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COMMON FIXEDPOINT THEOREMS IN COMPLETE INTUITIONISTIC FUZZY METRIC SPACES USING C – CONTRACTION

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Abstract In this paper we introduced weakly C- contractive in a complete intuitionistic fuzzy metric space, using weakly compatible mapping. We established a common fixed point theorem in a complete intuitionistic fuzzy metric space.

Keywords Complete Intuitionistic fuzzy metric space, Common Fixed Point,Self mapping, Weakly compatible, C – contraction, Weakly C- contractive.

1. Introduction

In 1986 Atanassov introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. In 2004 Park defined the notion of intuitionistic fuzzy metric space with the help of continuous t- norms and continuous t- conforms. Recently, (Alaca, Turkoglu, Yildiz 2004) defined the notion of intuitionistic fuzzy metric space by making use of intuitionistic fuzzy metric sets, with the help of continuous t- norm and continuous t- conforms as a generalization of fuzzy metric space. In 2008 Alaca, Altun Turkoglu, introduced compatible mappings in intuitionistic fuzzy metric space. In 2006, Turkoglu et al. extended the notion of compatible mappings to intuitionistic fuzzy metric space. Alaca weakened the notion of compatibility by using the notion of weakly compatible maps in intuitionistic fuzzy metric space and showed that every pair of compatible mappings is weakly compatible, but reverse is not true. Many authors have proved a number of fixedpoint theorems for different contractions in intuitionistic fuzzy metric space. The aim of this paper is to introduce C-contractive in a complete intuitionistic fuzzy metric space, using weakly compatible mapping.

2. Preliminaries

Definition 2.1 A mapping $T : X \to X$ where $(X, M, N.*,\diamond)$ is an intuitionistic fuzzy metric space is said to be a C-contration if there exists $\alpha \in \left(0, \frac{1}{2}\right)$ such that for all $x, y \in X$the following inequality holds:

$$M(Tx, Ty.kt) \geq \alpha\big(M(x, Ty, t) + M(y, Tx, t)\big)$$
$$N(Tx, Ty.kt) \geq \alpha\big(N(x, Ty, t) + N(y, Tx, t)\big).$$

Definition 2.2 (Alaca C, Altun I Turkoglu D. 2008) Let $(X, M, N.*,\Diamond)$ be intuitionistic fuzzy metric space and for all $x, y \in X$, $t > 0$ and if for a number $k \in (0,1)$, $M(x, y, kt) \geq M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$. Then x = y.

Definition 2.3 Let T and S be self- mappings of an intuitionistic fuzzy metric space: $(X, M, N.*,\diamond)$ and $C(T, S) := \{x \in X: T(x) = S(x)\}$. Then pair (T, S) is called weakly compatible if: $T(S(x)) = S(T(x))$ for all $x \in C(T, S)$.

3. Main Theorem:

Definition 3.1 A mapping $T: X \to X$ where $(X, M, N.*,\diamond)$ is an intuitionistic fuzzy metric space is said to be weakly C-contractive if for all $x, y \in X$,

$$M(Tx, Ty.kt) \geq \frac{1}{2}\Big(M(x, Ty, t) + M(y, Tx, t) - \phi\big(M(x, Ty, t), M(y, Tx, t)\big)\Big), \qquad (3.1)$$
$$N(Tx, Ty.kt) \leq \frac{1}{2}\Big(N(x, Ty, t) + N(y, Tx, t) - \phi\big(N(x, Ty, t), N(y, Tx, t)\big)\Big)$$

Where $\phi: [0, \infty)^2 \to [0, \infty)$ is a continuous function such that $\phi(x, y) = 0$if and only if $x = y = 0$.

