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INTRODUCTION TO STRONG SUPPORT DOMINATION IN FUZZY GRAPH $G(\sigma, \mu)$ BY STRONG ARC

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Abstract In this paper, definition of strong degree of a vertex and strong support of a vertex of the fuzzy graph are newly introduced. Further, definition of strong support dominating set and strong support domination number of the fuzzy graph are also newly introduced, moreover, some new results on strong support domination number for the standard fuzzy graph are found. Finally, strong domination and strong support domination of the fuzzy graph are compared with an example.

Keywords Fuzzy graph, Domination number, strong arc , non strong arc, strong degree , strong Support , strong Support domination.

1. Introduction

In1965, L.A Zadeh introduced the concept of fuzzy subset of a set as a way for representing uncertainity[19]. Zadeh's ideas stirred the interest of researchers worldwide. Monderson, J.N., Premchand, S.N. discussed fuzzy graph theory fuzzy hypergraph [4]. Fuzzy graph is the generalization of the ordinary graph. The formal mathematical definition of domination was given by Ore.O in 1962[10]. In 1975, A. Rosenfeld introduced the notion of fuzzy graph and several analogs of theoretic concepts such as path, cycle and connectedness[11]. A.Somasundaram and S. Somasundaram discussed the domination in fuzzy graph using effective arc[12]. A.Nagoorgani and V.T. Chandrasekarn discussed the strong arc in fuzzy graph[8,9]. Bhutani, K.R., and A.Rosenfeld have introduced the concept of Storng arcs in fuzzy graph[1,2].. Several works on fuzzy graph are also done by Mathumangal pal and Hossein Rashmanlou[6], Mathumangal pal[5] , Methew,S and M.S. sunitha[7], C.Y. Ponnappan, P. Surilinathan and S.Basheer Ahamed[13 ,17, 18], C.Y. Ponnappan and V.Senthilkumar[14,15,16]. Before discuss the study of strong Support domination of Fuzzy graphs, we are placed few preliminary.

2. Preliminaries [14,15,16]

Definition 2.1 Fuzzy graph $G(\sigma, \mu)$ is pair of function $V \to [0,1]$ and $\mu: V \times V \to [0,1]$, where for all u, v in V ,we have $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$.

Definition 2.2 The fuzzy graph $H(\tau, \rho)$ is called a fuzzy subgraph of $G(\sigma, \mu)$ if $\tau(u) \leq \sigma(u)$ for all u in V and $\rho(u, v) \leq \mu(u, v)$ for all u, v in V.

Definition 2.3 A fuzzy subgraph $H(\tau, \rho)$ is said to be a spanning sub graph of $G(\sigma, \mu)$ if $\tau(u) = \sigma(u)$ for all u in V. In this case the two graphs have the same fuzzy node set, they differ only in the arc weights.

Definition 2.4 Let $G(\sigma, \mu)$ be a fuzzy graph and τ be fuzzy subset of σ, that is, $\tau(u) \leq \sigma(u)$ for all u in V. Then the fuzzy subgrpah of $G(\sigma, \mu)$ induced by τ is the maximal fuzzy subgraph of $G(\sigma, \mu)$ that has fuzzy node set τ. Evidently, this is just the fuzzy graph $H(\tau, \rho)$ where $\rho(u, v) = \tau(u) \wedge \tau(v) \wedge \mu(u, v)$ for all u,v in V.

Definition 2.5 The underlying crisp graph of a fuzzy graph $G(\sigma, \mu)$ is denoted by $G^* = (\sigma^*, \mu^*)$, where $\sigma^* = \{ u \in V \mid \sigma(u) > 0 \}$ and $\mu^* = \{ (u, v) \in V \times V \mid \mu(u, v) > 0 \}$.

Definition 2.6 A fuzzy graph $G(\sigma, \mu)$ is a strong fuzzy graph if $\mu(u, v) = \mu^{\infty}(u, v)$ for all $u, v \in \mu^*$ and is a complete fuzzy graph if $\mu(u, v) > 0$ for all u, v in σ^*. Two nodes u and v are said to be neighbors if $\mu(u, v) > 0$.

Definition 2.7 A fuzzy graph G = (σ, μ) is said to be Bipartite if the node set V can be Partitioned into two non empty sets V1 and V2 such that $\mu(v1, v2) = 0$ if v1,v2 ϵ V_1 or v1,v2ϵV2.Further if $\mu(v_1, v_2) > 0$ for all v1ϵV1 and v2ϵV2 then G is called complete bipartite graph and it is denoted by $K_{\sigma 1, \sigma 2}$ where σ1 & σ2 are respectively the restriction of σ to V1 &V2

Definition 2.8 The complement of a fuzzy graph $G(\sigma, \mu)$ is a subgraph $\overline{G} = (\overline{\sigma}, \overline{\mu})$ where $\overline{\sigma} = \sigma$ and $\overline{\mu}(u, v) = \sigma(u) \wedge \sigma(v) - \mu(u, v)$ for all u, v in V. A fuzzy graph is self complementary if $G = \overline{G}$

Definition 2.9 The order p and size q of a fuzzy graph $G(\sigma, \mu)$ is defined as $p = \sum_{u \in V} \sigma(u)$ and $q = \sum_{(u,v) \in E} \mu(u, v)$.

Definition 2.10 The degree of the vertex u is defined as the sum of weight of arc incident at u, and is denoted by d(u).

Definition 2.11 A Path ρ of a fuzzy graph $G(\sigma, \mu)$ is a sequence of distinct nodes v1, v2, v3,…vn such that $\mu(v_{i-1}, v_i) > 0$ where $1 \leq i \leq n$. A path is called a cycle if $u_0 = u_n$ and $n \geq 3$

Definition 2.12 Let u,v be two nodes in $G(\sigma, \mu)$. If they are connected by means of a path ρ then strength of that path is $\wedge_{i=i}^{n} \mu(u_{i-1}, v_i)$.

