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Super exponential mean graphs

A. Rajesh Kannan[1] , R.Rathajeyalakshmi[2] and A. Durai Baskar[3]

[1,2]*Department of Mathematics*
Mepco Schlenk Engineering College, Sivakasi- 626 005, Tamil Nadu, INDIA.
[3]*Department of Mathematics C.S.I. Jayaraj Annapackiam College, Nallur - 627853, Tamil Nadu, INDIA.*
Email: rajmaths@gmail.com[1] &, rratha@mepcoeng.ac.in[2]& a.duraibaskar@gmail.com[3]

Abstract Let G be a graph and $\chi: V(G) \to \{1,2,3,\dots,p+q\}$ be an injection. For each uv, the induced edge labeling χ^* is defined as $\chi^*(uv) = \left\lfloor \frac{1}{e}\left(\frac{\chi(v)^{\chi(v)}}{\chi(u)^{\chi(u)}}\right)^{\frac{1}{\chi(v)-\chi(u)}} \right\rfloor$. Then χ is called a super exponential mean labeling if $\chi(V(G)) \cup \{f^*(uv): uv \in E(G)\} = \{1,2,3,\dots,p+q\}$. A graph that admits a super exponential mean labeling is called a super exponential mean graph. In this paper, the super exponential meanness of some standard graphs have been studied.

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1. Introduction

In this paper, only finite, simple and undirected graphs are considered. For terminology, definitions we follow [6] and for survey [5].

A path on n vertices is denoted by P_n. $G \odot S_m$ is the graph obtained from G by attaching m pendant vertices to each vertex of G. Let $v_1^{(i)}, v_2^{(i)}, v_3^{(i)}, \dots, v_{m+1}^{(i)}$ and $u_1, u_2, u_3, \dots, u_n$ be the vertices of the i^{th} copy of the star graph $S_m, 1 \le i \le n$ and the path P_n respectively. Then the graph $[P_n; S_m]$ is obtained from n copies of S_m and the path P_n by joining u_i with the central vertex $v_1^{(i)}$ of the i^{th} copy of S_m by means of an edge, for $1 \le i \le n$. An arbitrary subdivision of a graph G, is a graph obtained from G by a sequence of elementary subdivisions forming edges into paths through new vertices of degree 2. For a graph G, the graph $S(G)$ is obtained by subdividing each edge of G by a vertex. A square of a graph G, denoted by G^2, has the vertex set as in G and two vertices are adjacent in G^2 if they are at a distance either 1 or 2 apart in G.

The concept of exponential mean labeling was introduced [1] and developed the exponential mean labeling of some standard graphs [2] by by Rajesh Kannan et al.. The concept of super geometric labeling was first introduced by A. Durai Baskar et al. [3]. Arockiaraj et al. introduced the super F-root square mean labeling of graphs [4]. Motivated by the works on graph labeling, we introduced a new type of labeling called super exponential mean labeling.

Let G be a graph and $\chi: V(G) \to \{1,2,3,\ldots,p+q\}$ be an injection. For each uv, the induced edge labeling χ^* is defined as $\chi^*(uv) = \left\lceil \frac{1}{e}\left(\frac{\chi(v)^{\chi(v)}}{\chi(u)^{\chi(u)}}\right)^{\frac{1}{\chi(v)-\chi(u)}} \right\rceil$. Then χ is called a super exponential mean labeling if $\chi(V(G)) \cup \{f^*(uv): uv \in E(G)\} = \{1,2,3,\ldots,p+q\}$. A graph that admits a super exponential mean labeling is called a super exponential mean graph.

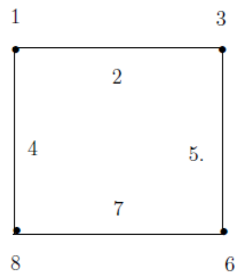

Figure 1. A super exponential mean labeling of C_4

In this paper, the super exponential meanness of some standard graphs have been studied.