Theorem 3.2 Let $(X, M, N.*,\diamond)$ be a complete intuitionistic fuzzy metric space and let E be a non empty closed subset of X. Let $T, S: E \to E$ be such that,

$$M(Tx, Sy.kt) \geq \frac{1}{2}\Big(M(Rx, Sy, t) + M(Ry, Tx, t) - \phi\big(M(Rx, Sy, t), M(Ry, Tx, t)\big)\Big),$$
$$N(Tx, Sy.kt) \leq \frac{1}{2}\Big(N(Rx, Sy, t) + N(Ry, Tx, t) - \phi\big(N(Rx, Sy, t), N(Ry, Tx, t)\big)\Big)$$

For every pair $(x, y) \in X \times X$, where $\phi: [0, \infty)^2 \to [0, \infty)$ is a continuous function such that $\phi(x, y) = 0$ if and only if $x = y = 0$and $R: E \to X$satisfying the following hypotheses:

(i) $TE \subseteq RE$ and $SE \subseteq RE$.

(ii) The pairs (T, R) and (S, R) are weakly compatible.

In addition, assume that $R(E)$ is a closed subset of X. Then, T and R and S have a unique common fixed point.

Proof: Let $x_0 \in E$ be an arbitrary.Using (i) there exist twosequences $\{x_n\}_{n=0}^{\infty}$ and $\{y_n\}_{n=0}^{\infty}$ such that $y_0 = Tx_0 = Rx_1, y_1 = Sx_1 = Rx_2, y_2 = Tx_2 = Rx_3, \ldots\ldots, y_{2n} = Tx_{2n} = Rx_{2n+1}, y_{2n+1} = Sx_{2n+1} = Rx_{2n+2}, \ldots..$

We complete the proof in three steps.

Step I. We will prove that $\lim_{n\to\infty} M(y_n, y_{n+1}.kt) = 0$ *and* $\lim_{n\to\infty} N(y_n, y_{n+1}.kt) = 0$.

Let $n = 2k$. Using condition (3.1), we obtain that

$$M(y_{2k+1}, y_{2k}, kt) = M(Tx_{2k}, Sx_{2k+1}, kt)$$
$$\geq \frac{1}{2}\big(M(Rx_{2k}, Sx_{2k+1}, t) + M(Rx_{2k+1}, Tx_{2k}, t)\big) - \phi\big(M(Rx_{2k}, Sx_{2k+1}, t), M(Rx_{2k+1}, Tx_{2k}, t)\big)$$

$= \frac{1}{2}\big(M(y_{2k-1}, y_{2k+1}, t) + M(y_{2k}, y_{2k}, t)\big) - \phi\big(M(y_{2k-1}, y_{2k+1}, t), M(y_{2k}, y_{2k}, t)\big)(2)$
$\geq \frac{1}{2} M(y_{2k-1}, y_{2k+1}, t)$
$\geq \frac{1}{2}\Big(M\big(y_{2k-1}, y_{2k,} t\big) + M(y_{2k}, y_{2k+1}, t)\Big).$

$N(y_{2k+1}, y_{2k}, kt) = N(Tx_{2k}, Sx_{2k+1}, kt)$
$\leq \frac{1}{2}\big(N(Rx_{2k}, Sx_{2k+1}, t) + N(Rx_{2k+1}, Tx_{2k}, t)\big) - \phi\big(N(Rx_{2k}, Sx_{2k+1}, t), N(Rx_{2k+1}, Tx_{2k}, t)\big)$
$= \frac{1}{2}\big(N(y_{2k-1}, y_{2k+1}, t) + N(y_{2k}, y_{2k}, t)\big) - \phi\big(N(y_{2k-1}, y_{2k+1}, t), N(y_{2k}, y_{2k}, t)\big)$ (3.2)
$\leq \frac{1}{2} N(y_{2k-1}, y_{2k+1}, t)$
$\leq \frac{1}{2}\Big(N\big(y_{2k-1}, y_{2k,} t\big) + N(y_{2k}, y_{2k+1}, t)\Big).$

Hence,
$$M\left(y_{2k+1}, y_{2k}, kt\right) \geq M\left(y_{2k}, y_{2k-1}, t\right)$$
$$N\left(y_{2k+1}, y_{2k}, kt\right) \leq N\left(y_{2k}, y_{2k-1}, t\right).$$

If $n = 2k+1$, similarly we can prove that
$$M\left(y_{2k+2}, y_{2k+1}, kt\right) \geq M\left(y_{2k+1}, y_{2k}, t\right)$$
$$N(y_{2k+2}, y_{2k+1}, kt) \leq N\left(y_{2k+1}, y_{2k}, t\right).$$

Thus $M(y_{n+1}, y_n, kt), N(y_{n+1}, y_n, kt)$ is a decreasing sequence of nonnegative real numbers and hence convergent.