Definition 2.13 Two nodes that are joined by a path are said to be connected. The relation connected is reflexive, symmetric and transitive. If u and v are connected by means of length k, then $\mu^k(u, v) = \sup\{\mu(u, v_1) \wedge \mu(v_1, v_2) \ldots \wedge \mu(v_{k-1}, v_k) \mid u, v_1, v_2, \ldots v \text{ in such path } \rho \}$

Definition 2.14 A Strongest path joining any two nodes u,v is a path corresponding to maximum strength between u and v. The strength of the strongest path is denoted by $\mu^{\infty}(u, v)$. $\mu^{\infty}(u, v) = \sup\{\mu^k(u, v) \mid k = 1,2,3\ldots\}$

Example 2.15

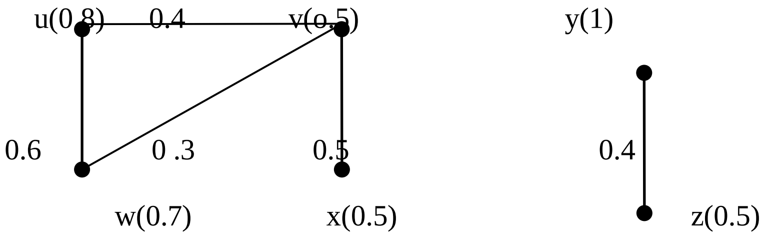


Fig (i)(a) (b)

In this fuzzy graph, fig(i)(a), u = w, v, x is a w-x path of length 2 and strength is 0.3.
Another path of w-x is w, u, v, x of length 3 and strength is 0.4.
But strength of the strongest path joining w and x is $\mu^{\infty}(w,x) = \sup\{0.3,0.4\} = 0.4$

Definition 2.16 Let G(σ, μ) be fuzzy graph. Let x, y be two distinct nodes and G' be the fuzzy subgraph obtained by deleting the arc(x,y) that is $G'(\sigma, \mu')$ where $\mu'(x,y) = 0$ and $\mu' = \mu$ for all other pairs. Then (x, y) is said to be fuzzy bridge in G if $\mu'^{\infty}(u,v) < \mu^{\infty}(u,v)$ for some u, v in V.

Definition 2.17 A node is a fuzzy cut node of $G(\sigma,\mu)$ if removal of it reduces the strength of the connectedness between some other pair of nodes. That is, w is a fuzzy cut node of $G(\sigma,\mu)$ iff there exist u,v such that w is on every strongest path from u to v .

Definition 2.18 An arc (u, v) of the fuzzy graph $G(\sigma,\mu)$ is called an effective edge if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ and effective edge neighborhood of $u \in V$ is $N_e(u) = \{v \in V: \text{edge}(u,v) \text{ is effective}\}$.
$N_e[u] = N_e(u) \cup \{u\}$ is the closed neighborhood of u.
The minimum cardinality of effective neighborhood $\delta_e(G) = \min\{|N_e(u)|\ u \in V(G)\}$.
Maximum cardinality of effective neighborhood $\Delta_e(G) = \max\{|N_e(u)|\ u \in V(G)\}$.

3. Main Results

3.1. Strong support domination in fuzzy graphs by strong arc

Here, Strong degree of a vertex, Strong support of a vertex and also Strong support domination of the fuzzy graph are newly introduced.

Definition 3.1 [14,15 ,16]An arc (u,v) of the fuzzy graph $G(\sigma,\mu)$ is called a strong arc if $\mu(u,v) = \mu^{\infty}(u,v)$ else arc(u,v) is called non strong.
Strong neighborhood of $u \in V$ is $N_s(u) = \{v \in V: \text{arc}(u,v) \text{ is strong}\}$.
$N_s[u] = N_s(u) \cup \{u\}$ is the closed neighborhood of u.
The minimum cardinality of strong neighborhood $\delta_s(G) = \min\{|N_s(u)|: u \in V(G)\}$. Maximum cardinality of strong neighborhood $\Delta_s(G) = \max\{|N_s(u)|: u \in V(G)\}$.

Definition 3.2 [14,15,16] Let $G(\sigma,\mu)$ be a fuzzy graph. Let u, v be two nodes of $G(\sigma,\mu)$. We say that u dominates v if edge (u,v) is a strong arc. A subset D of V is called a dominating set of $G(\sigma,\mu)$ if for every $v \in V - D$, there exists $u \in D$ such that u dominates v. A dominating set D is called a minimal dominating set if no proper subset of D is a dominating set.
The minimum fuzzy cardinality taken over all minimal dominating sets of a graph G is called the strong arc domination number and is denoted by $\gamma_s(G)$ and the corresponding dominating set is called minimum strong arc dominating set. The number of elements in the minimum strong arc dominating set is denoted by $n[\gamma_s(G)]$.

Note. The maximum fuzzy cardinality taken over all minimal dominating sets of a graph G is called the Γ -strong arc domination number and is denoted by $\Gamma_s(G)$ and cardinality of that minimal strong dominating set is denoted by n[$\Gamma_s(G)$].

Definition 3.3 (Strong degree of a vertex) The strong degree of a vertex u of a fuzzy graph is defined as the sum of strong neighborhood of u, and is denoted by Strong Deg(u).

Definition 3.4 (Strong support of a vertex) The strong support of a vertex is the sum of strong degree of strong neighborhood. It is denoted by Strong Supp(u).
In other words, Strong Supp (u) = $\sum_{v \in N_s(u)}$ Strong degree of v

Definition: 3.5 The (Open) strong support neighborhood of u ∈ V is $N_{ssupp}(u) = \{v \in V: arc(u, v)$ is strong and strong supp(u)≥ strong supp (v)$\}$.
$N_{ssupp}[u] = N_{ssupp}(u) \cup \{u\}$ is the (Closed) strong support neighborhood of u.
Similarly, the Anti (Open) strong support neighborhood of u ∈ V is
$N_{a-ssupp}(u) = \{v \in V: arc(u, v)$ is strong and strong supp(u)≤ strong supp (v)$\}$.
$N_{a-ssupp}[u] = N_{a-ssupp}(u) \cup \{u\}$ is the (Closed) strong support neighborhood of u.

