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FURTHER RESULTS ON FCM LABELING OF SOME GRAPHS AND ITS LINE GRAPH

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Abstract A function f is called an F-centroidal mean labeling of a graph $G(V, E)$ with p vertices and q edges if $f: V(G) \rightarrow \{1,2,3, \ldots, q+1\}$ is injective and the induced function $f^*: E(G) \rightarrow \{1,2,3, \ldots, q\}$ defined as

$$f^*(uv) = \left\lfloor \frac{2[f(u)^2 + f(u)f(v) + f(v)^2]}{3[f(u) + f(v)]} \right\rfloor,$$

for all $uv \in E(G)$, is bijective. A graph that admits an F-centroidal mean labeling (FCM labeling) is called an F-centroidal mean graph (FCM graph). The line graph is one among the graph operations. In this paper, we try to analyse that the line graph operation preserves the F-centroidal mean property for the graph $P_n \circ S_2$, the graph $[P_n; S_1]$, the graph $S(P_n \circ K_1)$, the ladder graph L_n and the slanting ladder graph SL_n.

Keywords Labeling, F-centroidal mean labeling, F-centroidal mean graph.

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1. Introduction

Throughout this paper, by a graph we mean a finite, undirected and simple graph. Let $\boldsymbol{G}(\boldsymbol{V}, \boldsymbol{E})$ be a graph with $\boldsymbol{p}$ vertices and $\boldsymbol{q}$ edges. For notations and terminology, we follow [8]. For a detailed survey on graph labeling, we refer [7].

The line graph $\boldsymbol{L}(\boldsymbol{G})$ of a graph $\boldsymbol{G}$ is defined to have as its vertices the edges of $\boldsymbol{G}$, with two being adjacent if the corresponding edges share a vertex in $\boldsymbol{G}$. Path on $\boldsymbol{n}$ vertices is denoted by $\boldsymbol{P_n}$. The graph $\boldsymbol{G} \circ \boldsymbol{S_m}$ is obtained from $\boldsymbol{G}$ by attaching $\boldsymbol{m}$ pendant vertices to each vertex of $\boldsymbol{G}$. If $\boldsymbol{v_1^{(i)}}, \boldsymbol{v_2^{(i)}}, \boldsymbol{v_3^{(i)}}, \ldots, \boldsymbol{v_{m+1}^{(i)}}$ and $\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}, \ldots, \boldsymbol{u_n}$ be the vertices of $\boldsymbol{i^{th}}$ copy of the star graph $\boldsymbol{S_m}$ and the path P_n respectively, then the graph $[P_n; S_m]$ is obtained from n copies of S_m and the path P_n by joining u_i with the central vertex $v_1^{(i)}$ of the i^{th} copy of S_m by means of an edge, for $1 \leq i \leq n$. A subdivision of a graph G, denoted by $S(G)$, is a graph obtained by subdividing edge of G by a vertex. Let G_1 and G_2 be any two graphs with p_1 and p_2 vertices respectively. Then the cartesian product $G_1 \times G_2$ has $p_1 p_2$ vertices which are $\{(u, v): u \in G_1, v \in G_2\}$ and the edges are obtained as follows: (u_1, v_1) and (u_2, v_2) are adjacent in $G_1 \times G_2$if either $u_1 = u_2$ and v_1 and v_2 are adjacent in G_2 or u_1 and u_2 are adjacent in G_1 and $v_1 = v_2$. A ladder graph L_n is the graph $P_2 \times P_n$. The slanting ladder SL_n is a graph obtained from two paths $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ by joining each v_i, with u_{i+1}, $1 \leq i \leq n - 1$.

The concept of geometric mean labeling was introduced by Durai Baskar and Arockiaraj [6]. In [5], Arockiaraj et al., introduced the concept of F-root square mean labeling of a graph. In [4], Arockiaraj et al., analyzed the line graph operation preserves the F-root square mean property for so many standard graphs. Arockiaraj et al., defined the F-centroidal mean labeling [1]. Motivated by the works of so many authors in the area of graph labeling, we try to analyse that the line graph operation preserves the F-centroidal mean property for some standard graphs.