Assume that, $\lim_{n\to\infty} M(y_{n+1}, y_n, kt) = r$, $\quad \lim_{n\to\infty} N(y_{n+1}, y_n, kt) = r$.

From the above argument we have
$$M(y_{n+1}, y_n, kt) \geq \frac{1}{2} M(y_{n-1}, y_{n+1}, t)$$
$$\geq \frac{1}{2}\big(M(y_{n-1}, y_n, t) + M(y_n, y_{n+1}, t)\big).$$
$$N(y_{n+1}, y_n, kt) \leq \frac{1}{2} N(y_{n-1}, y_{n+1}, t)$$
$$\leq \frac{1}{2}\big(N(y_{n-1}, y_n, t) + N(y_n, y_{n+1}, t)\big).$$

if $n \to \infty$, we have
$$r \leq \lim_{n\to\infty} \frac{1}{2} M(y_{n-1}, y_{n+1}, kt) \leq r$$
$r \leq \lim_{n\to\infty} \frac{1}{2} N(y_{n-1}, y_{n+1}, kt) \leq r,$

Therefore,
$$\lim_{n\to\infty} M(y_{n-1}, y_{n+1}, kt) = 2r$$
$$\lim_{n\to\infty} N(y_{n-1}, y_{n+1}, kt) = 2r.$$

We have proved in (3.2):
$$M(y_{2k+1}, y_{2k}, kt) = M(Tx_{2k}, Sx_{2k+1}, kt)$$
$$\geq \frac{1}{2}\big(M(y_{2k-1}, y_{2k+1}, t) + M(y_{2k}, y_{2k}, t)\big)$$
$$-\phi\big(M(y_{2k-1}, y_{2k+1}, t), M(y_{2k}, y_{2k}, t)\big).$$
$$N(y_{2k+1}, y_{2k}, kt) = N(Tx_{2k}, Sx_{2k+1}, kt)$$
$$\leq \frac{1}{2}\big(N(y_{2k-1}, y_{2k+1}, t) + N(y_{2k}, y_{2k}, t)\big)$$
$$-\phi\big(N(y_{2k-1}, y_{2k+1}, t), N(y_{2k}, y_{2k}, t)\big).$$

Now, if $k \to \infty$ and using the continuity of φ we obtain

$$r \leq \frac{1}{2} 2r - \phi(2r, 0),$$

and consequently, $\varphi(2r, 0) = 0$. This gives us that:

$$r = \lim_{n\to\infty} M(y_n, y_{n+1}, kt) = 0, r = \lim_{n\to\infty} N(y_n, y_{n+1}, kt) = 0 \qquad (3.3)$$

By our assumption about φ.

Step II. $\{y_n\}$ is Cauchy.

Since $M(y_{n+1}, y_{n+2}. kt) \geq M(y_n, y_{n+1}, kt)$, and $N(y_{n+1}, y_{n+2}. kt) \leq N(y_n, y_{n+1}, kt)$ it is sufficient to show that the subsequence $\{y_{2n}\}$ is a Cauchy sequence. Suppose that $\{y_{2n}\}$ is not a Cauchy sequence. Then there exists $\varepsilon > 0$ for which we can find subsequence $\{y_{2m(k)}\}$ and $\{y_{2n(k)}\}$ of $\{y_{2n}\}$ such that $n(k)$ is the least index for which $n(k) > m(k) > k$ and $M(y_{2m(k)}, y_{2n(k)}, kt) \geq \varepsilon$, $N(y_{2m(k)}, y_{2n(k)}, kt) \leq \varepsilon$.