Note. $deg_{a-ssupp}(v)$ = n[$N_{ssupp}(v)$]

$$\delta_{a-ssupp}(G) = \min_{v \in V(G)}\{ deg_{a-ssupp}(v)\}$$

$$\Delta_{a-ssupp}(G) = \max_{v \in V(G)}\{ deg_{a-ssupp}(v)\}$$

$deg_{ssupp}(v)$ = n[$N_{\ a-ssupp}(v)$]

$$\delta_{ssupp}(G) = \min_{v \in V(G)}\{ deg_{ssupp}(v)\}$$

$$\Delta_{ssupp}(G) = \max_{v \in V(G)}\{ deg_{ssupp}(v)$$

A vertex u is strong support if Strong Supp (u) ≥ Strong Supp (v) where v ∈ $N_{ssupp}(u)$ and a subset S of V is called Upper-strong support set if for every vertex u in S is Upper- strong support.

A vertex u is Anti-Strong support if Strong Supp (u) ≤Strong Supp (v), where v ∈ $N_{ssupp}(u)$and a subset S of V is called Anti -strong support set if for every vertex u in S is Anti- strong support.

Definition 3.6 (Strong support domination of the Fuzzy graph) Let $G(\sigma, \mu)$ be a fuzzy graph. Let u, v be two strong neighborhood nodes of $G(\sigma, \mu)$. We say that u strong support dominates v if Strong Supp (u) ≥ Strong Supp (v). (v anti- strong support dominates u)
A subset D of V is called a strong support dominating set of $G(\sigma, \mu)$ if for every $v \in V - D$, there exists u ∈ D such that u strong support dominates v.
(It is assymmetic domination, since if u strong support dominate v, then v need not strong support dominate u) .

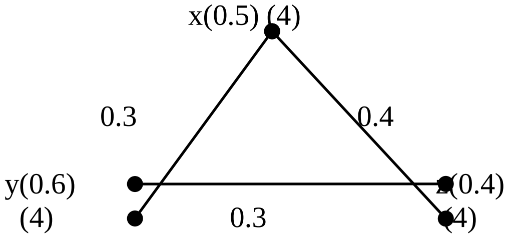
A strong support dominating set D is called a minimal strong support dominating set if no proper subset of D is a strong support dominating set.
The minimum strong support fuzzy cardinality taken over all minimal strong support dominating sets of a fuzzy graph G is called the Strong support domination number and is denoted by $\gamma_{ssupp}(G)$ and the corresponding dominating set is called minimum strong support dominating set. The number of elements in the minimum strong support dominating set is denoted by n[$\gamma_{ssupp}(G)$].

Here , Strong support domination number,

$\gamma_{ssupp}(G)$ =Min{ ∑{ strong support of u + σ(u) }, where u ∈ D }

Similarly, Anti - strong support dominates can be defined.

Example 3.7



Here , Arcs (x, y),(y, z) and(x, z) are all strong arc .
Strong support of all the nodes are same (say 4)
D1 ={ x} , D2= {y} and D3 = {z} are the strong support dominating set
n[γ_{ssupp}(G)] = 1 and
From D1 , we have $\gamma_{ssupp}(G) = 4 + \sigma(x) = 4+0.5 = 4.5$
From D2 , we have $\gamma_{ssupp}(G) = 4 + \sigma(y) = 4+0.6 = 4.6$
From D3 , we have $\gamma_{ssupp}(G) = 4 + \sigma(z) = 4+0.4 = 4.4$
$\gamma_{ssupp}(G)$ =Min{ ∑{ strong support of u + σ(u) }, where u ∈ D }
$\gamma_{ssupp}(G)$ =Min{ 4.5, 4.6, 4.4}= 4.4
That is, Strong support domination number of fuzzy cycle is 4.4
D3 = {z} is the minimum strong support dominating set

Example 3.8

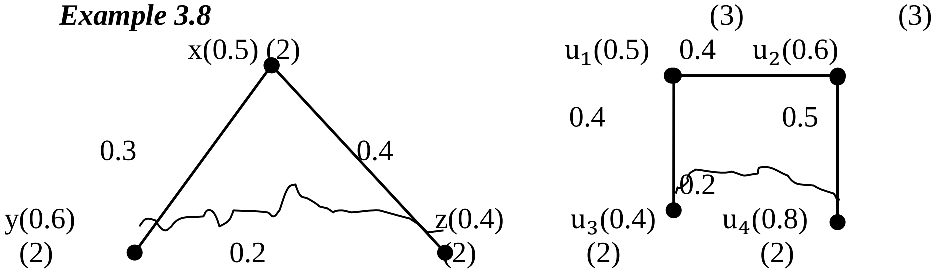
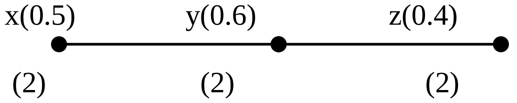


Fig (i) Fig (ii)

From fig(i), Here , Arcs (x, y),(x,z) are strong arc and (y, z) is non- strong arc .
Hence strong degree of vertices y and z are 1(one), Strong degree of vertex x is 2
Therefore strong support of vertices x, y and z are 2(two).
Minimal Strong support dominating set D = { x}.
n[γ_{ssupp}(G)] = 1 and $\gamma_{ssupp}(G) = 2 + \sigma(x) = 2+0.5 = 2.5$