A function f is called an F-centroidal mean labeling of a graph $G(V, E)$ with p vertices and q edges if $f: V(G) \rightarrow \{1, 2, 3, \ldots, q + 1\}$ is injective and the induced function $f^*: E(G) \rightarrow \{1, 2, 3, \ldots, q\}$ defined as

$$f^*(uv) = \left\lfloor \frac{2[f(u)^2 + f(u)f(v) + f(v)^2]}{3[f(u) + f(v)]} \right\rfloor,$$

for all $uv \in E(G)$, is bijective. A graph that admits an F-centroidal mean labeling (FCM labeling) is called an F-centroidal mean graph (FCM graph).

An FCM labeling of the graph is given in Figure 1.

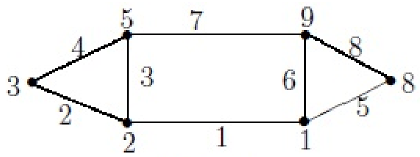

Figure 1

In this paper, we try to analyse that the line graph operation preserves the F-centroidal mean property for the graph $P_n \circ S_2$, the graph $[P_n; S_1]$, the graph $S(P_n \circ K_1)$, the ladder graph L_n and the slanting ladder graph SL_n.

Theorem 1.1. [2] The ladder graph L_n is an FCM graph, for $n \geq 1$.

Theorem 1.2. [2] The slanting ladder graph SL_n is an FCM graph, for $n \geq 2$.

Theorem 1.3. [3] Every path P_n is an FCM graph.

Theorem 1.4. [3] The graph $P_n \circ S_1$ is an FCM graph, for $n \geq 1$.

Theorem 1.5. [3] The graph $L(P_n \circ S_1)$ is an FCM graph, for $n \geq 2$.

2. Main Results

Theorem 2.1 The graph $P_n \circ S_2$ is an FCM graph, for $n \geq 1$.

Proof. Let $\boldsymbol{v_1, v_2, v_3, \dots, v_n}$ be the vertices of the path $\boldsymbol{P_n}$ and $\boldsymbol{u_1^{(i)}}$ be the pendant vertices at each $\boldsymbol{v_i}$, for $\boldsymbol{1 \le i \le n}$.

Define $\boldsymbol{f}: \boldsymbol{V(P_n \circ S_2)} \rightarrow \{\boldsymbol{1, 2, 3, \dots, 3n}\}$ as follows.

$$f(v_i) = 3i - 1, for\ 1 \le i \le n,$$

$$f(u_1^{(i)}) = 3i - 2, for\ 1 \le i \le n\ and$$

$$f(u_2^{(i)}) = 3i, for\ 1 \le i \le n.$$

Then the induced edge labeling $\boldsymbol{f^*}$ is obtained as follows.

$$f^*(v_i v_{i+1}) = 3i, for\ 1 \le i \le n - 1,$$

$$f^*(v_i u_1^{(i)}) = 3i - 2, for\ 1 \le i \le n\ and$$

$$f^*(v_i u_2^{(i)}) = 3i - 1, for\ 1 \le i \le n.$$

Hence, $\boldsymbol{f}$ is an FCM labeling of the graph $\boldsymbol{P_n \circ S_2}$. Thus the graph $\boldsymbol{P_n \circ S_2}$ is an FCM graph, for $\boldsymbol{n \ge 1}$.

Theorem 2.2 The graph $\boldsymbol{L(P_n \circ S_2)}$ is an FCM graph, for $\boldsymbol{n \ge 2}$.

Proof. Let $\boldsymbol{u_1, u_2, u_3, \dots, u_n}$ be the vertices of $\boldsymbol{P_n}$ and $\boldsymbol{v_i, w_i}$ be the pendant vertices attached at $\boldsymbol{u_i, 1 \le i \le n}$ in $\boldsymbol{P_n \circ S_2}$. The edge set of $\boldsymbol{P_n \circ S_2}$ is $\{\boldsymbol{x_i = u_i u_{i+1}: 1 \le i \le n - 1}\} \cup \{\boldsymbol{y_i = u_i v_i: 1 \le i \le n}\} \cup \{\boldsymbol{z_i = u_i w_i: 1 \le i \le n}\}$.

