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INDEPENDENT STRONG SUPPORT DOMINATION & INDEPENDENTANTI- STRONG SUPPORT DOMINATION IN FUZZY GRAPH G(σ, μ) BY USING STRONG ARC

V.Senthilkumar [1] and C.Y. Ponnappan [2]

[1] Assistant Professor of Mathematics, TKS College of Arts and Science, Theni ,Tamilnadu,India

[2] Research Supervisor and Assistant Professor of Mathematics, Govt. Arts college, Melur.

Email:[1]1984senthilvkumar@gmail.com

Abstract: In this paper, definition of independent strong support domination and independent anti- strong support domination in fuzzy graph G(σ, μ) are newly introduced by using strong arc. Moreover, some standard theorems on strong support domination and anti- strong support domination in fuzzy graph g(σ, μ) are proved with example. Finally, results on independent strong support domination and independent anti- strong support domination for the standard fuzzy graph like fuzzy Paths FP_n , fuzzy cycle FC_n, fuzzy complete .. are found.

Keywords: Fuzzy graph, , strong arc , non strong arc, independent set, strong degree , strong Support , strong Support domination , anti- strong Support domination.

1. Introduction

In 1965, L.A Zadeh introduced the concept of fuzzy subset of a set as a way for representing uncertainity [23]. Zadeh's ideas stirred the interest of researchers worldwide. Monderson, J.N., Premchand, S.N. discussed fuzzy graph theory fuzzy hypergraph [8]. Fuzzy graph is the generalization of the ordinary graph. The formal mathematical definition of domination was given by Ore.O in 1962 [14]. In 1975, A. Rosenfeld introduced the notion of fuzzy graph and several analogues of theoretic concepts such as path, cycle and connectedness[18]. A.Somasundaram and S. Somasundaram discussed the domination in fuzzy graph using effective arc [22]. A.Nagoorgani and V.T. Chandrasekarn discussed the strong arc in fuzzy graph [12,13]. Bhutani, K.R., and A.Rosenfeld have introduced the concept of Storng arcs in fuzzy graph [2,3].Theory of independent domination was formalized by Berge and Ore in 1962 [1,14], the independent domination number and notation i(G) were introduced by Cockayane and Hedetniemi [4,5]. Several works on fuzzy graph are C.Y. Ponnappan, and V.Senthilkumar [15,16,17,19,20,21]. Before discuss the study of independent strong Support of the Fuzzy graphs, we are placed few preliminary.

A set is independent(or stable) if no two vertices in it are adjacent and the independence number of G, denoted by $\boldsymbol{\beta(G)}$, is the maximum size of an independent set in G. Independent domination sets have been studied extensively in the literature. A dominating set of a graph G is a set S of vertices of G such that every vertex not in S is adjacent to vertices in S. The domination number of G is denoted by $\boldsymbol{\gamma(G)}$, is the minimum size of a dominating set. An independent dominating set of G is a set that is both dominating and independent in G. The independent domination number of G is denoted by i(G) , is the minimum size of an independent dominating set. The dominating set of G of size $\boldsymbol{\gamma(G)}$ is called a $\boldsymbol{\gamma}$- set, while an independent domination set of size i(G) is called i – set.

2. Preliminaries

Definition 2.1. Fuzzy graph G(σ,μ) is pair of function $V \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$, where for all $u, v \; in \; V$,we have $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$.

Definition 2.2 The fuzzy graph $H(\tau,\rho)$ is called a fuzzy subgraph of G(σ,μ) if $\tau(u) \leq \sigma(u)$ for all u in V and $\rho(u,v) \leq \mu(u,v)$ for all $u, v \; in \; V$.

Definition 2.3 A fuzzy subgraph $H(\tau,\rho)$ is said to be a spanning sub graph of G(σ,μ) if $\tau(u) = \sigma(u)$ for all u in V. In this case the two graphs have the same fuzzy node set, they differ only in the arc weights.

Definition 2.4 Let G(σ,μ) be a fuzzy graph and τ be fuzzy subset of σ, that is, $\tau(u) \leq \sigma(u)$ for all u in V. Then the fuzzy subgrpah of G(σ,μ) induced by τ is the maximal fuzzy subgraph of G(σ,μ) that has fuzzy node set τ. Evidently, this is just the fuzzy graph $H(\tau,\rho)$ where $\rho(u,v) = \tau(u) \wedge \tau(v) \wedge \mu(u,v)$ for all u,v in V.