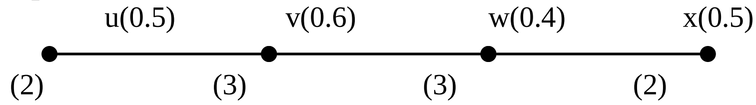
From fig(ii) Here , Arcs (u_3, u_4) is non- strong arc and all other strong arc .
Strong support of each vertex is pointed on it.
Minimal Strong support dominating set $D_1 = \{u_1, u_2\}$.
n[γ_{ssupp}(G)] = 2 and $\gamma_{ssupp}(G) = (3+3) + \sum \sigma(x) = 6+1.1 = 7.1$

Example 3.9



Here , arc (x, y) and arc (y,z) are strong arc (by strong arc definition)
Strong support of each vertex is 2 .
Now , D1 = { x, z } or D2 = { y } are support strong dominating set. But D1 is the minimal Support strong dominating set and $\gamma_{ssupp}(G) = 2 + \sigma(y) = 2+0.6 = 2.6$

Example 3.10



For the above fuzzy Path, We can find strong support dominating set as follows.
D1= {v ,w} is the strong support dominating set. D2 = { v, x } is a support strong dominating set , but D3 = { u ,x} is not minimum strong support dominating set since strong supp(x)< strong supp(w).
Therefore, n[$\gamma_{ssupp}(FP_4)$] = 2 and

$\gamma_{ssupp}(FP_4)$= $\sum$ strong supp(k) + σ(k),where k ∈ D1= {v ,w}
= (3+ σ(v))+ (3+ σ(w))
= (3+3)+($\sigma(v) + \sigma(w)$
= 6+(0.6 +0.4) = 7 and

$\gamma_{ssupp}(FP_4)$ = $\sum$ strong supp(k) + σ(k),where k ∈ D2= {v ,x}
= (3+ σ(v))+ (2+ σ(x))
= (3+2)+ ($\sigma(v) + \sigma(x)$)
= 5+(0.6 +0.4) = 6

Strong support domination number,
$\gamma_{ssupp}(G)$ =Min{ $\sum${ strong support of u + σ(u) }, where u ∈ D }
=Min { 7, 6 } = 6

From the above calculation, D2 = { v, x } is a minimum support strong dominating set.
Strong support domination number is 6.

Note. The maximum strong support fuzzy cardinality taken over all minimal strong support dominating sets of a fuzzy graph G is called the Γ-strong support domination number and is denoted by $\Gamma_{ssupp(G)}$ and the corresponding dominating set is called
Γ -strong support dominating set. The number of elements in Γ -strong support dominating set is denoted by n[$\Gamma_{ssupp}(G)$].

3.2.Standard Theorems and Results

Theorem 3.11 At least two vertices of any fuzzy path and fuzzy cycle have the same strong support degree.

Proof. Let FP_n be fuzzy path with n vertex and Let FC_n be fuzzy cycle with n vertex. Since any fuzzy path has two end vertices which are not adjacent with more than one adjacent vertex. Since all the arcs are strong arc in any strongest path, we can calculate strong support degree which is same in end vertices of that strongest fuzzy path.

In fuzzy cycle, all vertices are adjacent with exactly two adjacent vertices.
For this proof we can take two case .
Case:1

Suppose all the arcs of the fuzzy cycle are strong than clearly all vertices have same strong support degree. [Refer example 2]
Case :2

We know that any fuzzy cycle can have at most one non –strong arc.
[Refer 14, 15,16]

Suppose one non –strong exist in fuzzy cycle than by definition of strong support degree, we can't count the degree between non- strong arc of those two vertices.
Clearly these two vertices must have same strong support degree.
Hence at least two vertices of anyFP_n fuzzy path with n vertices and FC_n fuzzy cycle with n vertices have same strong support degree.

Theorem 3.12 A strong support dominating set D of V is minimal if and only if every vertex u in D satisfies one of the following conditions.

(i) u is a strong support of isolate of < D>
(ii) Sppn [u,D] $\neq \emptyset$
(Strong support private neighborhoods of vertex u with respect to a set D is non empty)

Proof. Suppose D is a minimal strong support dominating set .
Let u in D. Suppose u does not satisfy (i) and (ii).Then D-{u} is a strong support dominating set, a condradiction. Conversely, suppose a strong support dominating set D satisfies conditions (i) and (ii). If D is not minimal, then there exist a vertex u in D such that D-{u} is a strong support dominating set. In this case u cannot satisfy (i) as well as (ii), a contradiction . Hence the theorem.

Theorem 3.13 For any fuzzy graph G

$$\left[\frac{n}{1+\Delta_{ssupp}(G)}\right] \leq n[\gamma_{ssupp}(G)] \leq n- \Delta_{ssupp}(G)$$

Proof. Let u be a vertex in fuzzy graph G with support strength $\Delta_{ssupp}(G)$
Then V- $N_{a-ssupp}(u)$ is strong support dominating set of fuzzy graph G.

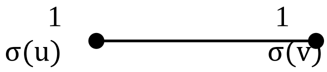
Result 1. Let FP_n be fuzzy path with n vertex.

$n[\gamma_{ssupp}(FP_n)] = \left[\frac{n}{3}\right] + 1$ where $n \geq 5$

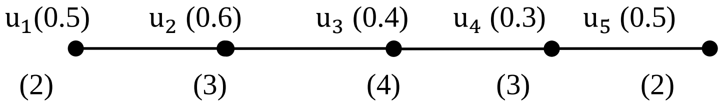
$\gamma_{ssupp}(FP_n) = n + n[\gamma_{ssupp}(FP_n)] + \sum_{k\in D} \sigma(k)$

$= n + \left[\frac{n}{3}\right] + 1 + \sum_{k\in D} \sigma(k)$, when $n \geq 5$

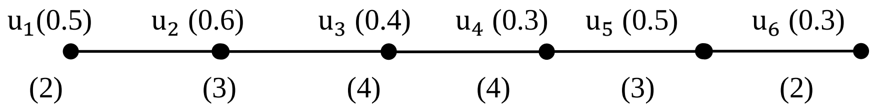
Where D is the minimum strong support dominating set

Proof. Let FP_n be fuzzy path with n vertex.
All the arc of fuzzy path FP_n are strong arc (Reference [14, 15, 19])
Since all arc strong, Strong support of vertex can be easily calculated
For n = 2


Here , each vertex have strong support namely one
For n = 3 and 4 (Refer example 2 and example 3)
For n = 5 , 6, 7 ….