Let $\boldsymbol{V(L(P_n \circ S_2))} = \{\boldsymbol{x_i: 1 \le i \le n - 1}\} \cup \{\boldsymbol{y_i, z_i: 1 \le i \le n}\}$ and

$$E(L(P_n \circ S_2)) = \{x_i z_i, x_i y_{i+1}, x_i z_{i+1}, x_i y_i: 1 \le i \le n - 1\} \cup \{x_i x_{i+1}: 1 \le i \le n - 2\} \cup \{y_i z_i: 1 \le i \le n\}$$

be the vertex set and edge set of the graph $\boldsymbol{L(P_n \circ S_2)}$. Define $\boldsymbol{f}: \boldsymbol{V(L(P_n \circ S_2))} \rightarrow \{\boldsymbol{1, 2, 3, \dots, 6n - 5}\}$ as follows.

$$f(x_i) = \begin{cases} 8i - 6, & 1 \le i \le 2 \\ 6i - 1, & 3 \le i \le n - 1, \end{cases}$$

$$f(y_i) = \begin{cases} 5i - 4, & 1 \le i \le 3 \\ 6i - 8, & 4 \le i \le n\ \ and \end{cases}$$

$$f(z_i) = \begin{cases} 4, & i = 1 \\ 5i - 2, & 2 \le i \le 3 \\ 6i - 5, & 4 \le i \le n. \end{cases}$$

Then the induced edge labeling $\boldsymbol{f^*}$ is obtained as follows.

$$f^*(x_i x_{i+1}) = \begin{cases} 7i - 1, & 1 \le i \le 2 \\ 6i + 2, & 3 \le i \le n - 2, \end{cases}$$

$$f^*(x_i y_i) = \begin{cases} 7i - 6, & 1 \le i \le 2 \\ 6i - 5, & 3 \le i \le n - 1, \end{cases}$$

$$f^*(x_i z_i) = 6i - 3, for\ 1 \le i \le n - 1,$$

$$f^*(x_i z_{i+1}) = \begin{cases} 6i-1, & 1 \le i \le 2 \\ 6i, & 3 \le i \le n-1, \end{cases}$$

$$f^*(x_i y_{i+1}) = 6i-2, for\ 1 \le i \le n-1 and$$

$$f^*(y_i z_i) = \begin{cases} 5i-3, & 1 \le i \le 3 \\ 6i-7, & 4 \le i \le n. \end{cases}$$

Hence f is an FCM labeling of the graph $L(P_n \circ S_2)$. Thus the graph $L(P_n \circ S_2)$ is an FCM graph, for $n \ge 2$.

Theorem 2.3 The graph $[P_n; S_1]$ is an FCM graph, for $n \ge 1$.

Proof. Let $u_1, u_2, u_3, \dots, u_n$ be the vertices of the path P_n and $v_1^{(i)}, v_2^{(i)}, v_3^{(i)}, \dots, v_{m+1}^{(i)}$ be the vertices of the star graph S_m such that $v_1^{(i)}$ is the central vertex of the star graph S_m, $1 \le i \le n$.

Define $f: V[P_n; S_1] \to \{1, 2, 3, \dots, 3n\}$ as follows.

$$f(u_i) = \begin{cases} 3i, & 1 \le i \le n \text{ and } i \text{ is odd} \\ 3i-2, & 1 \le i \le n \text{ and } i \text{ is even}, \end{cases}$$

$$f\left(v_1^{(i)}\right) = 3i-1, for\ 1 \le i \le n\ \text{ and}$$

$$f\left(v_2^{(i)}\right) = \begin{cases} 3i-2, & 1 \le i \le n \text{ and } i \text{ is odd} \\ 3i, & 1 \le i \le n \text{ and } i \text{ is even}. \end{cases}$$