Definition 2.5 The underlying crisp graph of a fuzzy graph G(σ,μ) is denoted by $G^* = (\sigma^*,\mu^*)$, where $\sigma^* = \{u \in V \mid \sigma(u) > 0\}$ and $\mu^* = \{(u,v) \in V \times V \mid \mu(u,v) > 0\}$.

Definition 2.6 A fuzzy graph G(σ,μ) is a strong fuzzy graph if $\mu(u,v) = (u,v)$ for all $u, v \in \mu^*$ and is a complete fuzzy graph if $\mu(u,v) > 0$ for all $u, v \; in \; \sigma^*$. Two nodes u and v are said to be neighbors if $\mu(u,v) > 0$.

Definition 2.7 A fuzzy graph G = (σ, μ) is said to be Bipartite if the node set V can be Partitioned into two non empty sets V_1 and V_2 such that $\mu(v1, v2) = 0$ if $v_1,v_2 \epsilon V_1$ or $v_1,v2 \epsilon V_2$. Further if $\mu(v_1, v_2) > 0$ for all $v_1 \epsilon V_1$ and $v_2 \epsilon V_2$ then G is called complete bipartite graph and it is denoted by $K_{\sigma 1,\sigma 2}$ where σ1 & σ2 are respectively the restriction of σ to V_1 &V_2.

Definition 2.8 The complement of a fuzzy graph G(σ,μ) is a subgraph $\bar{G} = (\bar{\sigma},\bar{\mu})$ where $\bar{\sigma} = \sigma$ and $\bar{\mu}(u,v) = \sigma(u) \wedge \sigma(v) - \mu(u,v)$ for all $u, v \; in \; V$. A fuzzy graph is self complementary if $G = \bar{G}$.

Definition 2.9 The order p and size q of a fuzzy graph G(σ,μ) is defined as $p = \sum_{u \in V} \sigma(u)$ and $q = \sum_{(u,v) \in E} \mu(u,v)$.

Definition 2.10 The degree of the vertex u is defined as the sum of weight of arc incident at u, and is denoted by d(u).

Definition 2.11 A Path ρ of a fuzzy graph G(σ,μ) is a sequence of distinct nodes v_1, v_2, v_3,…v_n such that $\mu(v_{i-1}, v_i) > 0$ where $1 \leq i \leq n$. A path is called a cycle if $u_0 = u_n$ and $n \geq 3$

Definition 2.12 Let u,v be two nodes in G(σ, μ). If they are connected by means of a path ρ then strength of that path is $\wedge_{i=i}^{n} \mu(u_{i-1}, v_i)$.

Definition 2.13 Two nodes that are joined by a path are said to be connected. The relation connected is reflexive, symmetric and transitive. If u and v are connected by means of length k, then $\mu^k(u,v) = \sup\{\mu(u,v_1) \wedge \mu(v_1,v_2) \ldots \wedge \mu(v_{k-1},v_k) \mid u, v_1, v_2, \ldots v$ in such path ρ }

Definition 2.14 A Strongest path joining any two nodes u,v is a path corresponding to maximum strength between u and v. The strength of the strongest path is denoted by $\mu^\infty(u,v)$. $\mu^\infty(u,v) = \sup\{\mu^k(u,v)$ | k = 1,2,3…}

Example 2.15

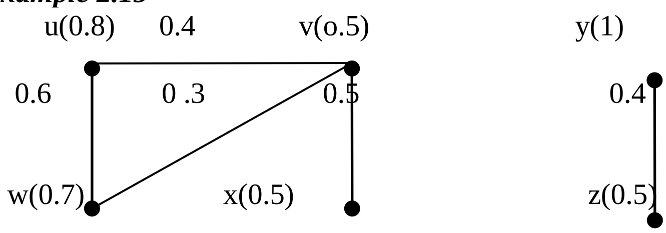


Fig (i)(a) (b)

In this fuzzy graph, fig(i)(a), u = w, v, x is a w-x path of length 2 and strength is 0.3.
Another path of w-x is w, u, v, x of length 3 and strength is 0.4.
But strength of the strongest path joining w and x is $\mu^\infty(w,x) = \sup\{0.3, 0.4\} = 0.4$

Definition 2.16 Let G(σ, μ) be fuzzy graph. Let x, y be two distinct nodes and G' be the fuzzy subgraph obtained by deleting the arc(x,y) that is $G'(\sigma, \mu')$ where $\mu'(x,y) = 0$ and $\mu' = \mu$ for all other pairs. Then (x, y) is said to be fuzzy bridge in G if $\mu'^\infty(u,v) < \mu^\infty(u,v)$ for some u, v in V.