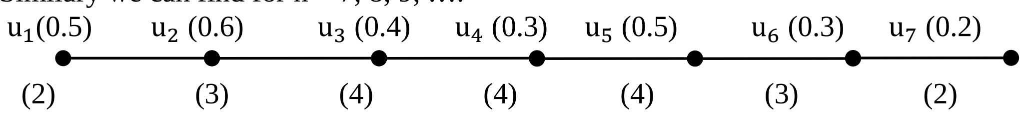


D = { u_1, u_3, u_5 } , $n[\gamma_{ssupp}(FP_5)] = 3$ and $\gamma_{ssupp}(FP_5) = 8 + \sum_{k\in D} \sigma(k)$



D = { u_1, u_3, u_5 } , $n[\gamma_{ssupp}(FP_6)] = 3$ and $\gamma_{ssupp}(FP_6) = 9 + \sum_{k\in D} \sigma(k)$

Similary we can find for n = 7, 8, 9, ….



D = { u_2, u_4, u_6 }, $n[\gamma_{ssupp}(FP_7)] = 3$ and $\gamma_{ssupp}(FP_7) = 10 + \sum_{k\in D} \sigma(k)$
Now, we can list for different n

$n[\gamma_{ssupp}(FP_5)] = 3$ and $\gamma_{ssupp}(FP_n) = 8 + \sum_{k\in D} \sigma(k)$
$n[\gamma_{ssupp}(FP_6)] = 3$ and $\gamma_{ssupp}(FP_6) = 9 + \sum_{k\in D} \sigma(k)$
$n[\gamma_{ssupp}(FP_7)] = 3$ and $\gamma_{ssupp}(FP_7) = 10 + \sum_{k\in D} \sigma(k)$
$n[\gamma_{ssupp}(FP_8)] = 4$ and $\gamma_{ssupp}(FP_8) = 12 + \sum_{k\in D} \sigma(k)$
$n[\gamma_{ssupp}(FP_9)] = 4$ and $\gamma_{ssupp}(FP_9) = 13 + \sum_{k\in D} \sigma(k)$

$n[\gamma_{ssupp}(FP_{10})] = 4$ and $\gamma_{ssupp}(FP_{10}) = 14 + \sum_{k\in D} \sigma(k)$
$n[\gamma_{ssupp}(FP_{11})] = 5$ and $\gamma_{ssupp}(FP_{11}) = 16 + \sum_{k\in D} \sigma(k)$
$n[\gamma_{ssupp}(FP_{12})] = 5$ and $\gamma_{ssupp}(FP_{12}) = 17 + \sum_{k\in D} \sigma(k)$
$n[\gamma_{ssupp}(FP_{13})] = 5$ and $\gamma_{ssupp}(FP_{13}) = 18 + \sum_{k\in D} \sigma(k)$

Thus

Number of elements in the minimum strong support dominating set

$$n[\gamma_{ssupp}(FP_n)] = \left\lceil \frac{n}{3} \right\rceil + 1 \quad \text{where } n \geq 5$$

Strong support domination number

$$\gamma_{ssupp}(FP_n) = n + n[\gamma_{ssupp}(FP_n)] + \sum_{k\in D} \sigma(k)$$
$$= n + \left\lceil \frac{n}{3} \right\rceil + 1 + \sum_{k\in D} \sigma(k), \text{ when } n \geq 5$$

Where D is the minimum strong support dominating set.

Result 2. Let FC_n be fuzzy cycle with n vertex and assume that all arcs of FC_n are strong arc.

(i) $n[\gamma_{ssupp}(FC_3)] = 1$ and $\gamma_{ssupp}(FC_3) = 4 + \sum_{k\in D} \sigma(k)$

(ii) $n[\gamma_{ssupp}(FC_n)] = \left\lceil \frac{n}{3} \right\rceil$ when $n \geq 4$ and

$\gamma_{ssupp}(FC_n) =$ 4 times of $n[\gamma_{ssupp}(FC_n)] + \sum_{k\in D} \sigma(k)$

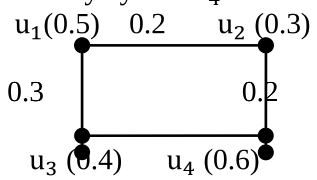
$= 4\left\lceil \frac{n}{3} \right\rceil + \sum_{k\in D} \sigma(k)$ when $n \geq 4$

Where D is the minimum strong support dominating set

Proof. Let FC_n be fuzzy cycle with n vertex and assume that all arcs of FC_n are strong arc . Since all arcs of FC_n are strong the strong support of each vertex can be easily calculated .

For the fuzz graph FC_3 (Refer example 1)

When n = 4, the fuzzy cycle FC_4 can be drawn



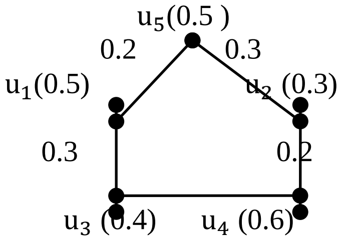
D = { u_2, u_3 },

$n[\gamma_{ssupp}(FC_4)] = \left\lceil \frac{4}{3} \right\rceil = 2$

And $\gamma_{ssupp}(FC_4) = 4\left\lceil \frac{n}{3} \right\rceil + \sum_{k\in D} \sigma(k) = 4(2) + (0.3+0.4)$
$= 8+0.7$
$= 8.7$

Note that, suppose if we select D as other than this, it will not be minimum strong support dominating set.