2. Main Results

Theorem 2.1 Union of number of path $\mathrm{P_n}$ is a super exponential mean graph, for $\mathrm{n} \geq 2$.

Proof. Let the graph G be the union of k paths. Let $\{v_\beta^{(\alpha)}: 1 \leq \beta \leq p_\alpha\}$ be the vertices of the α^{th} path P_{p_α} with $p_\alpha \geq 2$ and $1 \leq \alpha \leq k$.
Define $\chi: V(G) \to \left\{1,2,3,\ldots,\sum_{\alpha=1}^{\gamma} 2p_\alpha - \gamma\right\}$ as follows:

$\chi\left(v_\beta^{(1)}\right) = 2\beta - 1$, for $1 \leq \beta \leq p_1$ and

$\chi\left(v_\beta^{(\alpha)}\right) = f\left(v_{p_{\alpha-1}}^{(\alpha-1)}\right) + 2\beta - 1$, for $2 \leq \alpha \leq k$ and $1 \leq \beta \leq p_\alpha$.

The induced edge labeling is as follows:

$\chi^*\left(v_\beta^{(1)} v_{\beta+1}^{(1)}\right) = 2\beta$, for $1 \leq \beta \leq p_1 - 1$ and

$\chi^*\left(v_\beta^{(\alpha)} v_{\beta+1}^{(\alpha)}\right) = f\left(v_{p_{\alpha-1}}^{(\alpha-1)}\right) + 2\beta$, for $2 \leq \alpha \leq \gamma$ and $1 \leq \beta \leq p_\alpha - 1$.

Hence, χ is a super exponential mean labeling of G. Thus the graph G is a super exponential mean graph.

Corollary 2.2 Every path $\mathrm{P_n}$ is a super exponential mean graph, for $\mathrm{n} \geq 1$.

Theorem 2.3 The graph $\mathrm{P_n} \odot \mathrm{S_m}$ is a super exponential mean graph, for $\mathrm{n} \geq 1$ and $\mathrm{m} \leq 3$.

Proof. Let $u_1, u_2, \ldots, u_n$ be the vertices of the path P_n and $v_1^{(\alpha)}, v_2^{(\alpha)}, \ldots, v_m^{(\alpha)}$ be the pendant vertices at each vertex u_α of the path P_n, for $1 \leq \alpha \leq n$.
Case i. $m = 1$.
Define $\chi: V(P_n \odot S_1) \to \{1,2,3,\ldots,4n-1\}$ as follows:

$\chi(u_\alpha) = 4\alpha - 1$, for $1 \leq \alpha \leq n$ and

$$\chi\left(v_1^{(\alpha)}\right) = \begin{cases} 1 & \alpha = 1 \\ 4\alpha - 4 & 2 \leq \alpha \leq n. \end{cases}$$

The induced edge labeling is as follows:

$\chi^*(u_\alpha u_{i+1}) = 4\alpha + 1$, for $1 \leq \alpha \leq n - 1$ and

$\chi^*\left(v_1^{(\alpha)} u_\alpha\right) = 4\alpha - 2$, for $1 \leq \alpha \leq n$.

Case ii. $m = 2$.
Define $\chi: V(P_n \odot S_2) \rightarrow \{1,2,3,\ldots,6n-1\}$ as follows:

$\chi(u_\alpha) = 6\alpha - 3$, for $1 \leq \alpha \leq n$,

$\chi(v_1^{(\alpha)}) = 6\alpha - 5$, for $1 \leq \alpha \leq n$ and

$\chi\left(v_2^{(\alpha)}\right) = 6\alpha - 1$, for $1 \leq \alpha \leq n$.

The induced edge labeling is as follows:

$\chi^*(u_\alpha u_\alpha + 1) = 6\alpha$, for $1 \leq \alpha \leq n-1$,

$\chi^*(v_1^{(\alpha)} u_\alpha) = 6\alpha - 4$, for $1 \leq \alpha \leq n$ and

$\chi^*\left(v_2^{(\alpha)} u_\alpha\right) = 6\alpha - 2$, for $1 \leq \alpha \leq n$.