Definition 2.17 A node is a fuzzy cut node of G(σ, μ) if removal of it reduces the strength of the connectedness between some other pair of nodes. That is, w is a fuzzy cut node of G(σ, μ) iff there exist u,v such that w is on every strongest path from u to v .

Definition 2.18 An arc (u, v) of the fuzzy graph G(σ, μ) is called an effective edge if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ and effective edge neighborhood of u $\in V$ is $N_e(u) = \{v \in V: edge(u,v)\ is\ effective\}$. $N_e[u] = N_e(u) \cup \{u\}$ is the closed neighborhood of u.
The minimum cardinality of effective neighborhood $\delta_e(G) = \min\{|N_e(u)|\ u \in V(G)\}$.
Maximum cardinality of effective neighborhood $\Delta_e(G) = \max\{|N_e(u)|\ u \in V(G)\}$.

Definition 2.19 An arc (u,v) of the fuzzy graph G(σ, μ) is called a strong arc if $\mu(u,v) = \mu^\infty(u,v)$ else arc(u,v) is called non strong.
Strong neighborhood of u $\in V$ is $N_s(u) = \{v \in V: arc(u,v)$ is strong} .
$N_s[u] = N_s(u) \cup \{u\}$ is the closed neighborhood of u.

Definition 2.20 (Independent number) A subset S of the vertex set V(G) in a graph is said to be independent if no two vertices in S are adjacent. The maximum cardinality of the independent set of a graph is called the independence number and is denoted by $\beta_o(G)$ or $\beta(G)$.

Definition 2.21 (Support independent number) A subset S of V(G) is called support independent if for any u,v ∈ S, either u and v are not adjacent and supp(u) ≠ supp(v). The maximum cardinality of such set of a graph G is called support independent and is denoted by β_{o-supp}.

3. Main Result

Definition 3.1 An arc (u,v) of the fuzzy graph G(σ, μ) is called a strong arc if $\mu(u,v) = \mu^{\infty}(u,v)$ else arc(u,v) is called non strong.
Strong neighborhood of u ∈ V is $N_s(u) = \{v \in V: arc(u,v)$ is strong} .
$N_s[u] = N_s(u) \cup \{u\}$ is the closed neighborhood of u.

Definition 3. 2 The strong degree of a vertex u of a fuzzy graph is defined as the sum of strong neighborhood of u, and is denoted by Strong Deg(u).

Definition 3.3 The support of a vertex u of a graph is defined as the sum of degree of neighborhood of u, and is denoted by supp(u).
In other words, Supp (u) = $\sum_{v \in N(u)} degree\ of\ v$

Definition 3.4 The strong support of a vertex is the sum of strong degree of strong neighborhood. It is denoted by Strong Supp(u).
In other words, Strong Supp (u) = $\sum_{v \in N_s(u)} Strong\ degree\ of\ v$

Definition 3.5 The (Open) Strong support neighborhood of u ∈ V is $N_{ssupp}(u) = \{v \in V: arc(u,v)$ is strong and strong supp(u)≥ strong supp (v)}.
$N_{ssupp}[u] = N_{ssupp}(u) \cup \{u\}$ is the (Closed) strong support neighborhood of u.

Definition 3.6 (Strong support set and Anti-strong support set) A vertex u is strong support if Strong Supp (u) ≥ Strong Supp (v) where $v \in N_{ssupp}(u)$. A subset S of V is called strong support set if for every vertex u in S is strong support.

A vertex u is Anti-Strong support if Strong Supp (u) ≤Strong Supp (v), where $v \in N_{ssupp}(u)$. A subset S of V is called Anti-strong support set if for every vertex u in S is Anti- strong support.

Independent strong support domination number and Anti - strong support domination number in fuzzy graph

Definition 3.7 (Independent a strong support set)
A strong support set D is said to be independent if no two vertices of D are adjacent in G by strong arc. Similarly independent anti –strong support set can be defined.

