

LOCALLY CLOSED SETS IN INTUITIONISTIC TOPOLOGICAL SPACES

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Abstract The purpose of this paper is to introduce intuitionistic locally closed sets and some intuitionistic separation axioms using the notions of intuitionistic locally closed sets and study the characterization of intuitionistic locally closed sets with the new intuitionistic separation axioms.

Keywords . intuitionistic locally closed sets , intuitionistic separation axioms.

Mathematics Subject Classification: 54A99

1. Introduction

The concept of intuitionistic sets in topological space was first introduced by Coker [3]. He has studied some fundamental topological properties on intuitionistic sets. Later, he studied connectedness [4] and separation axioms [1] in intuitionistic topological spaces. In this paper, we have defined intuitionistic locally closed sets and using the notion of intuitionistic locally closed sets, some new intuitionistic separation axioms are introduced. Also, the characterization of intuitionistic locally closed set with the new intuitionistic separation axioms are investigated.

2. Preliminaries

Throughout this paper, (X, τ) and (Y, σ) represent intuitionistic topological spaces on which no separation axioms are defined. We recall the following definitions, which are useful in the sequel.

Definition 2.1 [2] Let X be a nonempty set. An intuitionistic set A is an object having the form $A = (A_1, A_2)$, where A_1 and A_2 are disjoint subsets of X . Also, A_1 is called the set of members of A and A_2 is the set of nonmembers of A .

Definition 2.2 [2] Let X be a nonempty set, and the intuitionistic sets A and B be in the form $A = (X, A_1, A_2)$, $B = (X, B_1, B_2)$ respectively. Then:

$A \subseteq B$ if and only if $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$

$\overline{A} = (X, A_2, A_1)$

$A \cap B = (X, A_1 \cap B_1, A_2 \cup B_2)$

$A \cup B = (X, A_1 \cup B_1, A_2 \cap B_2)$

$A - B = A \cap \overline{B}$

$\tilde{\phi} = (X, \phi, X)$ and $\tilde{X} = (X, X, \phi)$

Definition 2.3 [2] Let X be a nonempty set, $a \in X$ and let $A = (A_1, A_2)$ be an intuitionistic subset of X . The intuitionistic set $\tilde{a}$ defined by $\tilde{a} = (\{a\}, \{a\}^c)$ is called an intuitionistic point in X .

Definition 2.4 [3] An intuitionistic topology on a nonempty set X is a family τ of intuitionistic subsets of X containing $\tilde{\phi} = (\phi, X)$, $\tilde{X} = (X, \phi)$ and closed under finite infima and arbitrary suprema. Then the pair (X, τ) is called an intuitionistic topological space.

Every member of τ is known as an intuitionistic open set in X . The complement A^c of an intuitionistic open set A in an intuitionistic topological space (X, τ) is called intuitionistic closed.

Definition 2.5 [3] Let X be a nonempty set and let A be an intuitionistic subset of an intuitionistic topological space (X, τ) . Then the closure of A is defined by $\text{cl}(A) = \bigcap \{K / K \text{ is an intuitionistic closed set of } X \text{ and } A \subseteq K\}$.

Definition 2.6 [3] Let X be a nonempty set and let A be an intuitionistic subset of an intuitionistic topological space (X, τ) . Then the interior of A is defined by $\text{int}(A) = \bigcup \{K / K \text{ is an intuitionistic open set of } X \text{ and } K \subseteq A\}$.

Definition 2.7 [6] Let (X, τ) be an intuitionistic topological space and A be an intuitionistic set in X . The intuitionistic set A is called intuitionistic dense in X if $\text{cl}(A) = \tilde{X}$.

Definition 2.8 [5] An intuitionistic topological space (X, τ) is intuitionistic submaximal if every intuitionistic dense subset of X is intuitionistic open.

3. Intuitionistic Locally Closed Sets

Definition 3.1 An intuitionistic set A of an intuitionistic topological space (X, τ) is said to be intuitionistic locally closed if $A = U \cap V$, where $U, V^c \in \tau$.

Lemma 3.2 Let A be an intuitionistic set of an intuitionistic topological space (X, τ) . The subsequent statements are equivalent:

- (i) A is intuitionistic locally closed.
- (ii) $A = U \cap \text{cl}(V)$, where U is an intuitionistic open set.

