-closed set is called θ-open.

Similarly, the θ-interior of a set A in X, written int_θ (A), consists of those points x of A such that for some open set U containing x, cl(U) ⊆ A. A set A is θ-open if and only if A = int_θ (A), or equivalently, X \ A is θ-closed.

A point x of a space X is called a δ-adherent point of a subset A of X if int(cl(U)) ∩ A ≠ ϕ, for every open set U containing x. The set of all δ-adherent points of A is called the δ-closure of A and is denoted by cl_δ (A). A subset A of a space X is called δ-closed if and only if A = cl_δ (A). The complement of a δ-closed set is called δ-open. Similarly, the δ-interior of a set A in X, written int_δ (A), consists of those points x of A such that for some regularly open set U containing x, U ⊆ A. A set A is δ-open if and only if A = int_δ (A), or equivalently, X \ A is δ-closed.

The family of all θ-open (resp. δ-open) subsets of (X, τ) forms a topology on X and is denoted by τ_θ (resp. τ_δ). From the definitions it follows immediately that $\tau_\theta \subseteq \tau_\delta \subseteq \tau$. [9].

Definition 2.3

A point x ∈ X is called a semi θ-cluster [9] point of A if A ∩ scl(U) ≠ ϕ for each semi-open set U containing x.

The set of all semi θ-cluster points of A is called the semi-θ-cluster of A and is denoted by sclθ (A). Hence, a subset A is called semi-θ-closed if sclθ (A) = A. The complement of a semi-θ-closed set is called semi-θ-open set.

Recall that a subset A of a space (X, τ) is said to be δ-semi-open [20] if A ⊆ cl(intδ (A)).

Definition 2.4

A subset A of a space (X, τ) is called:

(i) a generalized closed (briefly, g-closed) set [13] if cl(A) ⊆ U, whenever A ⊆ U and U is open in (X, τ).

(ii) a generalized semi-closed (briefly, gs-closed) set [12] if scl(A) ⊆ U , whenever A ⊆ U and U is open in (X, τ).

(ii) an α-generalized closed (briefly, αg-closed) set [5] if αcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

(iv) a generalized semi-preclosed (briefly, gsp-closed) set [10] if spcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

(v) a generalized preclosed (briefly, gp-closed) set [12] if pcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

(vi) a regular generalized closed (briefly, rg-closed) set [13] if cl(A) ⊆ U whenever A ⊆ U and U is regular open in (X, τ).

(vii) a δ-generalized closed (briefly, δg-closed) set [9] if cl_δ (A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

(viii) a ĝ-closed set [23] (= ω-closed set [16]) if cl(A) ⊆ U whenever A ⊆ U and U is semi-open in (X, τ).

The complement of $\hat{g}$ −closed set is called $\hat{g}$ −open set. The collection of all $\hat{g}$ −open sets is denoted by $\hat{G}O(X)$.

Remark: The collection of all δg-closed (resp. ω-closed, g-closed, δ-closed, α-closed, semi-closed) sets of X is denoted by δGC(X) (resp. ωC(X), GC(X), δC(X), αC(X), SC(X)).
We denote the power set of X by P(X).

Definition 2.5 *[9]*
A space (X, τ) is said to be sub weakly T_2 if cl_δ ({x}) = cl({x}) for each x ∈ X.

Remark: We have the following diagram in which the converses of the implications need not be true.

$$\begin{tikzcd}
\theta\text{-open} \arrow[r] \arrow[d] & \delta\text{-open} \arrow[r] \arrow[d] & \text{open} \arrow[d] \\
\text{semi-}\theta\text{-open} \arrow[r] & \delta\text{-semi-open} \arrow[r] & \text{semi-open}
\end{tikzcd}$$

Theorem 2.8 *[9]*
Let (X, τ) be a space. The following hold.
(i) Every δ-closed set is δg-closed.
(ii) Every δg-closed set is g-closed and hence αg-closed, gs-closed, gsp-closed and rg-closed.

Remark*: [9,16]* δg-closed sets and ω-closed sets are independent.

Definition 2.9 *[9]*
A space (X, τ) is called semi-regular if τ_δ= τ.

Definition 2.10 *[16]*
A space X is called τω if ω-closed set in X is closed in X.

Proposition 2.11 *[9]* Let (X, τ) be a space. If A ⊆ X is preopen then cl(A) = αcl(A) = cl_δ (A).

Lemma 2.12 *[9]*
In any space, a singleton is δ-open if and only if it is regular open.

3. $\delta\omega$-closed sets
We introduce the following definition.

Definition 3.1
A subset A of X is called a $\delta\omega$-closed set if cl_δ (A) ⊆ U whenever A ⊆ U and U is semi-open in (X, τ). The complement of $\delta\omega$-closed set is called $\delta\omega$-open set.
The collection of all $\delta\omega$-closed sets of X is denoted by $\delta\omega$C(X).

Proposition 3.2 Every δ-closed set is $\delta\omega$-closed.
Proof: Let A be a δ-closed set and G be any semi-open set containing A. Since A is δ-closed, cl_δ (A) = A for every subset A of X. Therefore cl_δ (A) ⊆ G and hence A is $\delta\omega$-closed set.
The converse of Proposition 3.2 need not be true as seen from the following example.