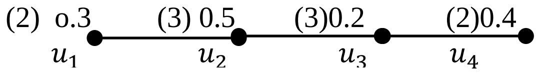
Definition 3. 8 (Independent strong support dominating set)
An Independent strong support set D of V is said to be an independent strong support dominating set of G(σ, μ) if D is strong support dominating set and independent set.
(It is same to the definition of strong support dominating set and differ only by strong independence property.)

It is asymmetric domination, since if u strong support dominate v, then v need not strong support dominates u, in other word v is a anti -strong support dominates u in G
A independent strong support dominating set D is called a minimal independent strong support dominating set if no proper subset of D is a independent strong support dominating set.
The minimum strong support fuzzy cardinality taken over all minimal independent strong support dominating sets of a fuzzy graph G is called the independent Strong support domination number and is denoted by $i - \gamma_{ssupp}(G)$ and the corresponding dominating set is called minimum independent strong support dominating set (or i$-\gamma_{ssupp}$set). The number of elements in the i- γ_{ssupp} -set is denoted by n[$i - \gamma_{ssupp}(G)$].
Here , Strong support domination number,

i- $\gamma_{ssupp}(G)$ =Min{ $\sum$\{ strong support of u + $\sigma(u)$ }, where u ∈ D }

Where D is the i- γ_{ssupp}set
Similarly, Independent Anti - strong support domination number i- $\gamma_{a-ssupp}(G)$ can be defined.
That is, it is same as above and only differ by consideration of anti strong support set.

Example 3.9. Let FP_n be a fuzzy paths with n vertices.
For n = 4,

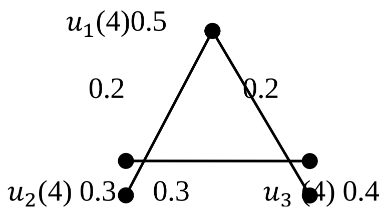


Independent set S = { u_1 , u_3 } and { u_2 , u_4}
Independent Strong set S = { u_1 , u_3 } and { u_2 , u_4}
Independent Strong support set S = { u_1 , u_3 } and { u_2 , u_4 }
Independent Anti -strong support set S = { u_1 , u_4 }.

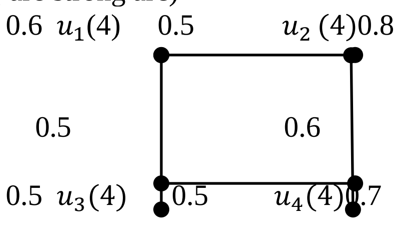
Independent strong support dominating set S = { u_1 , u_3 } and { u_2 , u_4 }
Independent Anti- strong support dominating set S = { u_1 , u_4 }.

Minimal Independent strong support dominating set S_1 = { u_1 , u_3 } and S_2= { u_2 , u_4 }
From S_1 = { u_1 , u_3 } ,$i-\gamma_{ssupp}(G)$ = (2 +0.3) + (3 +0.2) = (2+3) +(0.3+0.2) = 5.5
From S_2= { u_2 , u_4 } , $i-\gamma_{ssupp}(G)$ = (2 +0.5) + (3 +0.4) = (2+3) +(0.3+0.2) = 5.7
Therefore S_1 = { u_1 , u_3 } is i-γ_{ssupp}set and
Independent strong support dominating number $i-\gamma_{ssupp}(G)$ = 5.5 .
Further n[$i-\gamma_{ssupp}(G)$] = 2
Here S_2= { u_2 , u_4 } is not i- γ_{ssupp}set , since its sum of fuzzy cardinality is greater than other.
Note : From the above example we note that all independent strong support dominating set is not i-γ_{ssupp}set .

Example 3.10 Let FC_n be a fuzzy Cycle with n vertices
For n = 3 , 4, (Assume that the entire arcs of FC_n are strong arc)



Figure(i)

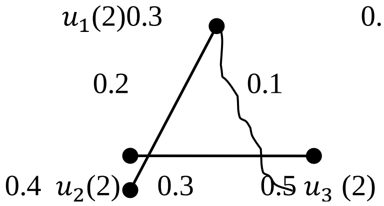


Figure(ii)