Remark. An intuitionistic set A of an intuitionistic topological space (X, τ) is intuitionistic locally closed if and only if A^c is the union of an intuitionistic open set and an intuitionistic closed set. Every intuitionistic open set of an intuitionistic topological space (X, τ) is intuitionistic locally closed.

The complement of an intuitionistic locally closed set need not be intuitionistic locally closed.

An intuitionistic dense set is intuitionistic open if and only if it is intuitionistic locally closed.

Definition 3.3 Two intuitionistic subsets A and B of an intuitionistic topological space (X, τ) are said to be intuitionistic separated if and only if $A \cap \text{cl}(B) = \tilde{\emptyset}$ and $\text{cl}(A) \cap B = \tilde{\emptyset}$.

Proposition 3.4 Let A and B be intuitionistic locally closed sets of an intuitionistic topological space (X, τ) . If A and B are separated, then $A \cup B \in \text{LC}(X, \tau)$ where $\text{LC}(X, \tau)$ denotes the set of all locally closed sets of (X, τ) .

Proof. Let U and V be intuitionistic open sets such that A is equal to U intersection $\text{cl}(A)$ and B is equal to V intersection $\text{cl}(B)$. As A and B are separated, $U \cap \text{cl}(B) = V \cap \text{cl}(A) = \tilde{\emptyset}$. $A \cup B = (U \cap \text{cl}(A)) \cup (V \cap \text{cl}(B)) = (U \cup V) \cap \text{cl}(A \cup B)$. Therefore, $A \cup B \in \text{LC}(X, \tau)$.

Proposition 3.5 For an intuitionistic set A of an intuitionistic topological space (X, τ) , the following statements are equivalent:

- A is intuitionistic locally closed;
- $A = U \cap \text{cl}(A)$ for some intuitionistic open set U ;
- $\text{Cl}(A) - A$ is intuitionistic closed;
- $A \cup (\text{cl } A)^c$ is intuitionistic open;
- $A \subseteq \text{int}(A \cup (\text{cl } A)^c)$.

Proof. The proof is obvious.

Definition 3.6 The intuitionistic local closure of an intuitionistic set A of an intuitionistic topological space (X, τ) is the intersection of all intuitionistic locally closed sets containing A and is denoted by $\text{lcl}(A)$.

Lemma 3.7 Let A, B be intuitionistic sets of an intuitionistic topological space (X, τ) . Then the following properties hold:

- A is intuitionistic locally closed if and only if $A = \text{lcl}(A)$.
- $A \subseteq \text{lcl}(A) \subseteq \text{cl}(A)$.
- If $A \subseteq B$, then $\text{lcl}(A) \subseteq \text{lcl}(B)$.
- $\text{lcl}(A)$ is intuitionistic locally closed.

Corollary 3.8 An intuitionistic topological space (X, τ) is intuitionistic submaximal if and only if every intuitionistic subset of (X, τ) is intuitionistic locally closed.

Theorem 3.9 Let A be an intuitionistic set of an intuitionistic topological space (X, τ) . Then the following statements are equivalent :

- (i) A is intuitionistic locally open;
- (ii) A is the union of T and C where T is an intuitionistic closed set and C is an intuitionistic open set.

Definition 3.10 An intuitionistic set A of intuitionistic topological space (X, τ) is called an intuitionistic local difference set (intuitionistic local D-set) if there are two intuitionistic locally open sets U, V in (X, τ) where U is not equal to $\tilde{X}$ and A is equal to $U - V$.

Definition 3.11 An intuitionistic topological space (X, τ) is intuitionistic local D_0 (intuitionistic local D_1) if for $x, y \in X$ such that $\tilde{x} \neq \tilde{y}$ there exists an intuitionistic local D-set of (X, τ) containing $\tilde{x}$ but not $\tilde{y}$ or (and) an intuitionistic local D-set containing $\tilde{y}$ but not $\tilde{x}$.

Intuitionistic local D_2 if for $x, y \in X$ such that $\tilde{x} \neq \tilde{y}$ there exist disjoint intuitionistic local D-sets A and B such that $\tilde{x} \in A$ and $\tilde{y} \in B$.