Then the induced edge labeling f^* is obtained as follows.

$$f^*(u_i u_{i+1}) = 3i, for\ 1 \le i \le n-1,$$

$$f^*\left(u_i v_1^{(i)}\right) = \begin{cases} 3i-1, & 1 \le i \le n \text{ and } i \text{ is odd} \\ 3i-2, & 1 \le i \le n \text{ and } i \text{ is even} \end{cases} \text{ and}$$

$$f^*\left(v_1^{(i)} v_2^{(i)}\right) = \begin{cases} 3i-2, & 1 \le i \le n \text{ and } i \text{ is odd} \\ 3i-1, & 1 \le i \le n \text{ and } i \text{ is even}. \end{cases}$$

Hence f is an FCM labeling of the graph $[P_n; S_1]$. Thus the graph $[P_n; S_1]$ is an FCM graph, for $n \ge 1$.

Theorem 2.4 The graph $L([P_n; S_1])$ is an FCM graph, for $n \ge 1$.

Proof. Let $V(L([P_n; S_1])) = \{u_i, v_j, w_j : 1 \le i \le n-1, 1 \le j \le n\}$ and

$$E(L([P_n; S_1])) = \{u_i u_{i+1} : 1 \le i \le n-2\} \cup \{u_i v_{i+1} : 1 \le i \le n-1\} \cup \{u_j v_j : 1 \le j \le n-1\} \cup \{v_j w_j : 1 \le j \le n\}$$

be the vertex set and edge set of the graph $L([P_n; S_1])$.

Assume that $n \ge 3$.

Define $f: V(L([P_n; S_1])) \to \{1, 2, 3, \dots, 4n-3\}$ as follows.

$$f(u_i) = \begin{cases} 2, & i = 1 \\ 4i, & 2 \le i \le n-1, \end{cases}$$

$$f(v_i) = \begin{cases} 3i-2, & 1 \leq i \leq 2 \\ 4i-5, & 3 \leq i \leq n \ \ and \end{cases}$$

$$f(w_i) = \begin{cases} 3, & i = 1 \\ 4i-3, & 2 \leq i \leq n. \end{cases}$$

Then the induced edge labeling f^* is obtained as follows.

$$f^*(u_i u_{i+1}) = \begin{cases} 5, & i = 1 \\ 4i+2, & 2 \leq i \leq n-2, \end{cases}$$

$$f^*(u_i v_{i+1}) = 4i-1, for \ 1 \leq i \leq n-1,$$

$$f^*(u_i v_i) = \begin{cases} 5i-4, & 1 \leq i \leq 2 \\ 4i-3, & 3 \leq i \leq n-1 and \end{cases}$$

$$f^*(v_i w_i) = \begin{cases} 2, & i = 1 \\ 4i-4, & 2 \leq i \leq n. \end{cases}$$

Hence f is an FCM labeling of the graph $L([P_n; S_1])$, for $n \geq 3$. For $1 \leq n \leq 2$, the graph $L([P_n; S_1])$ is a path and by Theorem 2.1, the result follows. Thus the graph $L([P_n; S_1])$ is an FCM graph, for any $n \geq 1$.

Theorem 2.5 The graph $S(P_n \circ K_1)$ is an FCM graph, for $n \geq 1$.

Proof. In $P_n \circ K_1$, let $u_i, 1 \leq i \leq n$, be the vertices on the path P_n and v_i be the vertex attached at each vertex $u_i, 1 \leq i \leq n$.

Let x_i be the vertex which divides the edge $u_i v_i$, for $1 \leq i \leq n$ and y_i be the vertex which divides the edge $u_i u_{i+1}$, for $1 \leq i \leq n-1$. Then $V(S(P_n \circ K_1)) = \{u_i, v_i, x_i, y_j : 1 \leq i \leq n, 1 \leq j \leq n-1\}$ and $E(S(P_n \circ K_1)) = \{u_i x_i, v_i x_i : 1 \leq i \leq n\} \cup \{u_i y_i, y_i u_{i+1} : 1 \leq i \leq n-1\}$.

