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STOCHASTIC ANALYSIS OF DIABETIC WITH TWO ORGANS AND DIFFERENT DYSFUNCTIONING PROCESSES

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Abstract. This article we investigate the damage of the diabetic patient's of two organs A and B. These organs ' dysfunction can cause the person to die.
Organ A is subjected to accumulated cycle of damage and organ B has a steady risk of failure. Organ A damage occurs according to the Revised Erlang cycle and organ B has a system of da mage of varying rates of damage.Considering treatment of all the damage caused by the two or gans of a diabetic individual.Finally, to illustrate the main results, the numerical example is given.

Keywords: Diabetic nephropathy, Dysfunction, Erlang distribution, SCBZ.

1. Introduction

Diabetes is a disorder of metabolism caused by inadequate action of insulin, characterized by elevated blood sugar levels. Cardio-vascular disease causes people with diabetes to die. Diabetes nephropathy can cause kidney failure. Diabetes is the main cause of kidney dysfunction and the huge cost of dialysis. We are researching a diabetic person's case. This paper contains two diabetic organs subject to the damage process.

Either the two organs are equivalent to pancreas, head, liver or arms. Modified Erlang process and S CBZ failure characteristics of organ B. We apply a new concept that Raja Rao has introduced as setting the Clock Back to Zero (SCBZ) [10] and studied for damage process by Murthy. et. all [9]. Due to the fact that gestational diabetes mellitus is temporary condition that happens during pregnancy and goes away after birth, there is an exponential distribution of the level of damage from one frequency to another. The time for hospitalization of the patient (T) is T= min{T1,T2}, in which T1 and T2 are the times which affect organ A and organ B. The models are obtained from Joint

Laplace Stieltjes that transforms time into hospitalization and diagnosis time distributions. The numerical explanations were given at various times at the end of the session

2. Mathematical Model and Assumptions

In this model, we find the system with SCBZ dysfunctioning property in which organ A has operating time given by updated Erlang distribution organ B.

(i) Organ A is given with an exponential distribution with the parameter ' μ ' before hospitalization at most k observation times. When the first cumulative observation period is finished, α-probability or second observation treatment starts with β-probability where α+β=1 ends.
The process is repeated for i observation for 1≤i≤k-1.
Subsequently completion of the study of k^{th}, treatment is performed with probability equal to 1.
If T_1 is the time for treatment due to organ A, where $T_1=\sum_{j=1}^{k} X_j$ with probability to $\alpha\,\beta^{k-1}$for 1≤i≤k-1.

(ii) Organ B has a damage cycle with varying damage levels after an exponential time, organ B dam age level is a period with parameter c.Let T 0 be the point of truncation which changes the parameter.

(iii) Let T_2 be the time when organ B hospital treatment needs immediate treatment.

(iv) When the procedure is done due to organ a dysfunction, the treatment time corresponding to i^{th} observation is R_i , $1 \le i \le k$ where hospitalization is due to organ B dysfunction(y) such that$\int_0^y y\, dR(y) < \infty$. The treatments are done one by one. Treatment starts at the time T={T 1,T 2} based on the assumptions. The pdf of time T1 is used to calculate the exponential step time of the modified Erlangian

$$f(x) = \alpha e^{-\mu x} \mu \sum_{i=0}^{k-2} \frac{(\mu x)^i}{i!} \beta_i + \beta^{k-1} \mu \frac{(\mu x)^{k-1}}{(k-1)!} e^{-\mu x} \qquad (2.1)$$

The pdf of time T_2 is given by$h(x) = \begin{cases} a\, e^{-ax}, & \text{if } x \le T_0 \\ b e^{-bx} e^{(b-a)T_0}, & if\ x > T_0 \end{cases}$

T_0 Is random variable with Pdf =ce^{-cT_0}.Now

$$h(x) = a e^{-ax} e^{cx} + b e^{-bx} \int_0^x e^{(b-a)T_0}\, c e^{-cT_0} dT_0 = \frac{(a-b)(c+a)}{(c+a-b)} e^{-(c+a)x} + \frac{cb\ e^{-bx}}{(c+a-b)} \qquad (2.2)$$

With distribution function

$$H(X) = 1 - p\, e^{-(c+a)x} - q e^{-bx} \qquad (2.3)$$

Where $p = \frac{a-b}{c+a-b}$ $q = \frac{c}{(c+a-b)}$ and p+q=1.