Intuitionistic local T_0 (intuitionistic local T_1) if for $x, y \in X$ such that $\tilde{x} \neq \tilde{y}$ there exists an intuitionistic locally open set of (X, τ) containing $\tilde{x}$ but not $\tilde{y}$ or (and) an intuitionistic locally open set containing $\tilde{y}$ but not $\tilde{x}$.

intuitionistic local T_2 if for $x, y \in X$ such that $\tilde{x} \neq \tilde{y}$ there exist disjoint intuitionistic locally open sets A and B such that $\tilde{x} \in A$ and $\tilde{y} \in B$.

Remark. If (X, τ) is intuitionistic local- T_i , then it is intuitionistic local- T_{i-1} , $i = 1, 2$.

If (X, τ) is intuitionistic local- T_i , then it is intuitionistic local- D_i , $i = 0, 1, 2$.

If (X, τ) is intuitionistic local- D_i , then it is intuitionistic local- D_{i-1} , $i = 1, 2$.

Theorem 3.12 Let (X, τ) be an intuitionistic topological space. The subsequent statements hold:

- (i) (X, τ) is intuitionistic local D_0 if and only if (X, τ) is intuitionistic local T_0 .
- (ii) (X, τ) is intuitionistic local D_1 if and only if (X, τ) is intuitionistic local D_2 .

Proof. The sufficiency for (i) and (ii) follows from the last Remark. Necessary condition for (i):

Let (X, τ) be intuitionistic local D_0 . Hence for any different pair of points $\tilde{x}, \tilde{y}$ of X , atleast one belongs to an intuitionistic local D-set O . Hence $\tilde{x} \in O$ and $\tilde{y} \notin O$. Suppose $O = U - V$ for which $U \neq \tilde{X}$ and U and V are intuitionistic locally open sets in (X, τ) . This implies that $\tilde{x} \in U$. If $\tilde{y} \notin O$, we have (i) $\tilde{y} \notin U$ (ii) $\tilde{y} \notin U$ and $\tilde{y} \in V$. For (i), (X, τ) is intuitionistic local- T_0 since $\tilde{x} \in U$ and $\tilde{y} \notin U$. For (ii), (X, τ) is also intuitionistic local- T_0 since $\tilde{y} \in V$ but $\tilde{x} \notin V$.

Necessary condition for (ii): Suppose that (X, τ) is intuitionistic local D_1 . Hence for any different points $\tilde{x}$ and $\tilde{y}$ in X there exists intuitionistic local-D sets G and E such that G contains $\tilde{x}$ but not $\tilde{y}$ and E contains $\tilde{y}$ not $\tilde{x}$. Let $G = U - V$ and $E = W - D$, where U, V, W and D are intuitionistic locally open sets in (X, τ) . By the fact that $\tilde{x} \notin E$, we have two cases, that is either $\tilde{x} \notin W$ or both W and D contain $\tilde{x}$. If $\tilde{x} \notin W$, then from $\tilde{y} \notin G$ either (i) $\tilde{y} \notin U$ or (ii) $\tilde{y} \in U$ and $\tilde{y} \in V$. If (i) is the case, then it follows from $\tilde{x} \in U - V$ that $\tilde{x} \in U - (V \cup W)$ and also it follows from $\tilde{y} \in W - D$ that $\tilde{y} \in W - (U \cup D)$. Thus, $U - (V \cup W)$ and $W - (U \cup D)$ are disjoint. If (ii) is the case, it follows that $\tilde{x} \in U - V$ and $\tilde{y} \in V$ since $\tilde{y} \in U$ and $\tilde{y} \in V$. Therefore, $(U - V) \cap V = \emptyset$. If $\tilde{x} \in W$ and $\tilde{x} \in D$, we have $\tilde{y} \in W - D$ and $\tilde{x} \in D$. Hence, $(W - D) \cap D = \emptyset$. Therefore, (X, τ) is intuitionistic local D_2 .

Theorem 3.13 If an intuitionistic topological space is intuitionistic local D_1 , then it is intuitionistic local T_0 .

Proof. The proof is derived from Remark and also from Theorem 3.12.

Definition 3.14 An intuitionistic subset B of an intuitionistic topological space (X, τ) is said to be an intuitionistic local neighbourhood of a point $\tilde{x}$ iff there exists an intuitionistic locally open set A such that $\tilde{x} \in A \subset B$.

Definition 3.15 Let $\tilde{x}$ be an intuitionistic point in (X, τ) . If $\tilde{x}$ does not have an intuitionistic local neighborhood other than $\tilde{X}$, then $\tilde{x}$ is an intuitionistic local neat point.