Case iii. $m = 3$.
Define $\chi: V(P_n \odot S_3) \rightarrow \{1,2,3,\ldots,8n-1\}$ as follows:

$\chi(u_\alpha) = 8\alpha - 3$, for $1 \leq \alpha \leq n$,

$$\chi(v_1^{(\alpha)}) = \begin{cases} 1 & \alpha = 1 \\ 8\alpha - 8 & 2 \leq \alpha \leq n \end{cases}$$

$\chi(v_2^{(\alpha)}) = 8\alpha - 6$, for $1 \leq \alpha \leq n$ and

$\chi\left(v_3^{(\alpha)}\right) = 8\alpha - 1$, for $1 \leq \alpha \leq n$.

The induced edge labeling is as follows:

$\chi^*(u_\alpha u_{\alpha+1}) = 8\alpha + 1$, for $1 \leq \alpha \leq n-1$,

$\chi^*(v_1^{(\alpha)} u_\alpha) = 8\alpha - 5$, for $1 \leq \alpha \leq n$,

$\chi^*(v_2^{(\alpha)} u_i) = 8\alpha - 4$, for $1 \leq \alpha \leq n$ and

$\chi^*\left(v_3^{(\alpha)} u_\alpha\right) = 8\alpha - 2$, for $1 \leq \alpha \leq n$.

Hence, χ is a super exponential mean labeling of $P_n \odot S_m$. Thus the graph $P_n \odot S_m$ is a super exponential mean graph, for $n \geq 1$ and $m \leq 3$.

Theorem 2.4 $[P_n; S_m]$ is a super exponential mean graph, for $n \geq 1$ and $m \leq 2$.

Proof. Let $u_1, u_2, \ldots, u_n$ be the vertices of the path P_n and $v_1^{(\alpha)}, v_2^{(\alpha)}, \ldots, v_m^{(\alpha)}$ be the pendant vertices at each vertex u_α of the path P_n, for $1 \leq \alpha \leq n$.
Case i. $m = 1$.
Define $\chi: V([P_n; S_1]) \rightarrow \{1,2,3,\ldots,6n-1\}$ as follows:

$$\chi(u_\alpha) = \begin{cases} 5 & \alpha = 1 \\ 6\alpha - 5 & 2 \leq \alpha \leq n, \end{cases}$$

$\chi\left(v_1^{(\alpha)}\right) = 6\alpha - 3$ for $1 \leq \alpha \leq n$,

$\chi\left(v_2^{(n)}\right) = 6n - 1$

and

$$\chi\left(v_2^{(\alpha)}\right) = \begin{cases} 1 & \alpha = 1 \\ 6\alpha & 2 \leq \alpha \leq n-1. \end{cases}$$

The induced edge labeling is as follows:

$$\chi^*(u_\alpha u_{\alpha+1}) = \begin{cases} 6 & \alpha = 1 \\ 6\alpha - 2 & 2 \leq \alpha \leq n-1, \end{cases}$$

$$\chi^*\left(u_\alpha v_1^{(\alpha)}\right) = \begin{cases} 4 & \alpha = 1 \\ 6\alpha - 4 & 2 \leq \alpha \leq n, \end{cases}$$

$$\chi^*\left(v_1^{(\alpha)} v_2^{(\alpha)}\right) = \begin{cases} 2 & \alpha = 1 \\ 6\alpha - 1 & 2 \leq \alpha \leq n-1 \end{cases}$$

and $\chi^*\left(v_1^{(n)} v_2^{(n)}\right) = 6n - 2$.