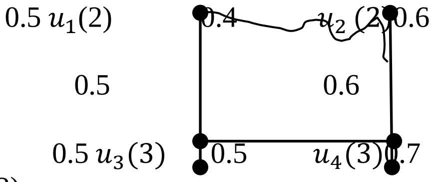
From Figure (i)
Independent set S = { u_1 } or{ u_2} or{ u_3}
Independent Strong set S = { u_1 } or{ u_2} or{ u_3}
Independent Strong support set S = { u_1 } or{ u_2} or{ u_3}
Independent Anti -strong support set S = { u_1 } or{ u_2} or{ u_3}
Independent Strong support dominating set S = { u_1 } and { u_2} and { u_3}
Sum fuzzy cardinality of { u_2} is lesser than { u_1 } and { u_3}.
Therefore { u_2} is i- γ_{ssupp}set and other are not.
Its domination number $i_{-\gamma ssupp}(G)$ = 4+0.3 =4.3 and also n[$i-\gamma_{ssupp}(G)$] =1
From Figure (ii)
Independent set S = { u_1, u_4} or{ $u_{2\,,}u_3$}
Independent strong set S = { u_1, u_4} and{ $u_{2\,,}u_3$}
Independent Strong support set S = { u_1, u_4} and { $u_{2\,,}u_3$}
Independent Anti strong –support set S = { u_1, u_4} and { $u_{2\,,}u_3$}
Now, { $u_{2\,,}u_3$} is i- γ_{ssupp}set and other are not.
Independent strong support domination number $i-\gamma_{ssupp}(G)$ =(4+0.8)+(4+0.5) = 9.3 and
n[$i-\gamma_{ssupp}(G)$] =2

Example 3.11 (Assume that FC_n has a non- strong arc)

For n =3,4



Figure(i)



Figure(ii)

From Figure (i)

There is no adjacency between u_1 and u_3 , since arc (u_1 , u_3) is a non-strong arc

Independent set S = { u_1 , u_3}

Independent strong set S = { u_1 , u_3}

Independent Strong support set S = { u_1 , u_3}

Independent Anti strong –support set S = { u_1 , u_3}

i- γ_{ssupp}set = { u_1 , u_3} and

$i-\gamma_{ssupp}(G)$ = (2+0.3)+(2+0.5) = (2+2)+(0.3+0.5) = 4.8

and also n[$i-\gamma_{ssupp}(G)$] =2

From figure(ii)

Independent set S = { u_1, u_2} or{ u_2, u_3} or { u_1, u_4}

Independent Strong set S = { u_1, u_2} or{ u_2, u_3} or { u_1, u_4}

Independent Strong support set S = { u_1, u_4} or{ u_2, u_3}

Independent Anti- strong support set S = { u_1, u_2} or{ u_2, u_3}or { u_1, u_4}

Independent Strong support dominating set S = { u_1, u_4} or{ u_2, u_3}

Here, $i-\gamma_{ssupp}(G)$ =(2+3)+(0.5+0.6) = 6.1 and

{u_2, u_3} is a i- γ_{ssupp}set, where as {u_1, u_4} is not i- γ_{ssupp}set.

Note. From the above three example, we noticed that all minimal independent strong support set need not be i- γ_{ssupp} set. Independence will be used to minimize strong support domination number.

Theorem 3.12 Every simple graph G has an independent strong support dominating set.

Proof. Choose an independent set U such that ω (U) = $\sum_{x\in U}\{strong\ supp(x)+1\}$! is maximum. Clearly U is a strong support dominating set of G. (Suppose that if y∈ V-U and y is not adjacent to any point of U then U-{y} in independent and ω (U ∪ {y}) = ω (y) +{ strong supp(y) +1}! $>$ ω(U), a contradiction). Suppose U is not a strong support dominating set of G, then there exist is a vertex v∈ V(G) –U such that strong supp(u)≥ strong supp(v) for all u ∈ N_s(v)∩ U. Suppose U* = U- (N_s(v)∩ U)∪ {v} , is independent .

ω(U*) = $\omega(U)$ +{ strong supp(v) +1}! – ω(N_s(v)∩ U))

≥ $\omega(U)$ +{ strong supp(v) +1}! – deg(v)(strong supp(v))!)

= $\omega(U)$ + {strong supp(v) !}(strong supp(v)+1- deg(v))

$>$ $\omega(U)$, is a contradiction. (Since strong supp(v) +1 $>$ deg(v) and strong supp(v)≥ 1 for any v∈ G – U)

Therefore U is an independent strong support dominating set.

Remark. Let G be a fuzzy graph with finite number of vertices n.