This means that:

$$M(y_{2m(k)}, y_{2n(k)-2}, kt) < \varepsilon, N(y_{2m(k)}, y_{2n(k)-2}, kt) < \varepsilon \ .$$

From triangle inequality:

$$\varepsilon \leq M(y_{2m(k)}, y_{2n(k)}, kt) > M(y_{2m(k)}, y_{2n(k)-2}, kt)$$
$$+M(y_{2n(k)-2}, y_{2n(k)-1}, kt) + M(y_{2n(k)-1}, y_{2n(k)}, kt)(7)$$
$$\geq \varepsilon + M(y_{2n(k)-2}, y_{2n(k)-1}, kt) + M(y_{2n(k)-1}, y_{2n(k)}, kt).$$
$$\varepsilon \leq N(y_{2m(k)}, y_{2n(k)}, kt) < N(y_{2m(k)}, y_{2n(k)-2}, kt)$$
$$+N(y_{2n(k)-2}, y_{2n(k)-1}, kt) + N(y_{2n(k)-1}, y_{2n(k)}, kt)$$
$$\leq \varepsilon + N(y_{2n(k)-2}, y_{2n(k)-1}, kt) + N(y_{2n(k)-1}, y_{2n(k)}, kt).$$

Letting $k \to \infty$ and using (3.3) we can conclude that :

$$\lim_{k\to\infty} M(y_{2m(k)}, y_{2n(k)}, kt) = \varepsilon, \quad \lim_{k\to\infty} N(y_{2m(k)}, y_{2n(k)}, kt) = \varepsilon \ . \qquad (3.4)$$

Moreover, we have

$$|M(y_{2m(k)}, y_{2n(k)+1}, kt) - M(y_{2m(k)}, y_{2n(k)}, kt)| \geq M(y_{2n(k)}, y_{2n(k)+1} t) \quad (3.5)$$
$$|N(y_{2m(k)}, y_{2n(k)+1}, kt) - N(y_{2m(k)}, y_{2n(k)}, kt)| \leq N(y_{2n(k)}, y_{2n(k)+1} t)$$

and

$$|M(y_{2n(k)}, y_{2m(k)-1}, kt) - M(y_{2n(k)}, y_{2m(k)}, kt)| \geq M(y_{2m(k)}, y_{2m(k)-1}, t) \quad (3.6)$$
$$|N(y_{2n(k)}, y_{2m(k)-1}, kt) - N(y_{2n(k)}, y_{2m(k)}, kt)| \leq N(y_{2m(k)}, y_{2m(k)-1}, t)$$

and

$$|M(y_{2n(k)}, y_{2m(k)-2}, kt) - M(y_{2n(k)}, y_{2m(k)-1}, kt)| \geq M(y_{2m(k)-2}, y_{2m(k)-1}, t) \quad (3.7)$$
$$|N(y_{2n(k)}, y_{2m(k)-2}, kt) - N(y_{2n(k)}, y_{2m(k)-1}, kt)| \leq N(y_{2m(k)-2}, y_{2m(k)-1}, t)$$

Using (3.3), (3.4) , (3.5), (3.6) and (3.7) we get:

$$\lim_{k\to\infty} M(y_{2m(k)-1}, y_{2n(k)}, kt) = \lim_{k\to\infty} M(y_{2m(k)-1}, y_{2n(k)-1}, kt)$$
$$= \lim_{k\to\infty} M(y_{2m(k)-2}, y_{2n(k)}, kt) = \varepsilon \qquad (3.8)$$
$$\lim_{k\to\infty} N(y_{2m(k)-1}, y_{2n(k)}, kt) = \lim_{k\to\infty} N(y_{2m(k)-1}, y_{2n(k)-1}, kt)$$
$$= \lim_{k\to\infty} N(y_{2m(k)-2}, y_{2n(k)}, kt) = \varepsilon.$$