Case ii. $m = 2$.
Define $\chi: V([P_n; S_2]) \rightarrow \{1,2,3,\ldots,8n-1\}$ as follows:

$$\chi(u_\alpha) = \begin{cases} 3\alpha + 2 & 1 \leq \alpha \leq 2 \\ 8\alpha - 8 & 3 \leq \alpha \leq n, \end{cases}$$

$$\chi\left(v_1^{(\alpha)}\right)=\begin{cases}3 & \alpha=1\\ 8\alpha-5 & 2\le\alpha\le n-1,\end{cases}$$
$$\chi\left(v_1^{(n)}\right)=8n-3,$$
$$\chi\left(v_2^{(\alpha)}\right)=\begin{cases}1 & \alpha=1\\ 8\alpha-1 & 2\le\alpha\le n-1,\end{cases}$$
$$\chi\left(v_2^{(n)}\right)=8n-6,$$
$$\chi\left(v_3^{(\alpha)}\right)=\begin{cases}9 & \alpha=1\\ 8\alpha+1 & 2\le\alpha\le n-1\end{cases}$$
and $\chi\left(v_3^{(n)}\right)=8n-1$. The induced edge labeling is as follows:
$$\chi^*(u_i u_{\alpha+1})=\begin{cases}8 & \alpha=1\\ 8\alpha-4 & 2\le\alpha\le n-1,\end{cases}$$
$$\chi^*\left(u_\alpha v_1^{(\alpha)}\right)=\begin{cases}4 & \alpha=1\\ 8\alpha-6 & 2\le\alpha\le n-1,\end{cases}$$
$$\chi^*\left(u_n v_1^{(n)}\right)=8n-5,\quad \chi^*\left(v_1^{(n)}v_2^{(n)}\right)=8n-4,$$
$$\chi^*\left(v_1^{(\alpha)}v_2^{(\alpha)}\right)=\begin{cases}2 & i=1\\ 8\alpha-3 & 2\le\alpha\le n-1\end{cases}$$
and $\chi^*\left(v_1^{(\alpha)}v_3^{(\alpha)}\right)=\begin{cases}6 & \alpha=1\\ 8\alpha-2 & 2\le\alpha\le n.\end{cases}$

Hence, χ is a super exponential mean labeling of $[P_n;S_m]$. Thus the graph $[P_n;S_m]$ is a super exponential mean graph, for $n\ge 1$ and $m\le 2$.

Theorem 2.5 Arbitrary subdivision of $K_{1,3}$ is a super exponential mean graph.

Proof. Let G be an arbitrary subdivision of $K_{1,3}$. Let v_0, v_1, v_2 and v_3 be the vertices of G in which v_0 is the central vertex and v_1, v_2 and v_3 are the pendant vertices of $K_{1,3}$.

Let the edges v_0v_1, v_0v_2 and v_0v_3 of $K_{1,3}$ be subdivided by p_1, p_2 and p_3 number of vertices respectively. Let
$$v_0, v_1^{(1)}, v_2^{(1)}, v_3^{(1)}, \dots, v_{p_1+1}^{(1)}(=v_1), v_0, v_1^{(2)}, v_2^{(2)}, v_3^{(2)}, \dots, v_{p_2+1}^{(2)}(=v_2)$$
and $v_0, v_1^{(3)}, v_2^{(3)}, v_3^{(3)}, \dots, v_{p_3+1}^{(3)}(=v_3)$ be the vertices of $S(K_{1,3})$ and $v_0=v_0^{(i)}$, for $1\le\alpha\le 3$.

Let $e_\beta^{(\alpha)}=v_{\beta-1}^{(\alpha)}v_\beta^{(\alpha)}, 1\le\beta\le p_\alpha+1$ and $1\le\alpha\le 3$ be the edges of $S(K_{1,3})$ and it has $p_1+p_2+p_3+4$ vertices and $p_1+p_2+p_3+3$ edges with $p_1\le p_2\le p_3$.