When n = 5, the fuzzy cycle FC_5 can be drawn



D = { u_2, u_3 },

By assumption all arcs of this fuzzy cycle are strong arc .

Hence $n[\gamma_{ssupp}(FC_5)] = \left\lceil \frac{5}{3} \right\rceil = 2$

And $\gamma_{ssupp}(FC_5) = 4\left\lceil \frac{5}{3} \right\rceil + \sum_{k\in D} \sigma(k) = 4(2) + (0.3+0.4)$
$= 8+0.7$

$= 8.7$

Similarly can prove for n =6, 7 ,8… as given below

$n[\gamma_{ssupp}(FC_4)] = \left\lceil\frac{4}{3}\right\rceil = 2$ and $\gamma_{ssupp}(FC_4) = 8 + \sum_{k\in D}\sigma(k)$

$n[\gamma_{ssupp}(FC_5)] = \left\lceil\frac{5}{3}\right\rceil = 2$ and $\gamma_{ssupp}(FC_5) = 8 + \sum_{k\in D}\sigma(k)$

$n[\gamma_{ssupp}(FC_6)] = \left\lceil\frac{6}{3}\right\rceil = 2$ and $\gamma_{ssupp}(FC_6) = 8 + \sum_{k\in D}\sigma(k)$

$n[\gamma_{ssupp}(FC_7)] = \left\lceil\frac{7}{3}\right\rceil = 3$ and $\gamma_{ssupp}(FC_7) = 12 + \sum_{k\in D}\sigma(k)$

$n[\gamma_{ssupp}(FC_8)] = \left\lceil\frac{8}{3}\right\rceil = 3$ and $\gamma_{ssupp}(FC_8) = 12 + \sum_{k\in D}\sigma(k)$

$n[\gamma_{ssupp}(FC_9)] = \left\lceil\frac{9}{3}\right\rceil = 3$ and $\gamma_{ssupp}(FC_9) = 12 + \sum_{k\in D}\sigma(k)$

$n[\gamma_{ssupp}(FC_{10})] = \left\lceil\frac{10}{3}\right\rceil = 4$ and $\gamma_{ssupp}(FC_{10}) = 16 + \sum_{k\in D}\sigma(k)$

Hence

$n[\gamma_{ssupp}(FC_n)] = \left\lceil\frac{n}{3}\right\rceil$ when $n \geq 4$

$\gamma_{ssupp}(FC_n) = 4$ times of $n[\gamma_{ssupp}(FC_n)] + \sum_{k\in D}\sigma(k)$

$= 4\left\lceil\frac{n}{3}\right\rceil + \sum_{k\in D}\sigma(k)$ when $n \geq 4$

Where D is the minimum strong support dominating set.

Result 3. Let FC_n be fuzzy cycle with n vertex .
If at least one non- strong arc exist in FC_n then the following result is true.

(i) $n[\gamma_{ssupp}(FC_3)] = 1$ and $\gamma_{ssupp}(FC_3) = 2 + \sum_{k\in D}\sigma(k)$

(ii) $n[\gamma_{ssupp}(FC_4)] = 2$ and $\gamma_{ssupp}(FC_4) = 5 + \sum_{k\in D}\sigma(k)$ and

(iii) $n[\gamma_{ssupp}(FC_n)] = \left\lceil\frac{n}{3}\right\rceil + 1 = n[\gamma_{ssupp}(FP_n)]$ when $n > 4$

$\gamma_{ssupp}(FC_n) = n + \left\lceil\frac{n}{3}\right\rceil + 1 + \sum_{k\in D}\sigma(k)$

$= n + n[\gamma_{ssupp}(FC_n)] + \sum_{k\in D}\sigma(k)$, when $n > 4$

$= \gamma_{ssupp}(FP_n)$

Where D is the minimum strong support dominating set.

Proof . Let FC_n be fuzzy cycle with n vertex .
We know that at most a non –strong arc can exist in any fuzzy cycle.
For this proof, assume that fuzzy cycle FC_n has a non- strong arc .
When n =3 ,we have $n[\gamma_{ssupp}(FC_3)] = 1$ (Refer example 3)
When n = 4 , we have $n[\gamma_{ssupp}(FC_4)] = 2$ (Refer graph FC4 in result 2)
Similarly, we have the following result for n = 5, 6, 7….

$n[\gamma_{ssupp}(FC_5)] = 3$ and $\gamma_{ssupp}(FC_5) = 8 + \sum_{k\in D}\sigma(k)$ (Refer graph FC5 in result 2)

$n[\gamma_{ssupp}(FC_6)] = 3$ and $\gamma_{ssupp}(FC_6) = 9 + \sum_{k\in D}\sigma(k)$

$n[\gamma_{ssupp}(FC_7)] = 3$ and $\gamma_{ssupp}(FC_7) = 10 + \sum_{k\in D}\sigma(k)$

$n[\gamma_{ssupp}(FC_8)] = 4$ and $\gamma_{ssupp}(FC_8) = 12 + \sum_{k\in D}\sigma(k)$

$n[\gamma_{ssupp}(FC_9)] = 4$ and $\gamma_{ssupp}(FC_9) = 13 + \sum_{k\in D}\sigma(k)$

$n[\gamma_{ssupp}(FC_{10})] = 4$ and $\gamma_{ssupp}(FC_{10}) = 14 + \sum_{k\in D}\sigma(k)$

$n[\gamma_{ssupp}(FC_{11})] = 5$ and $\gamma_{ssupp}(FC_{11}) = 16 + \sum_{k\in D}\sigma(k)$