Define $f: V(S(P_n \circ K_1)) \to \{1, 2, 3, \dots, 4n-1\}$ as follows.

$$f(u_i) = 4i-3, for \ 1 \leq i \leq n,$$

$$f(y_i) = 4i-1, for \ 1 \leq i \leq n-1,$$

$$f(x_i) = 4i-2, for \ 1 \leq i \leq n,$$

$$f(v_i) = 4i, for \ 1 \leq i \leq n-1 and$$

$$f(v_n) = 4n-1.$$

Then the induced edge labeling f^* is obtained as follows.

$$f^*(u_i y_i) = 4i-2, for \ 1 \leq i \leq n-1,$$

$$f^*(y_i u_{i+1}) = 4i, for \ 1 \leq i \leq n-1,$$

$$f^*(u_i x_i) = 4i-3, for \ 1 \leq i \leq n,$$

$$f^*(x_i v_i) = 4i-1, for \ 1 \leq i \leq n-1 and$$

$$f^*(x_n v_n) = 4n-2.$$

Hence f is an FCM labeling of the graph $S(P_n \circ K_1)$. Thus the graph $S(P_n \circ K_1)$ is an FCM graph, for $n \geq 1$.

Theorem 2.6 The graph $L(S(P_n \circ K_1))$ is an FCM graph, for $n \geq 1$.

Proof. The vertex set and edge set of the line graph of $S(P_n \circ K_1)$ are as given below.

$V(L(S(P_n \circ K_1))) = \{u_i, u_{j'}, v_i, w_i : 1 \leq i \leq n, 1 \leq j \leq n-2\}$ and

$$E(L(S(P_n \circ K_1))) = \{u_i v_i, v_i w_i : 1 \leq i \leq n\} \cup \{u_{i'} v_{i+1} : 1 \leq i \leq n-2\} \cup \{u_i u_{i-1}' : 2 \leq i \leq n-1\} \cup \{u_{i'} u_{i+2} : 1 \leq i \leq n-2\} \cup u_1 u_2.$$

Assume that $n \geq 3$.

Define $f : V(L(S(P_n \circ K_1))) \to \{1, 2, 3, \ldots, 5n-4\}$ as follows.

$$f(u_i) = \begin{cases} 3, & i = 1 \\ 5i-4, & 2 \leq i \leq n-1 \text{ and } i \text{ is odd} \\ 5i-6, & 2 \leq i \leq n-1 \text{ and } i \text{ is even}, \end{cases}$$

$$f(u_n) = \begin{cases} 5n-4, & n \text{ is odd} \\ 5n-6, & n \text{ is even}, \end{cases}$$

$$f(u_{i'}) = \begin{cases} 5i, & 1 \leq i \leq n-2 \text{ and } i \text{ is odd} \\ 5i+3, & 1 \leq i \leq n-2 \text{ and } i \text{ is even}, \end{cases}$$

$$f(v_i) = \begin{cases} 2, & i = 1 \\ 5i-5, & 2 \leq i \leq n-1 \text{ and } i \text{ is odd} \\ 5i-3, & 2 \leq i \leq n-1 \text{ and } i \text{ is even}, \end{cases}$$

$$f(v_n) = 5n-5,$$

$$f(w_i) = \begin{cases} 1, & i = 1 \\ 5i-6, & 2 \leq i \leq n-1 \text{ and } i \text{ is odd} \\ 5i-2, & 2 \leq i \leq n-1 \text{ and } i \text{ is even and} \end{cases}$$

$$f(w_n) = \begin{cases} 5n-6, & n \text{ is odd} \\ 5n-4, & n \text{ is even}. \end{cases}$$

Then the induced edge labeling f^* is obtained as follows.