Now, from (3.1) we have:

$$M(y_{2m(k)-1}, y_{2n(k)}, kt) = M(Tx_{2n(k)}, Sx_{2m(k)-1}, kt)$$
$$\geq \frac{1}{2}\left(M(Rx_{2n(k)}, Sx_{2m(k)-1}, t) + M(Rx_{2m(k)-1}, Tx_{2n(k)}, t)\right)$$
$$-\phi\left(M(Rx_{2n(k)}, Sx_{2m(k)-1}, t), M(Rx_{2m(k)-1}, Tx_{2n(k)}, t)\right)$$
$$= \frac{1}{2}\left(M(y_{2n(k)-1}, y_{2m(k)-1}, t) + M(y_{2m(k)-2}, y_{2n(k)}, t)\right)$$
$$-\phi\left((M(y_{2m(k)-1}, y_{2m(k)}, t), M(y_{2m(k)}, y_{2m(k)+1}, t)\right)$$
$$N(y_{2m(k)-1}, y_{2n(k)}, kt) = N(Tx_{2n(k)}, Sx_{2m(k)-1}, kt)$$

$$\le \frac{1}{2}\Big(N\big(Rx_{2n(k)},Sx_{2m(k)-1},t\big)+N\big(Rx_{2m(k)-1},Tx_{2n(k)},t\big)\Big)$$
$$-\phi\left(N\big(Rx_{2n(k)},Sx_{2m(k)-1},t\big),N\big(Rx_{2m(k)-1},Tx_{2n(k)},t\big)\right)$$
$$=\frac{1}{2}\Big(N\big(y_{2n(k)-1},y_{2m(k)-1},t\big)+N\big(y_{2m(k)-2},y_{2n(k)},t\big)\Big)$$
$$-\phi\left((N\big(y_{2m(k)-1},y_{2m(k)},t\big),N\big(y_{2m(k)},y_{2m(k)+1},t\big)\right).$$

if $k\to\infty$ in the above inequality, from (3.8) and the continuity of φ, we have:

$$\varepsilon \le \frac{1}{2}(\varepsilon+\varepsilon)-\varphi(\varepsilon,\varepsilon),$$

and from the last inequality $\varphi(\varepsilon,\varepsilon)=0$. By our assumption about φ, we have $\varepsilon=0$ which is a contradiction.

Step III. T,S and R have a common fixed point. Since $(X,M,N.*,\Diamond)$ is complete and $\{y_n\}$ is Cauchy, there exists $z\in X$ such that: $\lim_{n\to\infty} y_n=z.$ Since E is closed and $\{y_n\}\subseteq E,$ we have $z\in E.$ By assumption $R(E)$ is closed, so there exists $u\in E$ such that $z=Ru.$

For all $n\in N,$

$$M(Tu,y_{2n+1},kt)=M(Tu,Sx_{2n+1},kt)$$
$$\ge \frac{1}{2}\big(M(Ru,Sx_{2n+1},t)+M(Rx_{2n+1},Tu,t)\big)$$
$$-\phi\big(M(Ru,Sx_{2n+1},t),M(Rx_{2n+1},Tu,t)\big)$$
$$=\frac{1}{2}\big(M(z,y_{2n+1},t)+M(y_{2n},Tu,t)\big)$$
$$-\phi\big(M(Ru,Sx_{2n+1},t),M(Rx_{2n+1},Tu,t)\big)$$
$$N(Tu,y_{2n+1},kt)=N(Tu,Sx_{2n+1},kt)$$
$$\le \frac{1}{2}\big(N(Ru,Sx_{2n+1},t)+N(Rx_{2n+1},Tu,t)\big)$$
$$-\phi\big(N(Ru,Sx_{2n+1},t),N(Rx_{2n+1},Tu,t)\big)$$
$$=\frac{1}{2}\big(N(z,y_{2n+1},t)+N(y_{2n},Tu,t)\big)$$
$$-\phi\big(N(Ru,Sx_{2n+1},t),N(Rx_{2n+1},Tu,t)\big)$$

If $n\to\infty$,

$$M(Tu,z,kt)\ge \frac{1}{2}\big(M(z,z,t)+M(z,Tu,t)\big)-\phi\big(M(Ru,z,t),M(z,Tu,t)\big)$$
$$N(Tu,z,kt)\le \frac{1}{2}\big(N(z,z,t)+N(z,Tu,t)\big)-\phi\big(N(Ru,z,t),N(z,Tu,t)\big)$$

and hence

$$\phi(0,M(z,Tu,kt))\ge \frac{1}{2}(M(Tu,z,kt))\ge 0,$$
$$\phi(0,N(z,Tu,kt))\le \frac{1}{2}(N(Tu,z,kt))\le 0.$$