$n[\gamma_{ssupp}(FC_{12})] = 5$ and $\gamma_{ssupp}(FC_{12}) = 17 + \sum_{k\in D}\sigma(k)$

$n[\gamma_{ssupp}(FC_{13})] = 5$ and $\gamma_{ssupp}(FC_{13}) = 18 + \sum_{k\in D}\sigma(k)$

Formula derivation

$n[\gamma_{ssupp}(FC_5)] = 3 = 2 + 1 = \left\lceil\frac{5}{3}\right\rceil + 1 = \left\lceil\frac{n}{3}\right\rceil + 1$ and

$\gamma_{ssupp}(FC_5) = 8 + \sum_{k\in D}\sigma(k) = 5 + 3 + \sum_{k\in D}\sigma(k)$

$= 5 + \left\lceil\frac{5}{3}\right\rceil + 1 + \sum_{k\in D}\sigma(k)$

$= n + \left\lceil\frac{n}{3}\right\rceil + 1 + \sum_{k\in D}\sigma(k)$

Similarly we can derive the all the above result for various n.

$n[\gamma_{ssupp}(FC_n)] = \left[\frac{n}{3}\right] + 1 = n[\gamma_{ssupp}(FP_n)]$ when $n > 4$ and

$\gamma_{ssupp}(FC_n) =$ n + $n[\gamma_{ssupp}(FC_n)] + \sum_{k\in D}\sigma(k)$

$= n + \left[\frac{n}{3}\right] + 1 + \sum_{k\in D}\sigma(k)$

$= n + n[\gamma_{ssupp}(FP_n)] + \sum_{k\in D}\sigma(k)$

$= \gamma_{ssupp}(FP_n)$

$\gamma_{ssupp}(FC_n) = \gamma_{ssupp}(FP_n)$

Where D is the minimum strong support dominating set.

Note. If a non- strong arc exist in the fuzzy cycle, the strong support domination of corresponding the fuzzy cycle with n vertices is equal to the strong support domination of fuzzy path with n vertices.

Result 4. Let $FK_{1,n}$ be fuzzy star with n +1 vertices .

(i) $n[\gamma_{ssupp}(FK_{1,n})] = 1$ and

(ii) $\gamma_{ssupp}(FK_{1,n}) = n + \sum_{k\in D}\sigma(k)$

Proof. Since all the arcs are strong in $FK_{1,n}$, only one vertex k in D which is enough to dominate all other vertex of $FK_{1,n}$ Where D is the minimum strong support dominating set . By calculating strong support of that vertex is clearly n.

Hence $n[\gamma_{ssupp}(FK_{1,n})] = 1$ and $\gamma_{ssupp}(FK_{1,n}) = n + \sum_{k\in D}\sigma(k)$.

Result 5. Let FK_n be fuzzy complete with n vertices .

Case (a) If no non-strong arc exist ,then

(i) $n[\gamma_{ssupp}(FK_n)] = 1$

(ii) $\gamma_{ssupp}(FK_n) = (n-1)^2 + \sum_{k\in D}\sigma(k)$

Case(b) If non -strong arc exist, then

(iii) $n[\gamma_{ssupp}(FK_3)] = 1$ and $\gamma_{ssupp}(FK_3) = 2 + \sum_{k\in D}\sigma(k)$

(iv) $n[\gamma_{ssupp}(FC_4)] = 2$ and $\gamma_{ssupp}(FK_4) = 5 + \sum_{k\in D}\sigma(k)$ and

(v) $1 \leq n[\gamma_{ssupp}(FK_n)] \leq \left[\frac{n}{3}\right] + 1$, when $n > 4$

(vi) $n + \left[\frac{n}{3}\right] + 1 + \sum_{k\in D}\sigma(k) \leq \gamma_{ssupp}(FK_n) \leq (n-1)^2 + \sum_{k\in D}\sigma(k)$, when $n > 4$

Proof. Let FK_n be fuzzy complete with n vertices .

Proof of this result is discussed in two cases.

Case(a)

Assume that there is no non- strong arcs in a Fuzzy complete graph .

Therefore, only one vertex is enough to dominate that fuzzy graph.

Clearly, $n[\gamma_{ssupp}(FK_n)] = 1$ and Strong support of each vertex has same $(n-1)^2$.

By definition of strong support domination, $\gamma_{ssupp}(FK_n) = (n-1)^2 + \sum_{k\in D}\sigma(k)$

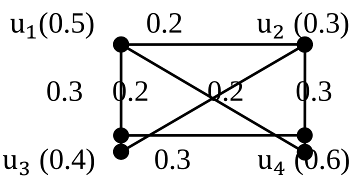
Case(b)

In this case, assume that fuzzy complete graph has at least one non-strong arc.

For n =3, Fuzzy complete graph FC_3 , [refer example 3]

$n[\gamma_{ssupp}(FK_3)] = 1$ and $\gamma_{ssupp}(Fk_3) = 2 + \sum_{k\in D}\sigma(k)$

When n = 4, the fuzzy complete graph FK_4 can be drawn



$D = \{ u_2, u_3 \}$,

In fuzzy complete graph FK_4, at most 3 non-strong arc can exist (Refer [14,15, 16])

Therefore, cardinality minimum strong support dominating set D is 2 (two).

That is, $n[\gamma_{ssupp}(FK_4)] = 2$

Also strong support of these two vertex in D are 3 and 2 .
By definition of strong support domination, $\gamma_{ssupp}(FK_4)$ = 5 +$\sum_{k \in D} \sigma(k)$
To prove (v) and (vi) of case (b), we have the following criteria
(i) Suppose no non-strong arc exist then clearly it will yield lower bound of cardinality of minimum strong support dominating set and also upper bound of strong support domination number.
(ii) Suppose maximum non –strong exist in FK_n it will yield upper bound of cardinality of minimum dominating set as well as lower bound strong support domination number.