Case i. $p_1=p_2$.
Define $\chi: V(S(K_{1,3}))\to\{1,2,3,\dots,2(p_1+p_2+p_3)+7\}$ as follows:
$$\chi(v_0)=2(p_1+p_2)+5,$$
$$\chi(v_\beta^{(1)})=2(p_1+p_2)+5-4j, \text{ for } 1\le\beta\le p_1+1,$$
$$\chi(v_\beta^{(2)})=2(p_1+p_2)+6-4j, \text{ for } 1\le\beta\le p_2+1 \text{ and}$$
$$\chi(v_\beta^{(3)})=2(p_1+p_2)+5+2j, \text{ for } 1\le\beta\le p_3+1.$$
The induced edge labeling is as follows:
$$\chi^*\left(v_\beta^{(1)}v_{\beta+1}^{(1)}\right)=2(p_1+p_2)+3-4\beta, \text{ for } 1\le\beta\le p_1,$$
$$\chi^*\left(v_\beta^{(2)}v_{\beta+1}^{(2)}\right)=2(p_1+p_2)+4-4\beta, \text{ for } 1\le\beta\le p_2,$$
$$\chi^*\left(v_\beta^{(3)}v_{\beta+1}^{(3)}\right)=2(p_1+p_2)+6+2\beta, \text{ for } 1\le\beta\le p_3,$$
$$\chi^*\left(v_0v_1^{(1)}\right)=2(p_1+p_2)+3,$$
$$\chi^*\left(v_0v_1^{(2)}\right)=2(p_1+p_2)+4$$
and
$$\chi^*\left(v_0v_1^{(3)}\right)=2(p_1+p_2)+6.$$
Case ii. $p_1<p_2<p_3$.
Define $\chi: V(S(K_{1,3}))\to\{1,2,3,\dots,2(p_1+p_2+p_3)+7\}$ as follows:

$\chi(v_0) = 2(p_1 + p_2) + 5,$
$\chi(v_\beta^{(1)}) = 2(p_1 + p_2) + 6 - 4\beta$, for $1 \le j \le p_1 + 1$,
$\chi(v_\beta^{(2)}) = \begin{cases} 2(p_1 + p_2) + 5 - 4j & 1 \le j \le p_1 + 1 \\ 2p_2 + 3 - 2j & p_1 + 2 \le \beta \le p_2 + 1 \end{cases}$

and

$\chi(v_\beta^{(3)}) = 2(p_1 + p_2) + 5 + 2\beta$, for $1 \le \beta \le p_3 + 1$.

The induced edge labeling is as follows:

$\chi^*\left(v_\beta^{(1)} v_{\beta+1}^{(1)}\right) = 2(p_1 + p_2) + 4 - 4\beta$, for $1 \le \beta \le p_1$,
$\chi^*\left(v_\beta^{(2)} v_{\beta+1}^{(2)}\right) = \begin{cases} 2(p_1 + p_2) + 3 - 4\beta & 1 \le \beta \le p_1 \\ 2p_2 + 2 - 2\beta & p_1 + 1 \le \beta \le p_2, \end{cases}$
$\chi^*\left(v_\beta^{(3)} v_{\beta+1}^{(3)}\right) = 2(p_1 + p_2) + 6 + 2\beta$, for $1 \le \beta \le p_3$,
$\chi^*\left(v_0 v_1^{(1)}\right) = 2(p_1 + p_2) + 4,$
$\chi^*\left(v_0 v_1^{(2)}\right) = 2(p_1 + p_2) + 3$

and

$\chi^*\left(v_0 v_1^{(3)}\right) = 2(p_1 + p_2) + 6.$

Hence, χ is a super exponential mean labeling of $S(K_{1,3})$. Thus the graph the graph $S(K_{1,3})$ is a super exponential mean graph.

Theorem 2.6 P_n^2 is a super exponential mean graph, for $n \ge 3$.