Theorem 3.16 An intuitionistic topological space (X, τ) is intuitionistic local T_1 iff the singletons are intuitionistic locally closed sets.

Proof. Let $\tilde{x}$ be any intuitionistic point of an intuitionistic local T_1 space and $\tilde{y} \in \tilde{X}^c$. Then $\tilde{x}$ is not equal to $\tilde{y}$. Then there exists an intuitionistic locally open set U where $\tilde{y} \in U$ but $\tilde{x} \notin U$. Therefore, $\tilde{y} \in U \subset \tilde{X}^c$. That is $\tilde{X}^c = \bigcup \{U / \tilde{y} \in \tilde{X}^c\}$ which is intuitionistic locally open. Hence, singletons are intuitionistic locally closed.

Conversely, let $x, y \in X$ with $\tilde{x} \neq \tilde{y}$. Now $\tilde{x} \neq \tilde{y}$ implies $\tilde{y} \in \tilde{X}^c$. Hence $\tilde{X}^c$ is intuitionistic locally open set containing $\tilde{y}$ but not $\tilde{x}$. Similarly, $\tilde{y}^c$ is an intuitionistic locally open set containing $\tilde{x}$ but not $\tilde{y}$. Therefore, X is intuitionistic local T_1 .

Theorem 3.17 An intuitionistic topological space (X, τ) is intuitionistic local T_2 iff the intersection of all intuitionistic locally closed local neighborhoods of each point of the space is reduced to that point.

Proof. Let (X, τ) be intuitionistic local T_2 and $x \in X$. Then for each $y \in X$, different from x , there exist intuitionistic locally open sets G and H such that $\tilde{x} \in G$, $\tilde{y} \in H$ and $G \cap H = \emptyset$. Since $\tilde{x} \in G \subset H^c$, then H^c is an intuitionistic locally closed local neighborhood of $\tilde{x}$ to which $\tilde{y}$ does not belong. Consequently, the intersection of all intuitionistic locally closed local neighborhoods of $\tilde{x}$ is reduced to $\tilde{x}$.

Conversely, let $x, y \in X$ with $\tilde{x}$ not equal to $\tilde{y}$. By hypothesis, there exists an intuitionistic locally closed local neighborhoods U of $\tilde{x}$ such that $\tilde{y}$ does not belongs to U . Thus, there is an intuitionistic locally open set G such that $\tilde{x} \in G \subset U$. Thus G and U^c are disjoint locally open sets containing $\tilde{x}$ and $\tilde{y}$ respectively. Therefore, (X, τ) is intuitionistic local T_2 .

Theorem 3.18 An intuitionistic topological space (X, τ) is intuitionistic local T_0 iff for each pair of different points $\tilde{x}, \tilde{y}$ of X , $\text{lcl}(\tilde{x}) \neq \text{lcl}(\tilde{y})$.

Proof. Sufficiency: Suppose that for $x, y \in X$, $\tilde{x} \neq \tilde{y}$ and $\text{lcl}(\tilde{x}) \neq \text{lcl}(\tilde{y})$. Let $\tilde{z}$ be a point of X such that $\tilde{z}$ belong to $\text{lcl}(\tilde{x})$ and $\tilde{z}$ does not belong to $\text{lcl}(\tilde{y})$. To prove that $\tilde{x} \notin \text{lcl}(\tilde{y})$. For if, $\tilde{x} \in \text{lcl}(\tilde{y})$, then $\text{lcl}(\tilde{x}) \subset \text{lcl}(\tilde{y})$. This contradicts the fact that $\tilde{z} \notin \text{lcl}(\tilde{y})$. Hence, $\tilde{x}$ belongs to the intuitionistic locally open set $(\text{lcl}(\tilde{y}))^c$ to which $\tilde{y}$ does not belong.

Necessity: Let (X, τ) be an intuitionistic local T_0 space and $\tilde{x}, \tilde{y}$ be any two different intuitionistic points of X . Then there exists an intuitionistic locally open set G containing $\tilde{x}$ or $\tilde{y}$, say $\tilde{x}$ but not $\tilde{y}$. Then G^c is an intuitionistic locally closed set which does not contain $\tilde{x}$ but contains $\tilde{y}$. Since $\text{lcl}(\tilde{y})$ is the smallest intuitionistic locally closed set containing $\tilde{y}$, $\text{lcl}(\tilde{y}) \subset G^c$ and hence $\tilde{x} \notin \text{lcl}(\tilde{y})$. Therefore, $\text{lcl}(\tilde{x}) \neq \text{lcl}(\tilde{y})$.

