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FIXED POINT THEOREMS FOR RECIPROCALLY CONTINUOUS MAPS IN GENERALIZED INTUITIONISTIC FUZZY METRIC SPACES

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Abstract: The purpose of this paper is to prove common fixed point theorems in generalized intuitionistic fuzzy metric spaces using weak compatibility, semi compatibility, and reciprocal continuity.

Key words: Fixed point, Generalized intuitionistic fuzzy metric spaces.

Mathematics subject classification: 54H25, 47H10.

1. Introduction

The concept of fuzzy sets, introduced by Zadeh [13] plays an important role in topology and analysis. Since then, there are many authors to study the fuzzy sets with applications. Especially Kramosil and Michlek [5] put forward a new concept of fuzzy metric space. George and Veeramani [3] revised the notion of fuzzy metric space with the help of continuous t-norm. As a result, many fixed point theorems for various forms of mappings are obtained in fuzzy metric spaces. Dhage [2] introduced the definition of D-metric space and proved many new fixed point theorems in D-metric spaces. In [12] Guangpeng Sun and Kai yang introduced the notion of Q- fuzzy metric space. In this study, we introduce the notion of generalized intuitionistic fuzzy metric space, which can be considered as a generalization of fuzzy metric space. The purpose of this paper is to prove common fixed point theorems in generalized intuitionistic fuzzy metric spaces using weak compatibility, semi compatibility, and reciprocal continuity. Our results extend, generalize and fuzzify several fixed point theorems in Q- fuzzy metric spaces.

2. Preliminaries

Definition 2.1:

A 5-tuple (X, Q, H, ∗, ◊) is said to be an generalized intuitionistic fuzzy metric space (for short GIFMS) if X is an arbitrary set, ∗ is a continuous t-norm, ◊ is a continuous t - conorm and Q, H are fuzzy set on $X^3 \to (0, \infty)$ satisfying the following conditions. For every x, y, z, a ∈ X and t, s > 0

i) Q(x, y, z, t) + H (x, y, z, t) ≤ 1

ii) Q(x, x, y, t) > 0, for all x ≠ y

iii) Q(x, x, y, t) ≤ Q (x, y, z, t) for y ≠ z

iv) Q (x, y, z, t) =1 iff x = y = z

v) Q (x, y, z, t) = Q {p (x, y, z), t}, where p is a permutation function.

vi) Q (x, a, a t) ∗ Q (a, y, z, s) ≤ Q (x, y, z, t+s)

vii) Q (x, y, z, .) : (0,∞) → [0,1] is continuous

viii) Q is non decreasing function on $R^+ \lim_{t\to\infty}$ Q(x,y,z,t) = 1 and $\lim_{t\to 0} Q(x,y,z,t) = 0$ for all x, y, z ϵ X, t > 0

ix) H (x, x, y, t) <1, for all x ≠ y

x) H (x, x, y, t) ≥ H (x, y, z, t) for y ≠ z

xi) H (x, y, z, t) = 0 iff x = y = z

xii) H (x, y, z, t) = H {p (x, y, z), t} where p is a permutation function.

xiii) H (x, a, a t) ◊ H (a, y, z, s) ≥ H (x, y, z, t + s)

xiv) H (x, y, z, .) : (0,∞) → [0,1] is continuous

xv) H is a non- increasing function on R+ $\lim_{t\to\infty}$ H (x, y, z, t) = 0 and $\lim_{t\to 0} H(x,y,z,t) = 1$ for all x, y, z ϵ X, t > 0

In this case, the pair (Q, H) is called a generalized intuitionistic fuzzy metric on X.