The treatment time $R^*t = \sum_{j=1}^{i} R_j$ when damage occurs

In case of damage due to group B and I Observations for organ A are completed due to organ A for 1 locality, $1 \le i \le k$, R^*t=$\sum_{j=0}^{i} R_j$+R . The pdf T and R*t is given by

$$\frac{\partial^2}{\partial x \partial y} P(T \le x, R^*t \le y) = (1-H(x)) [\mu e^{-\mu x} \alpha r^*(y) + \mu \frac{(\mu x)}{1!} e^{-\mu x} \alpha \beta r^{*2}(y) + \mu \frac{(\mu x)^{2)}}{2!} e^{-\mu x} \alpha \beta^2 r^{*3}(y) + \ldots$$

$+\mu \frac{(\mu x)^{k-2}}{(k-2)!} e^{-\mu x} \alpha\beta^{k-2} r^{*(k-1)}$(y) + $\mu \frac{(\mu x)^{k-1}}{(k-1)!} e^{-\mu x} \alpha\beta^{k-1} r^{*(k)}$(y)] +h(x) $\sum_{i=0}^{k-1} e^{-\mu x} \frac{(\mu x)^{i}}{i!} \beta^{i} r^{*(i+1)}$(y)
(2.4)

The first term is organ A damage and the second term is usually organ B hazard using (2.2) and (2.3).

We find $\frac{\partial^2}{\partial x \partial y}$ P (T$\leq x, R^*$t$\leq y$)= [p$e^{-(c+a)x}$+qe^{-bx}]$\sum_{i=0}^{k-2} \alpha\, e^{-\mu x} \mu \frac{(\mu x)^{i}}{i!} \beta^{i} r^{*(i+1)}$(y) + [p$e^{-(c+a)x}$+ q$e^{-bx}$] $e^{-\mu x} \mu \frac{(\mu x)^{k-1}}{(k-1)!} \beta^{k-1} r^{*(k)}$(y)+[p(c+a)$e^{-(c+a)x}$+qb$e^{-bx}$] $\sum_{i=0}^{k-1} e^{-\mu x} \frac{(\mu x)^{i}}{i!} \beta^{i} r^{*(i+1)}$ (y) (2.5)

by double Laplace transform

$$E(e^{-\delta T} e^{\xi R^* t}) = \left\{\frac{Pr^*(\xi)}{\mu+\delta+c+a-\mu\beta r^*(\xi)}\right\} \times [c+a+\alpha\mu - \alpha\mu \left(\frac{\mu\beta r^*(\xi)}{\mu+\delta+c+a}\right)^{k-1} - (c+a)\left(\frac{\mu\beta r^*(\xi)}{\mu+\delta+c+a}\right)^{k}] +$$

$$\left(\frac{qr^*(\xi)}{\{\mu+\delta+b-\mu\beta r^*(\xi)\}}\right)[\mu\alpha + b - \mu\alpha\left(\left(\frac{\mu\beta r^*(\xi)}{\mu+\delta+b}\right)^{k-1} - b\left(\left(\frac{\mu\beta r^*(\xi)}{\mu+\delta+b}\right)^{k}\right] + p\, \beta^{k-1}\left(\frac{\mu r^*(\xi)}{\mu+\delta+b}\right)^{k}\right] \quad (2.6)$$

for $\xi = 0$ and $\delta = 0$ we obtain from (2.6)

$$E\,(e^{-\delta t}) = \frac{p\mu c x[1-(\frac{\mu\beta}{\delta+c+a+\mu\alpha})^{k-1}]}{(\delta+c+a+\mu\alpha)} + \frac{q\mu\alpha[1-(\frac{\mu\beta}{\mu+c+b})^{k-1}}{(c+b+\mu\alpha)}$$

$$=p\, \beta^{k-1}\left(\frac{\mu}{\mu+\delta+c+a}\right)^{k} + q\, \beta^{k-1}\left(\frac{\mu}{\mu+\delta+b}\right)^{k} + \frac{p(c+a)[1-(\frac{\mu\beta}{\mu+\delta+c+a})^{k}]}{(\delta+c+a+\mu\alpha)} + \frac{qb[1-(\frac{\mu\beta}{\mu+\delta+b})^{k}]}{(\delta+b+\mu\alpha)} \quad (2.7)$$