Proof. Let $v_1, v_2, \dots, v_n$ be the vertices of the path P_n. Define $\chi: V(P_n^2) \to \{1,2,3,\dots,3n-3\}$ as follows:

$\chi(v_1) = 1,$
$\chi(v_\alpha) = \begin{cases} 3i - 3 & 3 \le \alpha \le n-1 \text{ and } \alpha \text{ is odd} \\ 3\alpha - 2 & 2 \le \alpha \le n-1 \text{ and } i \text{ is even and} \end{cases}$
$\chi(v_n) = 3n - 3.$

The induced edge labeling is as follows:

$\chi^*(v_\alpha v_{\alpha+1}) = 3\alpha - 1$, for $1 \le \alpha \le n-1$ and
$\chi^*(v_\alpha v_{\alpha+2}) = \begin{cases} 3\alpha & 1 \le \alpha \le n-2 \text{ and } \alpha \text{ is odd} \\ 3\alpha + 1 & 2 \le \alpha \le n-2 \text{ and } \alpha \text{ is even.} \end{cases}$

Hence, χ is a super exponential mean labeling of P_n^2. Thus the graph P_n^2 is a super exponential mean graph, for $n \ge 3$.

Theorem 2.7 $S(P_n \odot K_1)$ is a super exponential mean graph, for $n \ge 1$.

Proof. Let $V(P_n \odot K_1) = \{u_i, v_i : 1 \le i \le n\}$. Let x_α be the vertex which divides the edge $u_\alpha v_\alpha$, for $1 \le \alpha \le n$ and y_α be the vertex which divides the edge $u_\alpha v_{\alpha+1}$, for $1 \le \alpha \le n-1$. Then

$V(S(P_n \odot K_1)) = \{u_\alpha, v_\alpha, x_\alpha, y_\beta : 1 \le \alpha \le n, 1 \le \beta \le n-1\}$
$E((P_n \odot K_1)) = \{u_\alpha x_\alpha, v_\alpha x_\alpha : 1 \le \alpha \le n\} \cup \{u_\alpha y_\alpha, y_\alpha u_{\alpha+1} : 1 \le \beta \le n-1\}$

Define $\chi: V(S(P_n \odot K_1)) \cup E(S(P_n \odot K_1)) \to \{1,2,3,\dots,8n-3\}$ as follows:

$\chi(u_\alpha) = \begin{cases} 5 & \alpha = 1 \\ 8\alpha - 7 & 2 \le \alpha \le n, \end{cases}$
$\chi(y_\alpha) = 8i - 1$ for $1 \le \alpha \le n-1$,
$\chi(x_\alpha) = 8i - 5$ for $1 \le \alpha \le n$,
$\chi(v_\alpha) = \begin{cases} 1 & i = 1 \\ 8\alpha - 2 & 2 \le \alpha \le n-1 \end{cases}$

and

$\chi(v_n) = 8n - 3.$

Then the induced edge labeling is as follows:

$$\chi^*(u_\alpha y_\alpha) = \begin{cases} 6 & i = 1 \\ 8i - 4 & 2 \leq \ i \leq \ n-1, \end{cases}$$

$$\chi^*(y_\alpha u_{\alpha+1}) = 8\alpha \text{ for } 1 \leq \alpha \leq n-1,$$

$$\chi^*(u_\alpha x_\alpha) = \begin{cases} 4 & \alpha = 1 \\ 8\alpha - 6 & 2 \leq \ \alpha \leq \ n, \end{cases}$$

$$\chi^*(x_\alpha v_\alpha) = \begin{cases} 2 & \alpha = 1 \\ 8\alpha - 3 & 2 \leq \ \alpha \leq \ n-1 \end{cases}$$

and $\chi^*(x_n v_n) = 8n - 4$.

Hence, χ is a super exponential mean labeling of $S(P_n \odot K_1)$. Thus the graph $S(P_n \odot K_1)$ is a super exponential mean graph, for $n \geq 1$.

3. Conclusion

In this paper, the super exponential meanness of some standard graphs have been studied. It is possible to investigate the super exponential meanness for other graphs.

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