Definition 2.2 :

Let (X, Q, H, ∗, ◊) be an generalized intuitionistic fuzzy metric space, then

i) A sequence {xn} in X is said to be convergent to x if $\lim_{n\to\infty} Q(x_n, x_n, x, t) = 1$ and $\lim_{n\to\infty} H(x_n, x_n, x, t) = 0$.

ii) A sequence {xn} in X is said to be Cauchy sequence if $\lim_{n,m\to\infty} Q(x_n, x_n, x_m, t) = 1 \quad and \quad \lim_{n,m\to\infty} H(x_n, x_n, x_m, t) = 0$

that is, for any ε> 0 and for each t > 0, there exists n_0∈ N such that $Q(x_n, x_n, x_m, t) > 1 - \varepsilon$ and $H(x_n, x_n, x_m, t) < \varepsilon$ for n, m ≥ n_0.

iii) A generalized intuitionistic fuzzy metric space (X, Q, H, ∗, ◊) is said to be complete if every Cauchy sequence in X is convergent.

Definition 2.3:

Let f and g be self maps on generalized intuitionsitic fuzzy metric space (X, Q, H, ∗, ◊). Then the mappings are said to be weakly compatible if they commute at their coincidence point, that is, fx = gx implies that fgx = gfx.

Definition 2.4:

Let f and g be self mapsof generalized intuitionistic fuzzy metric space (X, Q, H, ∗,◊). The pair (f, g) is said to be compatible if $\lim_{n\to\infty} Q(fgx_n, gfx_n, gfx_n, t) = 1$ and $\lim_{n\to\infty} H(fgx_n, gfx_n, gfx_n, t) = 0$.

Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some z ∈ X.

Definition 2.5:

Let A and S be self-maps on a generalized intuitionistic fuzzy metric space (X, Q, H, ∗, ◊). Then A and S are said to be reciprocally continuous if,

$\lim_{n\to\infty} Q(ASx_n, Ax, Ax, t) = 1$, $\lim_{n\to\infty} Q(SAx_n, Sx, Sx, t) = 1$ and
$\lim_{n\to\infty} H(ASx_n, Ax, Ax, t) = 0$, $\lim_{n\to\infty} H(SAx_n, Sx, Sx, t) = 0$
Whenever there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ some $x \in X$.

Definition2.6:

Two self-maps A and Son a generalized intuitionistic fuzzy metric space(X, Q, H, $*$, $\lozenge$) are said to be semi compatible if

$$\lim_{n\to\infty} Q(ASx_n, Sx, Sx, t) = 1 \text{and} \quad \lim_{n\to\infty} H(ASx_n, Sx, Sx, t) = 0$$

Whenever there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ for some $x \in X$.
Any continuous function is a reciprocally continuous, but the converse is not true

Example 2.7:

Let $X = [2,20]$ with usual metric and * be defined as $a * b = \min\{a, b\}$ *and* $a \lozenge b = \max\{a, b\}$. Define$Q(x, y, z, t) = \frac{t}{t+G(x,y,z)}$ and $H(x, y, z, t) = \frac{G(x,y,z)}{t+G(x,y,z)}$.
Where $G(x, y, z) = |x - y| + |y - z| + |z - x|$ is a usual generalized metric. Define

$$Ax = \begin{Bmatrix} 2 \text{ if } x = 2 \\ 3 \text{ if } x > 2 \end{Bmatrix} \text{ and } Sx = \begin{Bmatrix} 2 \text{ if } x = 2 \\ 3 \text{ if } x > 2 \end{Bmatrix}$$

Consider a sequence $\{x_n\}$ in [2,20] such that $x_n < 2$ for each n.
Then $\lim_{n\to\infty} Ax_n = 2, \lim_{n\to\infty} Sx_n = 2, Ax_n \to 2 = A2$ and $Sx_n \to 2 = S2$
Neither A nor S is continuous at 2 and A and S are reciprocally continuous. Indeed,
$\lim_{n\to\infty} Q(ASx_n, A2, A2, t) = \frac{t}{t+|ASx_n - A2|+|ASx_n - A2|+|A2-A2|}$ and
$\lim_{n\to\infty} Q(ASx_n, A2, A2, t) = \frac{t}{t + 2|ASx_n - A2|}$
Thus $\lim_{n\to\infty} Q(ASx_n, A2, A2, t) \to 1$ as $n \to \infty$

$$\lim_{n\to\infty} H(ASx_n, A2, A2, t) = \frac{|ASx_n - A2| + |ASx_n - A2| + |A2 - A2|}{t + |ASx_n - A2| + |ASx_n - A2| + |A2 - A2|} \lim_{n\to\infty} H(ASx_n, A2, A2, t)$$
$$= \frac{2|ASx_n - A2|}{t + 2|ASx_n - A2|}$$