$$E\,(e^{-\xi R^* t}) = [\frac{r^{\wedge *}(\xi)p}{\mu+c=a-\mu\beta r^*(\xi)}] \times [c+a+\mu\alpha - \mu\alpha\left(\left(\frac{\mu\beta r^*(\xi)}{\mu+c+a}\right)^{k-1}\right) - (c+a)\left(\frac{\mu\beta r^*(\xi)}{\mu+c+a}\right)^{k}] + [\frac{qr^*(\xi)}{\mu+b-\mu\beta r^*(\xi)}][b+\mu\alpha - \mu\alpha\left(\frac{\mu\beta r^*(\xi)}{\mu+b}\right)^{k-1} - b\left(\frac{\mu\beta r^*(\xi)}{\mu+b}\right)^{k}] + p\beta^{k-1}\left(\frac{\mu r^*(\xi)}{\mu+c+a}\right)^{k} + q\beta^{k-1}\left(\frac{\mu r^*(\xi)}{\mu+b}\right)^{k} \quad (2.8)$$

From (2.7) and (2.8) by differentiating

$$E\,(T) = \frac{p}{c+a+\mu\alpha}[1-(\frac{\mu\beta}{\mu+c+a})^{k}] + \frac{q}{b+\mu\alpha}[1-(\frac{\mu\beta}{\mu+b})^{k}\ ]$$

$$E(R^* t) = E\,(R^*)\ \left\{1 + \frac{p\beta\mu}{c+a+\mu\alpha}[1-(\frac{\mu\beta}{\mu+c+a})^{k-1}] + \frac{q\beta\mu}{b+\mu\alpha}\left[1-(\frac{\mu\beta}{b+\mu})^{k-1}\right]\right\} \quad (2.9)$$

Where p=$\frac{a-b}{c+a-b}$and q=$\frac{c}{c+a-b}$.

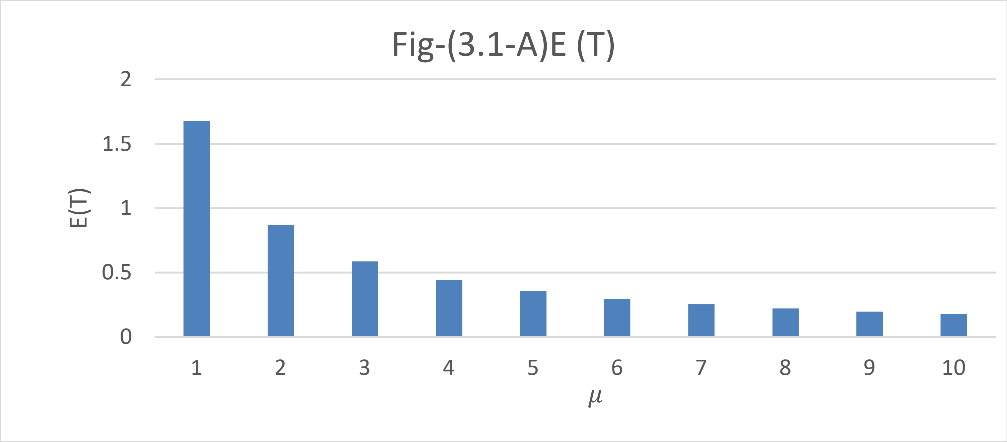
3. Mathematical Results with Numerical Examples

By giving the parameters in E (T) and E (R^*t) different values and varying μ from 1 to 10, we represent the E (T) and E (R^*t) graph. Table 3.1

$a = 0.05, b = 0.02, c = 0.01, E(R^*)$ =3, k=2,$\alpha = 0.2, \beta = 0.8$

μ	1	2	3	4	5	6	7	8	9	10
E (T)	1.679	0.869	0.586	0.442	0.355	0.296	0.255	0.223	0.198	0.179