$i-\gamma_{a-ssupp}(G) \leq \gamma_{ssupp}(G) \leq i-\gamma_{ssupp}(G)$

Where

$\gamma_{ssupp}(G)$ is strong support domination number

$i-\gamma_{ssupp}(G)$ is independent strong support domination number

$i-\gamma_{a-ssupp}(G)$ is independent anti -strong support domination number

In graph theory, a claw –free graph is a graph that does not have a claw as an induced sub graph. A claw is another name of complete bipartite graph $FK_{1,3}$ (That is star graph with three edges, three leaves and one central vertex.)
A claw -free graph is graph in which no induced sub graph is a claw.
Any subset of four vertices has other than only three edges connecting them in this pattern.
Equivalently, a claw free graph is a graph in which the neighborhood of any vertex is the complement of a triangle –free graph.
Claw free graph were initially studied as a generalization of line graphs , and gained additional motivation through three key discoveries about them. The fact that all claw free graph connected graphs of even order have perfect matching, the discovery of polynomial time algorithms for finding maximum independent set in claw- free graphs.

Theorem 3.13 If a fuzzy graph G does not contain $FK_{1,3}$is claw -free as an induced fuzzy sub graph then $\gamma_{ssupp}(G) \cong i - \gamma_{ssupp}(G)$.

That is, Strong support domination number ($\gamma_{ssupp}(G)$) is approximately equal to Independent strong support domination number ($i - \gamma_{ssupp}(G)$)

Proof. Let D be a strong support dominating set of fuzzy graph G and let $|D|$ = n[$\gamma_{ssupp}(G)$] = k .
If D is independent then n[$i - \gamma_{ssupp}(G)$] $\leq$ $|D|$ $=$ n[$\gamma_{ssupp}(G)$].
But n[$\gamma_{ssupp}(G)$] ≤ n[i- $\gamma_{ssupp}(G)$], Therefore $\gamma_{ssupp}(G) \cong i - \gamma_{ssupp}(G)$.
Suppose $< D >$ has strong arc(edges). Let S be a set all vertices in D which are not isolates in D. Let x be a vertex of S such that Strong supp(x) = max { strong supp(v) : v∈ S }.
Let xy be a strong edges in $< D >$, then strong supp(y)≤ strong supp(x). Note that all vertices in D with strong support greater than strong supp (x) are isolates in $< D >$.
Let N = { u ∈V-D: u is a strong support dominate only by y∈ D }.Since D is minimum strong support dominating set and since y is not an isolate of $< D >$, it follows that N≠ ∅. Note that no vertex in N is adjacent to any vertex z ∈ D with strong supp(z)≥ strong supp(y). Let u and v be two vertices in N .Since G is $FK_{1,3}$is claw free { uv,vx,ux}∩ E(G) ≠ ∅. Also since strong supp(x) ≥ strong supp(y) and u,v ∈N , it follows that ux and vx are not strong arc (edges) in G. Hence uv ∈E(G). This proves that any two vertices in N∪ y are adjacent. Let u_0 ∈ N such that strong supp(u_0) = max { strong supp(u): u ∈ N}. Consider the sset D' is equal to (D-{y}) ∪ u_0 . Clearly all vertices in N –{u_0} are strong support dominates by u_0 ∈ D' .Further vertices in V-D –{u_0 } are strong support dominated by vertices (D-{y}∪ u_0 in D'. Also y is strong support dominated by x inD'.
This proves that D' is a strong support dominating set with $|D'|$ = k. If there are more vertices in D' are adjacent to x', they are replaced one by one with vertices not adjacent to x until a minimum strong support dominating set D_1 is obtained such that x is isolated in $< D >$. If D_1 is independent , we are through. Otherwise the process is repeated and ultimately a minimum strong support dominating set $D_r (1 \leq r \leq k - 1)$ is obtained , which is independent .Therefore, $i - \gamma_{ssupp}(G) \leq \gamma_{ssupp}(G)$. But $i - \gamma_{ssupp}(G) \geq \gamma_{ssupp}(G)$. Hence $\gamma_{ssupp}(G) \cong i - \gamma_{ssupp}(G)$.

Note. $\gamma_{ssupp}(G) \cong i - \gamma_{ssupp}(G)$ becomes $\gamma_{ssupp}(G) = i - \gamma_{ssupp}(G)$ if entire arc of fuzzy G is strong arc .