Thus $\lim_{n\to\infty} H(ASx_n, A2, A2, t) \to 0$ as $n \to \infty$
This shows that$ASx_n \to A2$. In similar way we get$SAx_n \to S2$.
Therefore (A, S) is reciprocally continuous.

Proposition 2.8:

Let f,g be self-maps on a generalized intuitionistic Q-fuzzy metric space (X, Q, H, $*$, $\lozenge$). Assume that (f, g) is reciprocally continuous. Then (f, g) is semi compatible if and only if (f, g) is compatible.

Proof: Let $\{x_n\}$ be a sequence in X so that $fx_n \to z$ and $gx_n \to x$ (f, g) is reciprocally continuous.

$$\lim_{n\to\infty} Q(fgx_n, fx, fx, t) \to 1, \ \lim_{n\to\infty} Q(gfx_n, gx, gx, t) \to 1 \text{ and}$$
$$\lim_{n\to\infty} H(fgx_n, fx, fx, t) \to 0, \ \lim_{n\to\infty} H(gfx_n, gx, gx, t) \to 0.$$

Assume that (f, g) is semi compatible, then
$\lim_{n\to\infty} Q(fgx_n, gx, gx, t) = 1$ and $\lim_{n\to\infty} H(fgx_n, gx, gx, t) = 0$.
Consider,

$$Q(fgx_n, gfx_n, gfx_n, t) \geq Q(fgx_n, gx, gx, t/2) * Q(gx, gfx_n, gfx_n, t/2)$$
$$= Q(fgx_n, gx, gx, t/2) * Q(gfx_n, gfx_n, gx, t/2)$$

$$\geq Q(fgx_n, gx, gx, t/2) * Q(gfx_n, gx, gx, t/4) * Q(gx, gfx_n, gx, t/4)$$
$$H(fgx_n, gfx_n, gfx_n, t) \leq H(fgx_n, gx, gx, t/2) \lozenge H(gx, gfx_n, gfx_n, t/2)$$
$$= H(fgx_n, gx, gx, t/2) \lozenge H(gfx_n, gfx_n, gx, t/2)$$
$$\leq H(fgx_n, gx, gx, t/2) \lozenge H(gfx_n, gx, gx, t/4) \lozenge H(gx, gfx_n, gx, t/4)$$

Taking limits as $n \to \infty$ we obtain

$$\lim_{n\to\infty} Q(fgx_n, gfx_n, gfx_n, t) \geq 1 * 1 * 1 = 1 \text{ and } \lim_{n\to\infty} H(fgx_n, gfx_n, gfx_n, t) \leq 0 \lozenge 0 \lozenge 0 = 0$$

Therefore f and g are compatible. Conversely suppose that (f, g) is compatible. Then for t >0, $Q(fgx_n, gfx_n, gfx_n, t) \to 1$ and $H(fgx_n, gfx_n, gfx_n, t) \to 0$ as $n \to \infty$.

Next to show that (f, g) is semi compatible.

$$Q(fgx_n, gx, gx, t) \geq Q(fgx_n, gfx_n, gfx_n, t/2) * Q(gfx_n, gx, gx, t/2) \text{ and}$$

$$H(fgx_n, gx, gx, t) \leq H(fgx_n, gfx_n, gfx_n, t/2) \lozenge H(gfx_n, gx, gx, t/2)$$

Taking limit as $n \to \infty$, we get:

$$\lim_{n\to\infty} Q(fgx_n, gx, gx, t) \geq 1 * 1 = 1 \text{ and } \lim_{n\to\infty} H(fgx_n, gx, gx, t) \leq 0 \lozenge 0 = 0$$

Therefore (f, g) is semi compatible.