Example 3.3 Let X = {a, b, c} with τ = {ϕ, {a}, X}. Then $\delta\omega$C(X) = {ϕ, {b, c}, X} and δC(X) = {ϕ, X}. We have A = {b, c} is $\delta\omega$-closed but not δ-closed set in (X, τ).

Proposition 3.4 Every $\delta\omega$-closed set is g-closed.
Proof: Let A be a $\delta\omega$-closed set and G be any open set containing A. Since every open set is semi-open and A is $\delta\omega$-closed, cl_δ (A) $\subseteq$ G. Since cl(A) $\subseteq$ cl_δ (A) $\subseteq$ G , cl(A) $\subseteq$ G and hence A is g-closed.
The converse of Proposition 3.4 need not be true as seen from the following example.

Example 3.5 Let X and τ be as in the Example 3.3. Then $\delta\omega$C(X) = {ϕ, {b, c}, X} and GC(X) = {ϕ, {b}, {c}, {a, b}, {a, c}, {b, c}, X}. We have A = {a, b} is g-closed but not $\delta\omega$-closed set in (X, τ).

Proposition 3.6 Every $\delta\omega$-closed set is ω-closed.
Proof: Let A be a $\delta\omega$-closed and G be any semi-open set containing A.
Since cl(A) $\subseteq$ cl_δ (A) $\subseteq$ G and hence A is ω-closed.
The converse of Proposition 3.6 need not be true as seen from the following example.

Example 3.7 Let X = {a, b, c} with τ = {ϕ, {a}, {a, b}, X}. Then $\delta\omega$C(X) = {ϕ, {b, c}, X} and ωC(X) = {ϕ, {c}, {b, c}, X}. We have A = {c} is ω-closed but not $\delta\omega$-closed set in (X, τ).

Proposition 3.8 Every $\delta\omega$-closed set is δg-closed.
Proof: Let A be a $\delta\omega$-closed set and G be any open set containing A. Since every open set is semi-open and A is $\delta\omega$-closed, cl_δ (A) $\subseteq$ G. Therefore cl_δ (A) $\subseteq$ G and G is open. Hence A is δg-closed.
The converse of Proposition 3.8 need not be true as seen from the following example.

Example 3.9 Let X and τ be as in the Example 3.3. Then $\delta\omega$C(X) = {ϕ, {b, c}, X} and δGC(X) = {ϕ, {b}, {c}, {a, b}, {a, c}, {b, c}, X}. We have A = {a, c} is δg-closed but not $\delta\omega$-closed set in (X, τ).

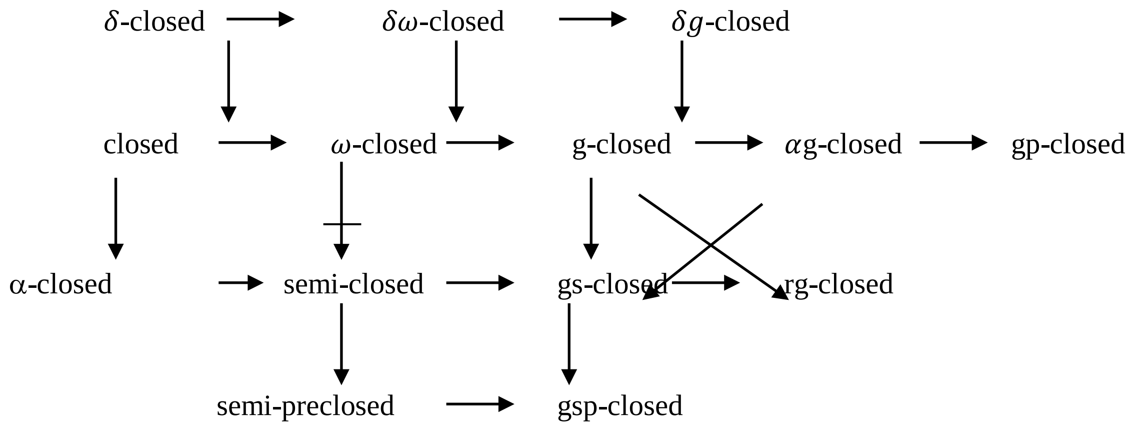
Remark: The following examples show that $\delta\omega$-closedness is independent of closedness, semi-closedness and α-closedness.

Example 3.10 Let X and τ be as in the Example 3.3. Then $\delta\omega$C(X) = {ϕ, {b, c}, X} and αC(X) = SC(X) = {ϕ, {b}, {c}, {b, c}, X}. We have A = {b} is α-closed as well as semi-closed in (X, τ) but it is not $\delta\omega$-closed set in (X, τ).

Example 3.11 Let X = {a, b, c} with τ = {ϕ, {a, b}, X}. Then $\delta\omega$C(X) = {ϕ, {c}, {a, c}, {b, c}, X} and αC(X) = SC(X) = {ϕ, {c}, X}. We have A = {a, c} is $\delta\omega$-closed but it is neither α-closed set nor semi-closed set in (X, τ).

Example 3.12 In Example 3.7, {c} is closed but not $\delta\omega$-closed set. In Example 3.12, {b, c} is $\delta\omega$-closed but not closed set.

Remark: From the above discussions and known results in [9, 10, 21, 24], we obtain the following diagram, where A $\rightarrow$ B (resp. A $\longleftrightarrow\!\!\!\!\!\!\!\!+\;\;\;$ B) represents A implies B but not conversely (resp. A and B are independent of each other).



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