Definition 3.19 An intuitionistic topological space (X, τ) is intuitionistic local regular if for each intuitionistic locally closed set A and any intuitionistic point $\tilde{x}$ belonging to $(\tilde{X} - A)$, there

exist disjoint intuitionistic locally open sets U and V such that A is contained in U and $\tilde{x}$ belongs to V .

Theorem 3.20 An intuitionistic topological space (X, τ) is intuitionistic local regular if for each $x \in X$ and each intuitionistic locally open set U containing $\tilde{x}$, there exists an intuitionistic locally open set V for which $\tilde{x}$ belongs to V contained in $\text{lcl}(V) \subset U$.

Proof. Let X be intuitionistic local regular and let $x \in X, U \in \text{LO}(X, \tilde{x})$. Then $\tilde{X} - U$ is intuitionistic locally closed and $\tilde{x} \notin \tilde{X} - U$. By intuitionistic local regularity of X , there exist disjoint intuitionistic locally open sets V and W such that $\tilde{x} \in V$ and $\tilde{X} - U \subset W$. (ie) $V \subset \tilde{X} - W$ and $\tilde{X} - W \subset U$. Then $\tilde{x} \in V \subset \text{lcl}(V) \subset \text{lcl}(\tilde{X} - W) = \tilde{X} - W \subset U$. Therefore, $\tilde{x} \in V \subset \text{lcl}(V) \subset U$.

Conversely, assume that for each $x \in X$ and for each intuitionistic locally open set U containing $\tilde{x}$, there exists an intuitionistic locally open set V such that $\tilde{x} \in V \subset \text{lcl}(V) \subset U$. Let F be an intuitionistic locally closed set of X and $\tilde{x} \in \tilde{X} - F$. Then there exists an intuitionistic locally open set V such that $\tilde{x} \in V \subset \text{lcl}(V) \subset \tilde{X} - F$. Therefore, $\tilde{x} \in V$ and $F \subset \tilde{X} - \text{lcl}(V)$. The intuitionistic sets V and $\tilde{X} - \text{lcl}(V)$ are intuitionistic locally open and are disjoint. Hence, X is intuitionistic local regular.

Definition 3.21 If every intuitionistic locally open set of an intuitionistic topological space contains the intuitionistic local closure of each of its singletons then the intuitionistic topological space X is said to be intuitionistic local R_0 .

Definition 3.22 An intuitionistic topological space X is said to be intuitionistic local $T_{\frac{1}{2}}$ if every singleton is intuitionistic locally open or intuitionistic locally closed.

Theorem 3.23 Let (X, τ) be an intuitionistic topological space. Then,

- (i) Every intuitionistic local- T_1 space is intuitionistic local R_0 .
- (ii) (X, τ) is intuitionistic local $T_{\frac{1}{2}}$ and intuitionistic local R_0 if and only if (X, τ) is intuitionistic local T_1 .

Proof. (i) Let U be any intuitionistic locally open set of X . Then, for each intuitionistic point $\tilde{x} \in U$, $\text{lcl}(\tilde{x}) = \tilde{x} \in U$. Hence, X is intuitionistic local R_0 .

(ii) Necessity: Let $x \in X$.

Case 1: $\tilde{x}$ is intuitionistic locally open. Since (X, τ) is intuitionistic local R_0 ,

$\text{lcl}(\tilde{x}) \subset \tilde{x}$ and hence $\tilde{x}$ is intuitionistic locally closed.

Case 2: $\tilde{x}$ is intuitionistic locally closed. Thus, every singleton is intuitionistic locally closed.

Sufficiency: Obvious from (i).

Theorem 3.24 For an intuitionistic local R_0 intuitionistic topological space (X, τ) , the following are equivalent:

- (i) (X, τ) is intuitionistic local T_0 ;
- (ii) (X, τ) is intuitionistic local T_1 ;
- (iii) $(X, \tau)(X, \tau)$ is intuitionistic local $T_{\frac{1}{2}}$.