Therefore $M(z,Tu,kt)=0, N(z,Tu,kt)=0$. Therefore $Tu=z.$

Similarly $Su=z.$ So $Tu=Su=Ru=z.$ Since the pairs (R,T) and (R,S) are weakly compatible, we have $Tz=Sz=Rz.$

Now we can have

$$M(Tz,y_{2n+1},kt)=M(Tz,Sx_{2n+1},kt)$$
$$\ge \frac{1}{2}\big(M(Rz,Sx_{2n+1},t)+M(Rx_{2n+1},Tz,t)\big)$$
$$-\phi\big(M(Rz,Sx_{2n+1},t),M(Rx_{2n+1},Tz,t)\big)$$
$$=\frac{1}{2}\big(M(Rz,y_{2n+1},t)+M(y_{2n},Tz,t)\big)$$

$$-\phi\big(M(Rz,y_{2n+1},t),M(y_{2n},Tz,t)\big)$$
$$N(Tz,y_{2n+1},kt)=N(Tz,Sx_{2n+1},kt)$$
$$\leq\frac{1}{2}\big(N(Rz,Sx_{2n+1},t)+N(Rx_{2n+1},Tz,t)\big)$$
$$-\phi\big(N(Rz,Sx_{2n+1},t),N(Rx_{2n+1},Tz,t)\big)$$
$$=\frac{1}{2}\big(N(Rz,y_{2n+1},t)+N(y_{2n},Tz,t)\big)$$
$$-\phi\big(N(Rz,y_{2n+1},t),N(y_{2n},Tz,t)\big).$$

If $n\to\infty$, since $Tz=Sz=Rz$, we obtain

$$M(Tz,z,kt)=\frac{1}{2}\big(M(Tz,z,t)+M(z,Tz,t)\big)-\phi\big(M(Tz,z,t),M(z,Tz,t)\big)(16)$$
$$N(Tz,z,kt)=\frac{1}{2}\big(N(Tz,z,t)+N(z,Tz,t)\big)-\phi\big(N(Tz,z,t),N(z,Tz,t)\big)$$

Hence, $\phi\big(M(Tz,z,t),M(z,Tz,t)\big)=0,\phi\big(N(Tz,z,t),N(z,Tz,t)\big)=0$ and so $M(Tz,z,t)=0,N(Tz,z,t)=0$. Therefore $Tz=z$ and from $Tz=Sz=Rz$ we conclude that $Tz=Sz=Rz=z$. Uniqueness of the common fixed point follows from (3.1).

Theorem 3.3 Let $(X,M,N.*,\Diamond)$ be a complete intuitionistic fuzzy metric space and T,S,φ and R verifying the conditions of Theorem 3.2. Assume that R is a continuous function on X. In addition for all $x\in X$:

$$M(RTx,TRx,kt)\geq(M(Rx,Tx,kt)\ \ and\ \ M(RSx,SRx,kt)\geq M(Rx,Sx,kt)$$
$$N(RTx,TRx,kt)\leq(N(Rx,Tx,kt)\ \ and\ \ N(RSx,SRx,kt)\leq N(Rx,Sx,kt)$$

Then, T and R and S have a unique common fixed point.

Proof: If we review the proof of Theorem 3.2, we obtain that $\{y_n\}$ is a Cauchy sequence converging to some $z\in X$.

We know that

$$z=\lim_{n\to\infty}y_{2n}=\lim_{n\to\infty}Tx_{2n}=\lim_{n\to\infty}Rx_{2n+1}=\lim_{n\to\infty}y_{2n+1}=\lim_{n\to\infty}Sx_{2n+1}=\lim_{n\to\infty}Rx_{2n+2}$$

Since R is continuous, Ry_n converges to Rz.