Theorem 3.14 For any Fuzzy line graph FL(G) , $\gamma_{ssupp}(FL(G)) \cong i - \gamma_{ssupp}(FL(G))$

Proof. Follows from the fact that any line graph is $FK_{1,3}$is claw free.
Hence by above theorem, $\gamma_{ssupp}(FL(G)) \cong i - \gamma_{ssupp}(FL(G))$

Definition 3.15 Let G be a simple fuzzy graph. Then fuzzy graph G is $i - \gamma_{ssupp} perfect$ set if $i - \gamma_{ssupp}(H)$ = $\gamma_{ssupp}(H)$ for every induced subgraph H of G.

Remark. Any $FK_{1,3}$is claw free is $i - \gamma_{ssupp}\ perfect$

Definition 3.16 A graph G is said to be domination perfect if $\gamma\ (H)$ = i(H) for every induced subgraph H of G.

Theorem 3.17 Any $i-\gamma_{ssupp}\, perfect$ fuzzy graph is domination perfect.

Proof. Let G be an $i-\gamma_{ssupp}\, perfect$ fuzzy graph. Suppose G is not domination perfect. Then there exists an induced sub graph H of G such that $\gamma\ (H) <$ i(H). Choose a minimum dominating set D of H such that the induced sub graph of D in H has minimum number of edges. Since $\gamma\ (H) <$ i(H) , the induced subgraph of D in H has at least one edge . Let e = uv an edge in the induced subgraph of D in H. Let A = {a$\in V(H)-\ D : N(a) \cap D = \{u\}$} and B = {b$\in V(H)-\ D : N(b) \cap D = \{v\}$}. Since D is a minimum dominating set of H, u and v are not isolates in the induced subgraph of D in H , A and B are not empty. The induced subgraph of A in H is not complete. For , otherwise (D- {u})- {a} is a minimum dominating set of H with fever edges than D contradicting the choice of D. Therefore, there are two non empty adjacent vertices $a_1\, , a_2$ in A. Similar argument shows that there are two non adjacent vertices $b_1\, , b_2$ in B. The subgraph induced in G by the points u, v, $a_1\, , a_2,\ b_1\, , b_2$ has $i-\gamma_{ssupp}(G)$ = 3 and γ_{ssupp} (G) = 2.which is contradiction to the fact that G is $i-\gamma_{ssupp}(G)$ perfect. Hence the proof follows.

Note. This theorem is only true if every arc in G is strong arc, otherwise there is small variation occurring in the $i-\gamma_{ssupp}$ perfect set. That is, if non strong existence in fuzzy graph the value of $i-\gamma_{ssupp}$ will be varied

4. Result on independent Strong support (Anti -strong support) Domination on various fuzzy graph

Independent strong support domination number

$i-\gamma_{ssupp}(G)$ =Min{ $\sum${ strong support of u + $\sigma(u)$ }, u ∈ D }

Where D = $i-\gamma_{ssupp}(G)$Set is the minimum independent strong support dominating set

$i-\gamma_{a-ssupp}(G)$ =Min{ $\sum${ strong support of u + $\sigma(u)$ }, u ∈ D}

Where D = $i-\gamma_{a-ssupp}(G)$Set is the minimum independent anti -strong support dominating set

Result 4.1 Let FP_n be fuzzy path with n vertices.

Number of elements in the minimum independent strong support dominating set

n[$i-\gamma_{ssupp}(FP_n)$] = $\left\lceil\frac{n}{3}\right\rceil + 1$ where n ≥ 5

Independent Strong support domination number

i- $\gamma_{ssupp}(FP_n) \leq$ n + n[$\gamma_{ssupp}(FP_n)$] + $\sum_{k\in D}\sigma(k)$

= n + $\left\lceil\frac{n}{3}\right\rceil + 1 + \sum_{k\in D}\sigma(k)$, when $n \geq 5$

Where D is the minimum independent strong support dominating set.

Result 4.2 LetFC_n be fuzzy cycle with n vertices and assume that all arcs of FC_n are strong arc.

(i) n[$i-\gamma_{ssupp}(FC_3)$] =1 and $i-\ \gamma_{ssupp}(FC_3)$ = 4 + $\sum_{k\in D}\sigma(k)$

(ii) n[$i{-}\gamma_{ssupp}(FC_n)$] = $\left\lceil\frac{n}{3}\right\rceil$ when n ≥ 4 and

$i{-}\gamma_{ssupp}(FC_n) \leq$ 4 times of n[$i-\gamma_{ssupp}(FC_n)$] + $\sum_{k\in D}\sigma(k)$

= 4 $\left\lceil\frac{n}{3}\right\rceil + \sum_{k\in D}\sigma(k)$ when $n \geq 4$

Where D is the minimum independent strong support dominating set

If at least one non- strong arc exist in FC_n then the following result is true.