Proof. It suffices to prove only that (i) $\Rightarrow$ (ii): Let $\tilde{x} \neq \tilde{y}$ and as (X, τ) is intuitionistic local T_0 , $\tilde{x} \in U \subset \tilde{X} - \tilde{Y}$ for some $U \in LO(X, \tau)$. Then $\tilde{x} \notin \text{lcl}(\tilde{y})$ and hence $\tilde{y} \notin \text{lcl}(\tilde{x})$. Hence there exists $V \in LO(X, \tau)$ such that $\tilde{y} \in V \subset \tilde{X} - \tilde{x}$ and thus (X, τ) is intuitionistic local T_1 .

Definition 3.25 An intuitionistic topological space (X, τ) is said to be intuitionistic local R_1 if for x, y in X with $\text{lcl}(\tilde{x}) \neq \text{lcl}(\tilde{y})$, there exist disjoint intuitionistic locally open sets U and V such that $\text{lcl}(\tilde{x}) \subset U$ and $\text{lcl}(\tilde{y}) \subset V$.

Proposition 3.26 If (X, τ) is intuitionistic local R_1 , then (X, τ) is intuitionistic local R_0 .

Proof. Let U be intuitionistic locally open and $\tilde{x} \in U$. If $\tilde{y} \notin U$, then since $\tilde{x} \notin \text{lcl}(\tilde{y})$, $\text{lcl}(\tilde{x}) \neq \text{lcl}(\tilde{y})$. Hence, there exists an intuitionistic locally open set V such that $\text{lcl}(\tilde{y}) \subset V$ and $\tilde{x} \notin V$ which implies $\tilde{y} \notin \text{lcl}(\tilde{x})$. Thus $\text{lcl}(\tilde{x}) \subset U$. Hence, (X, τ) is intuitionistic local R_0 .

Theorem 3.27 Let (X, τ) be an intuitionistic topological space. Then (X, τ) is intuitionistic local $T_{\frac{1}{2}}$ and intuitionistic local R_1 if and only if (X, τ) is intuitionistic local T_2 .

Proof. Necessity: Let $\tilde{x}$ and $\tilde{y}$ be two distinct intuitionistic points of X . Since (X, τ) is intuitionistic local $T_{\frac{1}{2}}$, we consider the following cases:

Case (i): Let $\tilde{x}$ and $\tilde{y}$ be intuitionistic locally closed. From assumptions, there exist disjoint intuitionistic locally open sets U and V such that $\tilde{x} = \text{lcl}(\tilde{x}) \subset U$ and $\tilde{y} = \text{lcl}(\tilde{y}) \subset V$. Hence (X, τ) is intuitionistic local T_2 .

Case (ii): $\tilde{x}$ is intuitionistic locally closed and $\tilde{y}$ is intuitionistic locally open. Let $U = \tilde{y}$. If $\tilde{z} \notin U$, then since $\tilde{y} \notin \text{lcl}(\tilde{z})$, $\text{lcl}(\tilde{y}) \neq \text{lcl}(\tilde{z})$. Since (X, τ) is intuitionistic local R_1 , there exists an intuitionistic locally open set V such that $\text{lcl}(\tilde{z}) \subset V$ and $\tilde{y} \notin V$, which implies $\tilde{z} \notin \text{lcl}(\tilde{y})$. Thus $\text{lcl}(\tilde{y}) \subset U = \tilde{y}$ and so $\tilde{y}$ is intuitionistic locally closed. Hence this case reduces to case (i).

Case (iii): $\tilde{x}$ is intuitionistic locally open and $\tilde{y}$ is intuitionistic locally closed. Hence this reduces to case (i).

Case (iv): $\tilde{x}$ and $\tilde{y}$ are intuitionistic locally open. Thus, (X, τ) is intuitionistic local T_2 .

Sufficiency: Every intuitionistic local T_2 is intuitionistic local T_1 and every intuitionistic local T_1 is intuitionistic local $T_{\frac{1}{2}}$. Let $\tilde{x}$ and $\tilde{y}$ be intuitionistic points such that $\text{lcl}(\tilde{x}) \neq \text{lcl}(\tilde{y})$. Then, by intuitionistic local $T_{\frac{1}{2}}$, there exist intuitionistic locally open sets U and V such that $\text{lcl}(\tilde{x}) = \tilde{x} \subset U$ and $\text{lcl}(\tilde{y}) = \tilde{y} \subset V$. Thus, (X, τ) is intuitionistic local R_1 .

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