3. Existence of unique common fixed point for four self maps

Theorem 3.1:

Let A, B, S, T be self-maps on a completegeneralized intuitionistic fuzzy metric space (X, Q, H,∗, ◊) where * is a continuous t-norm and ◊ is a continuous t-conorm, satisfying:

1. AX⊆TX, BX ⊆SX
2. (B, T) is weak compatible
3. For each x, y, z∈X and t > 0, Q(Ax, By, Bz, t) ≥ Φ (Q(Sx, Ty, Tz, t)) and H(Ax, By, Bz, t) ≤ ψ (H(Sx, Ty, Tz, t)), Where Φ,ψ: $[0, 1] \to [0, 1]$ is a continuous function such that Φ(1) = 1 and ψ(0) = 0 and Φ(a) > a , ψ (a) < a , for each 0 < a < 1.

If (A, S) is semi compatible and reciprocally continuous, then A, B, S and T have unique common fixed point.

Proof: Let $x_0 \in X$ be an arbitrary point. Then there exists $x_1, x_2 \in X$ such that $Ax_0 = Tx_1$ and $Bx_1 = Sx_2$. Thus we can construct sequences $\{y_n\}$ and $\{x_n\}$ in x such that $y_{2n+1} = Ax_{2n} = Tx_{2n+1}$, $y_{2n+2} = Bx_{2n+1} = Sx_{2n+2}$ for n = 0,1,2,… .By contractive condition, we get,

$$Q(y_{2n+1}, y_{2n+2,} y_{2n+3}, t) = Q(Ax_{2n}, Bx_{2n+1}, Bx_{2n+2}, t)$$
$$\geq \Phi(Q(Sx_{2n}, Tx_{2n+1}, Tx_{2n+2}, t))$$
$$> Q(y_{2n}, y_{2n+1}, y_{2n+2}, t)$$
$$H(y_{2n+1}, y_{2n+2,} y_{2n+3}, t) = H(Ax_{2n}, Bx_{2n+1}, Bx_{2n+2}, t)$$
$$\leq \psi(H(Sx_{2n}, Tx_{2n+1}, Tx_{2n+2}, t))$$
$$< H(y_{2n}, y_{2n+1}, y_{2n+2}, t)$$

Similarly we can have $Q(y_{2n+2}, y_{2n+3,} y_{2n+4}, t) > Q(y_{2n+1}, y_{2n+2}, y_{2n+3}, t)$ and $H(y_{2n+2}, y_{2n+3,} y_{2n+4}, t) < H(y_{2n+1}, y_{2n+2}, y_{2n+3}, t)$

In general, we can write

$Q(y_{n+2}, y_{n+1,} y_n, t) > Q(y_{n+1}, y_n, y_{n-1}, t)$ and $H(y_{n+2}, y_{n+1,} y_n, t) < H(y_{n+1}, y_n, y_{n-1}, t)$

Therefore $\{Q(y_{n+1}, y_n, y_{n-1}, t)\}$ is an increasing sequence and $\{H(y_{n+1}, y_n, y_{n-1}, t)\}$ is a decreasing sequence of positive real numbers in [0, 1] and tends to limit $l \leq 1$.

If $l < 1$ then

$Q(y_{n+2}, y_{n+1}, y_n, t) \geq \Phi(Q(y_{n+1}, y_n, y_{n-1}, t))$ and $H(y_{n+2}, y_{n+1}, y_n, t) \leq \psi(H(y_{n+1}, y_n, y_{n-1}, t))$.

On letting $n \to \infty$ we get,

$$\lim_{n\to\infty} Q(y_{n+2}, y_{n+1}, y_n, t) \geq \Phi(\lim_{n\to\infty} Q(y_{n+1}, y_n, y_{n-1}, t)) \text{ and}$$
$$\lim_{n\to\infty} H(y_{n+2}, y_{n+1}, y_n, t) \leq \psi\left(\lim_{n\to\infty} H(y_{n+1}, y_n, y_{n-1}, t)\right).$$

That is $l \geq \Phi(l) > l$ and $l \leq \psi(l) < l$ a contradiction. Thus $l = 1$.