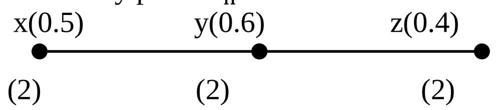
Hence, above two criteria we have

(v) $1 \le n[\gamma_{ssupp}(FK_n)] \le \left[\frac{n}{3}\right] + 1$, when $n > 4$

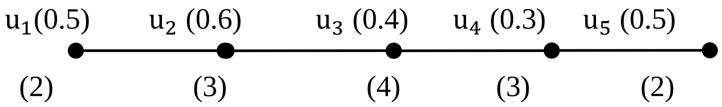
(vi) $n + \left[\frac{n}{3}\right] + 1 + \sum_{k \in D} \sigma(k) \le \gamma_{ssupp}(FK_n) \le (n-1)^2 + \sum_{k \in D} \sigma(k)$, when $n > 4$

Note. Existence of non-strong arc of any fuzzy graph , that will be reduced strong degree and strong support of the vertices , Further it affects the strong support domination number. That is, if non-strong arc exist in the fuzzy graph that will be altered the strong support domination number.

Result 6. Let G be a fuzzy graph with n vertices, then $n[\gamma_{ssupp}(G) \ge n\,[\gamma_s(G)\,]$
That is, Cardinality of minimum strong support dominating set are greater than or equal to cardinality of minimum strong dominating set .
Proof. We can easily prove this result by fuzzy path
Let G be a fuzzy path FP_n
For n = 3



We know that all arcs are strong in any fuzzy path (Refer [14,15,16])
Minimum Strong support dominating set D = { y }
And minimum strong dominating set D = { y }
Here ,$n[\gamma_{ssupp}(G)] = n\,[\gamma_s(G)\,]$ = 1 (one)
For n = 5 , the fuzzy path FP_5 are drawn below.



We know that all arcs are strong arc in any fuzzy path (Refer [14,15])
Therefore, minimum Strong support dominating set D = { u_1, u_3, u_5 }
And minimum strong dominating set D = { u_2, u_5 }
Therefore $n[\gamma_{ssupp}(G)$ = 3 and $n\,[\gamma_s(G)\,]$ =2
Hence $n[\gamma_{ssupp}(G) \ge n\,[\gamma_s(G)\,]$.

4. Conclusion

In this paper, definition of strong degree of a vertex and strong support of a vertex of the fuzzy graph are newly generalized. Further, definition of strong support dominating set and strong support domination number of the fuzzy graph are newly introduced, Moreover, some new results on strong support domination number for standard fuzzy graph are found and compared with strong dominating set.

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References

[1] Bhutani, K.R., and Rosenfeld, A., Storng arcs in fuzzy graph, Information sciences, 152(2003),319-322

[2] Bhutani, K.R.,and Mordeson, J., and Rosenfeld, A., On degrees of end node and cut node in fuzzy graphs, Iranian Journal of Fuzzy systems, 1(1) (2004), 57-64

[3] Harary, E., 1969. Graph Theory. Addison Wesley, Reading, MA. McAlester, M.L.N., 1988. Fuzzy intersection graphs. Comp. Math. Appl. 15 _10., 871–886.

[4] Monderson, J.N., Premchand, S.N., fuzzy graphs and fuzzy hypergraphs, physica –verlag.

[5] Mathumangal pal, Intersection graphs: An intersection, Anals of Pure and Applied Mathematics ,vol.4, No.1, 2013,43-91.

[6] Mathumangal pal and Hossein Rashmanlou, Irregular interval –valued Fuzzy graph, Anals of Pure and Applied Mathematics, vol.3, No.1, 2013,56-66.

[7] Methew,S., and M.S. sunitha, Types of arcs in fuzzy graph, Information sciences, 179(2009), 1760-1768.

[8] Nagoorgani, A ., Chandrasekaran, V.T , First look at Fuzzy Graph Theory.

[9] Nagoorgani, A ., Chandrasekaran, V.T,2006. Domination in fuzzy graph, Adv. In fuzzy set & systems ,Allahabad, India ,Volume 1(1), page 17-26.

[10] Ore, O., 1962. Theory of Graphs. Amer. Math. Soc. Colloq. Publ. 38, Providence.

[11] Rosenfeld, A., 1975. Fuzzy graphs. In: Zadeh, L.A., Fu, K.S., Shimura, M. _Eds.., Fuzzy Sets and Their Applications. Academic Press, New York.

[12] Somasundaram, A., somasundaram, S., 1998. Domination in fuzzy graph-1, pattern recognition letter,19 (9), 787-791.

[13] Ponnappan, C.Y., Surilinathan, P., Basheer Ahamed, S., 2015. Edge domination in fuzzy graph, New approach,International journal of IT, Engg. applied sciences research vol 4,no 1.

[14] Senthilkumar.V, Ponnappan. C.Y.,and Selvam .A., A note on Domination in fuzzy graph using arc , Journal of Science and Computation, Vol. 5(6) (2018) page.No.84

[15] Ponnappan, C.Y., Senthilkumar.V., Case study of non strong arc in Cartesian product fuzzy graphs, Annls of Pure and Applied Mathematics,(2018)

[16] Ponnappan, C.Y., Senthilkumar.V., Domination in Cartesian product fuzzy graphs by strong arc, Bulletin of pure and applied mathematics,Vol(2)-2018, 321-336.

[17] C.V.R.Harinarayan, C.Y. Ponnappan,V.Swaminathan.,Just excellance and very excellance in a graphs with respect to strong domination, Tamkang journal of mathematics,Volume 38(1),167-175,summer 2007.

[18] C.Y.Ponnappan, Surilinathan.P, Basheer Ahamed.S,. The strong split domination number of fuzzy graphs, International journal of computer and organization trends, volume 8(1)-may2014.

[19] Zadeh, L.A, Fuzzy set, Information and control,8(1965),338-353.