(i) n[$i-\gamma_{ssupp}(FC_3)$] = 1 and $i-\gamma_{ssupp}(FC_3)$ = 2 + $\sum_{k\in D}\sigma(k)$
(ii) n[$i-\gamma_{ssupp}(FC_4)$] = 2 and $i-\gamma_{ssupp}(FC_4)$ = 5 +$\sum_{k\in D}\sigma(k)$ and
(iii) n[$i-\gamma_{ssupp}(FC_n)$] = $\left[\frac{n}{3}\right]+1$ when n > 4

i− $\gamma_{ssupp}(FC_n)$ ≤ n + $\left[\frac{n}{3}\right]+1+\sum_{k\in D}\sigma(k)$

= n + n[$i-\gamma_{ssupp}(FC_n)$] + $\sum_{k\in D}\sigma(k)$, when $n>4$

Where D is the minimum independent strong support dominating set

Result 4.3 Let FK_n be fuzzy complete with n vertices .

Case (a)

If no non-strong arc exist ,then

(i) n[$i-\gamma_{ssupp}(FK_n)$] = 1
(ii) i −$\gamma_{ssupp}(FK_n)$ = $(n-1)^2$ + $\sum_{k\in D}\sigma(k)$

Case(b)

If non -strong arc exist, then

(iii) n[$i-\gamma_{ssupp}(FK_3)$] = 1 and $i-\gamma_{ssupp}(FK_3)$ = 2 + $\sum_{k\in D}\sigma(k)$
(iv) n[$i-\gamma_{ssupp}(FC_4)$] = 2 and $i-\gamma_{ssupp}(FK_4)$ = 5 +$\sum_{k\in D}\sigma(k)$ and
(v) 1 ≤ n[$i-\gamma_{ssupp}(FK_n)$] ≤ $\left[\frac{n}{3}\right]+1$, when n > 4
(vi) n + $\left[\frac{n}{3}\right]+1+\sum_{k\in D}\sigma(k)\ \le i-\gamma_{ssupp}(FK_n)\ \le\ (n-1)^2$ + $\sum_{k\in D}\sigma(k)$, when $n>4$

Result 4.4 Let $FK_{1,n}$ be fuzzy star with n +1 vertices .

(i) n[$i-\gamma_{ssupp}(FK_{1,n})$] = 1 and
(ii) i - $\gamma_{ssupp}(FK_{1,n})$ ≤ n + $\sum_{k\in D}\sigma(k)$

Note that the above results are verified by the reference [16,17,19, 20, 21]

Result 4.5 Let $FK_{m,n}$ be a fuzzy complete bipartite graph , where m, n are positive integer.

Then

n[i - γ_{ssupp} ($FK_{m,n}$)] = min { m, n},

where m , n are positive integer such that 2≤ $m\le n$.

Independent strong support domination number can approximately identified that as given below in the two cases.

Case i

No non strong in the fuzzy complete graph.

(a) i - γ_{ssupp} ($FK_{m,n}$) = m(m^2) + $\sum_{u\in D}\sigma(u)$ if m = n
(b) i - γ_{ssupp} ($FK_{m,n}$) = m(m× n) = $\sum_{u\in D}\sigma(u)$ if m≠ n and m< n

Case ii

Maximum number non -strong appear in the fuzzy complete graph

(c) i - γ_{ssupp} ($FK_{m,n}$) = (2m-1) +m(m-1) + $\sum_{u\in D}\sigma(u)$ if m = n
(d) i - γ_{ssupp} ($FK_{m,n}$) = (m+n-1) +m(m-1) + $\sum_{u\in D}\sigma(u)$ if m≠ n and m< n

Lower and upper bounds of i - γ_{ssupp} ($FK_{m,n}$)

(e) (2m-1) +m(m-1) + $\sum_{u\in iD}\sigma(u)$ ≤ i($FK_{m,n}$)≤ m(m^2) + $\sum_{u\in iD}\sigma(u)$ m = n
(d) (m+n-1) +m(m-1) + $\sum_{u\in iD}\sigma(u)$ ≤ i($FK_{m,n}$) ≤ m(m× n) = $\sum_{u\in iD}\sigma(u)$
if m≠ n and m< n .

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