Now for positive integer p,

$$Q(y_n, y_{n+p,} y_{n+p}, t) \geq Q(y_n, y_{n+1,} y_{n+1}, t/2) * Q(y_{n+1}, y_{n+p,} y_{n+p}, t/2)$$

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$$\geq Q\left(y_n, y_{n+1}, \; y_{n+1}, {}^{t}/_{p}\right) * Q\left(y_{n+1}, y_{n+2}, y_{n+2}, {}^{t}/_{p}\right) * \dots * Q\left(y_{n+p-1}, y_{n+p}, \; y_{n+p}, {}^{t}/_{p}\right)$$

$$H\left(y_n, y_{n+p}, y_{n+p}, t\right) \leq H\left(y_n, y_{n+1}, y_{n+1}, {}^{t}/_{2}\right) \lozenge H\left(y_{n+1}, y_{n+p}, y_{n+p}, {}^{t}/_{2}\right)$$

.

$$\leq H\left(y_n, y_{n+1}, \; y_{n+1}, {}^{t}/_{p}\right) \lozenge H\left(y_{n+1}, y_{n+2}, y_{n+2}, {}^{t}/_{p}\right) \lozenge \dots \lozenge H\left(y_{n+p-1}, y_{n+p}, \; y_{n+p}, {}^{t}/_{p}\right)$$

Taking limit $\lim_{n\to\infty} Q\left(y_n, y_{n+p}, y_{n+p}, t\right) = 1$ and $\lim_{n\to\infty} H\left(y_n, y_{n+p}, y_{n+p}, t\right) = 0$

$\lim_{n\to\infty} Q\left(y_n, y_{n+p}, y_{n+p}, t\right) \geq 1 * 1 * \dots * 1 = 1$ and $\lim_{n\to\infty} H\left(y_n, y_{n+p}, y_{n+p}, t\right) \leq 0 \lozenge 0 \dots \lozenge 0 = 0$

Thus $\{y_n\}$ is a Cauchy sequence in X. Since X is complete $y_n \to u$ in X.

That is, $\{Ax_{2n}\}$, $\{Tx_{2n+1}\}, \{Bx_{2n+1}\}, \{Sx_{2n+2}\}$ also converges to u in X.

Thus $\lim_{n\to\infty} Sx_{2n} = u$ and $\lim_{n\to\infty} Ax_{2n} = u$.

Since (A, S) is reciprocally continuous and semi compatible.

$\lim_{n\to\infty} ASx_{2n} = Au$, $\lim_{n\to\infty} SAx_{2n} = Su$ and $\lim_{n\to\infty} Q(ASx_{2n}, Su, Su, t) = 1$, $\lim_{n\to\infty} H(ASx_{2n}, Su, Su, t) = 0$.

Thus Au = Su.

Now we will show that Au = u. Suppose Au ≠ u . Then by contractive condition, we obtain

$Q(Au, Bx_{2n+1}, Bx_{2n+1}, t) \geq \left(Q(Su, Tx_{2n+1}, Tx_{2n+1}, t)\right)$and $H(Au, Bx_{2n+1}, Bx_{2n+1}, t) \leq \psi(H(Su, Tx_{2n+1}, Tx_{2n+1}, t))$

Letting $n \to \infty$,

Q(Au, u, u, t) ≥ Φ(Q(Su, u, u, t)) = Φ(Q(Au, u, u, t)) > Q (Au, u, u, t) and

H (Au, u, u, t) ≤ ψ(H (Su, u, u, t)) = ψ(H (Au, u, u, t)) < H (Au, u, u, t) a contradiction. Thus Au = u = Su.