E($R^{*}t$)	3.815	5.342	5.36	5.37	5.376	5.38	5.383	5.386	5.387	5.388



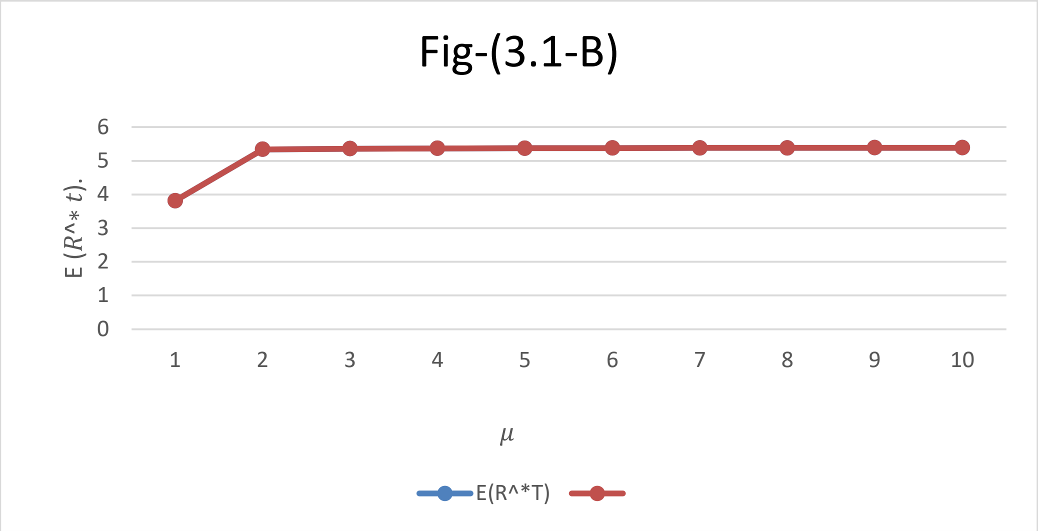
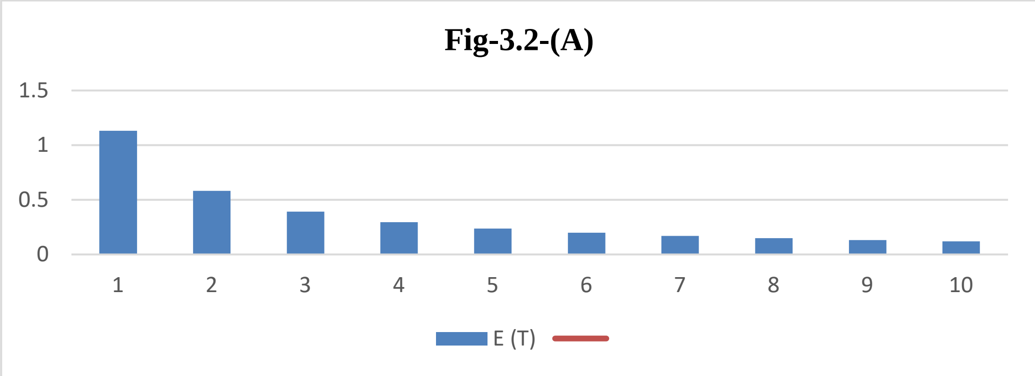
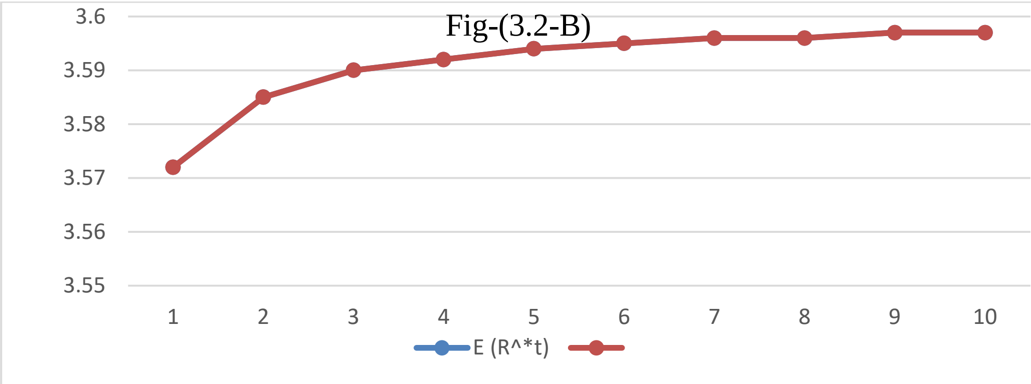


Table-3.2

$\alpha = 0.05, b = 0.02, c = 0.01, E(R^{*})$ =3,k=2,$\alpha = 0.8, \beta = 0.2$

μ	1	2	3	4	5	6	7	8	9	10
E (T)	1.13	0.582	0.392	0.296	0.236	0.198	0.17	0.149	0.132	0.119
E($R^{*}t$)	3.572	3.585	3.59	3.592	3.594	3.595	3.596	3.596	3.597	3.597





4. Conclusion

Finally, we conclude that, our mathematical model leads that from the table 3.1 and 3.2 shows that the functions of E(T) and E(R*t). Before mean time treatment for fixed values of arbitrary constants for E(R*t). Both cases the parameter μupturns then the value of E(T) also upturns and E(R*t) diminishes. When α increases both E(T) And E(R*t)diminishes. It will be use full for in the field of medicine.

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