On the other hand,

$$M(Ty_{2n+1},Rz,kt)\geq M(Ty_{2n+1},Ry_{2n+2},t)+M(Ry_{2n+2},Rz,t)$$
$$=M(TRx_{2n+2},RTx_{2n+2},t)+M(Ry_{2n+2},Rz,t)$$
$$\geq M(Tx_{2n+2},Rx_{2n+2},t)+M(Ry_{2n+2},Rz,t)$$
$$=M(y_{2n+2},y_{2n+1},t)+M(Ry_{2n+2},Rz,t)$$
$$N(Ty_{2n+1},Rz,kt)\leq N(Ty_{2n+1},Ry_{2n+2},t)+N(Ry_{2n+2},Rz,t)$$
$$=N(TRx_{2n+2},RTx_{2n+2},t)+N(Ry_{2n+2},Rz,t)$$
$$\leq N(Tx_{2n+2},Rx_{2n+2},t)+N(Ry_{2n+2},Rz,t)$$
$$=N(y_{2n+2},y_{2n+1},t)+N(Ry_{2n+2},Rz,t)$$

Therefore $\lim_{n\to\infty}M(Ty_{2n+1},Rz,kt)=0,\lim_{n\to\infty}N(Ty_{2n+1},Rz,kt)=0,$ and we can have

$$M(Ty_{2n+1},Sz,kt)$$
$$\geq\frac{1}{2}\big(M(Ry_{2n+1},Sz,t)+M(Rz,Ty_{2n+1},t)\big)$$
$$-\phi\big(M(Ry_{2n+1},Sz,t),M(Rz,Ty_{2n+1},t)\big)$$
$$N(Ty_{2n+1},Sz,kt)$$
$$\leq\frac{1}{2}\big(N(Ry_{2n+1},Sz,t)+N(Rz,Ty_{2n+1},t)\big)-\phi\big(N(Ry_{2n+1},Sz,t),N(Rz,Ty_{2n+1},t)\big)$$

If $n\to\infty$, we have

$$M(Rz,Sz,kt)\geq\frac{1}{2}\big(M(Rz,Sz,t)+M(Rz,Rz,t)\big)-\phi\big(M(Rz,Sz,t),M(Rz,Rz,t)\big)$$

$$N(Rz,Sz,kt) \le \frac{1}{2}\big(N(Rz,Sz,t)+N(Rz,Rz,t)\big)-\phi\big(N(Rz,Sz,t),N(Rz,Rz,t)\big)$$

So,

$$\frac{1}{2}\big(M(Rz,Sz,kt)\big) \ge -\phi((M(Rz,Sz,t),0)),$$
$$\frac{1}{2}\big(N(Rz,Sz,kt)\big) \le -\phi((N(Rz,Sz,t),0))$$

and hence $Sz = Rz$. We can analogously prove that $Tz = Rz$. That is, $Tz = Sz = Rz = a$.

Using weak compatibility of the pairs (T,R) and (S,R) we have $Ra = Ta = Sa$. So

$$M(Ta,a,kt) = M(Ta,Sz,kt) \ge \frac{1}{2}\big(M(Ra,Sz,t)+M(Ra,Ta,t)\big)-\phi\big((M(Ra,Sz,t),M(Rz,Ta,t)\big)$$
$$\ge \frac{1}{2}\big(M(Ta,a,t)+M(a,Ta,t)\big)-\phi\big((M(Ta,a,t),M(a,Ta,t)\big)$$
$$N(Ta,a,kt) = N(Ta,Sz,kt) \le \frac{1}{2}\big(N(Ra,Sz,t)+N(Ra,Ta,t)\big)-\phi\big((N(Ra,Sz,t),N(Rz,Ta,t)\big)$$
$$\le \frac{1}{2}\big(N(Ta,a,t)+N(a,Ta,t)\big)-\phi\big((N(Ta,a,t),N(a,Ta,t)\big).$$

That is $\varphi(M(Ta,a,t),N(a,Ta,t))=0, \varphi(N(Ta,a,t),N(a,Ta,t))=0$ and this implies that $Ta = a$. Therefore $Ra = Ta = Sa = a$.

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