Now AX ⊆ TX, then there exists a w ∈X such that u = Au = Tw. Then by substituting x = x_{2n} and y = z = w,we obtain:

$$Q(Ax_{2n}, Bw, Bw, t) \geq \Phi\,(Q(Sx_{2n}, Tw, Tw, t)) \quad \text{and}$$

$$H\,(Ax_{2n}, Bw, Bw, t) \leq \psi\,(H\,(Sx_{2n}, Tw, Tw, t))$$

Taking limit $n \to \infty$ we get,

$$Q(u, Bw, Bw, t) \geq \Phi(Q(u, Tw, Tw, t)) = \Phi\,(Q(u, u, u, t)) = \Phi(1) = 1$$

$$H\,(u, Bw, Bw, t) \leq \psi(H\,(u, Tw, Tw, t)) = \psi(H\,(u, u, u, t)) = \psi(0) = 0.$$

Thus u = Bw = Tw.

Also weak compatibility of (B, T) implies TBw = BTw. Thus Tu = Bu.

Now we claim that Au = Bu. If not,

$$Q(Au,Bu,Bu,t) \geq \Phi\,(Q(Su,Tu,Tu,t)) \text{and } H(Au, Bu, Bu, t) \leq \psi\,(H\,(Su, Tu, Tu, t))$$

$$Q(u,Bu, Bu,t) \geq \Phi\,(Q(u,Bu,Bu,t)) > Q(u,Bu,Bu,t) \quad \text{and}$$

$$H\,(u, Bu, Bu, t) \leq \psi\,(H(u, Bu, Bu, t)) < H\,(u,Bu, Bu, t)$$

a contradiction. Thus Au=Bu and hence Au= Bu=Tu=Su= u. To prove the uniqueness assume u,v are two distinct common fixed points of A,B,S and T.Then:

$$Q(Au,Bv,Bv,t) \geq \Phi\,(Q(Su,Tv,Tv,t))$$

$$Q(u,v,v,t) \geq \Phi\,(Q(u,v,v,t)) > Q(u,v,v,t) \quad \text{and}$$

$$H\,(Au, Bv, Bv, t) \leq \psi\,(H\,(Su, Tv, Tv, t))$$

$$H\,(u, v, v, t) \leq \psi\,(H(u, v, v, t)) < H\,(u, v, v, t)$$

a contradiction . Hence u= v.

As a consequence of the above theorem, the following corollaries are obtained.

Corollary 3.2:

Let A, B and S be self-maps on a complete generalized intuitionistic fuzzy metric space (X, Q, H,$*$, ◊) where * is a continuous t-norm and ◊ is a continuous t-conorm, satisfying:

1. AX⊆SX, BX ⊆SX
2. (B, S) is weak compatible
3. For each x, y, z∈X and t > 0, Q(Ax, By, Bz, t) ≥ Φ (Q(Sx, Sy, Sz, t)) and H(Ax, By, Bz, t) ≤ ψ (H(Sx, Sy, Sz, t)),

Where $\Phi, \psi : [0, 1] \rightarrow [0, 1]$ is a continuous function such that $\Phi(1) = 1$ and $\psi(0) = 0$ and $\Phi(a) > a$, $\psi(a) < a$ for each $0 < a < 1$.

If (A, S) is semi compatible and reciprocally continuous, then A, B and Shaveunique common fixed point.

Corollary 3.3:

Let A, S and T be self-maps on a complete generalized intuitionistic fuzzy metric space (X, Q, H, ∗, ◊) where * is a continuous t-norm and ◊ is a continuous t-conorm, satisfying:

1. $AX \subseteq SX$, $AX \subseteq TX$
2. (A, S) is weak compatible
3. For each x, y, z ∈ X and t > 0, $Q(Ax, Ay, Az, t) \geq \Phi(Q(Sx, Ty, Tz, t))$ and $H(Ax, Ay, Az, t) \leq \psi(H(Sx, Ty, Tz, t))$
   Where $\Phi, \psi: [0, 1] \rightarrow [0, 1]$ is a continuous function such that $\Phi(1) = 1$ and $\psi(0) = 0$ and $\Phi(a) > a$, $\psi(a) < a$ for each $0 < a < 1$. If (A, T) is semi compatible and reciprocally continuous, then A, S and T have a unique common fixed point.

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