$$f^*(u_i v_i) = \begin{cases} 2, & i = 1 \\ 5i-5, & 2 \leq i \leq n-1, \end{cases}$$

$$f^*(u_n v_n) = \begin{cases} 5n-5, & n \text{ is odd} \\ 5n-6, & n \text{ is even}, \end{cases}$$

$$f^*(v_i w_i) = \begin{cases} 1, & i = 1 \\ 5i-6, & 2 \leq i \leq n-1 \text{ and } i \text{ is odd} \\ 5i-3, & 2 \leq i \leq n-1 \text{ and } i \text{ is even}, \end{cases}$$

$$f^*(v_n w_n) = \begin{cases} 5n-6, & n \text{ is odd} \\ 5n-5, & n \text{ is even}, \end{cases}$$

$$f^*(u_i, v_{i+1}) = 5i + 1, \text{for } 1 \le i \le n-2,$$

$$f^*(u_i u_{i-1}') = \begin{cases} 5i-3, & 2 \le i \le n-1 \text{ and } i \text{ is odd} \\ 5i-6, & 2 \le i \le n-1 \text{ and } i \text{ is even}, \end{cases}$$

$$f^*(u_i, u_{i+2}) = 5i + 3, \text{for } 1 \le i \le n-2 \text{and}$$

$$f^*(u_1 u_2) = 3.$$

Hence f is an FCM labeling of the graph $L(S(P_n \circ K_1))$.

For $1 \le n \le 2$, the graph $L(S(P_n \circ K_1))$ is a path and by Theorem 1.3, the result follows. Thus the graph $L(S(P_n \circ K_1))$ is an FCM graph.

Theorem 2.7 The graph $L(L_n)$ is an FCM graph, for $n \ge 2$.

Proof. Let $V(L(L_n)) = \{u_i, v_j, w_i: 1 \le i \le n-1, 1 \le j \le n\}$ and

$$E(L(L_n)) = \{u_i u_{i+1}, w_i w_{i+1}: 1 \le i \le n-2\} \cup \{u_i v_i, v_i w_i: 1 \le i \le n-1\} \cup \{u_i v_{i+1}, w_i v_{i+1}: 1 \le i \le n-1\}$$

be the vertex set and edge set of the graph $L(L_n)$.

Define $f: V(L(L_n)) \to \{1, 2, 3, \dots, 6n-7\}$ as follows.

$$f(u_i) = \begin{cases} 8i-5, & 1 \le i \le 2 \\ 6i-1, & 3 \le i \le n-1, \end{cases}$$

$$f(v_i) = \begin{cases} 3i-1, & 1 \le i \le 2 \\ 6i-8, & 3 \le i \le n \text{ and} \end{cases}$$

$$f(w_i) = \begin{cases} 1, & i = 1 \\ 6i-4, & 2 \le i \le n-1. \end{cases}$$

Then the induced edge labeling f^* is obtained as follows.

$$f^*(u_i u_{i+1}) = \begin{cases} 7, & i = 1 \\ 6i+2, & 2 \le i \le n-2, \end{cases}$$

$$f^*(w_i w_{i+1}) = 6i - 1, \text{for } 1 \le i \le n-2,$$

$$f^*(u_i v_i) = \begin{cases} 6i-4, & 1 \le i \le 2 \\ 6i-5, & 3 \le i \le n-1, \end{cases}$$

$$f^*(v_i w_i) = \begin{cases} 1, & i = 1 \\ 6i-6, & 2 \le i \le n-1, \end{cases}$$

$$f^*(u_i v_{i+1}) = 6i - 2, \text{for } 1 \le i \le n-1 \text{and}$$

$$f^*(w_i v_{i+1}) = 6i - 3, \text{for } 1 \le i \le n-1.$$

Hence f is an FCM labeling of the graph $L(L_n)$. Thus the graph $L(L_n)$ is an FCM graph.

Theorem 2.8 The graph $L(SL_n)$ is an FCM graph, for $n \ge 2$.

roof. Let $\boldsymbol{V(L(SL_n)) = \{u_i, v_i, w_i : 1 \leq i \leq n\}}$ and

$$\boldsymbol{E(L(SL_n)) = \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{w_i w_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_{i-1} : 2 \leq i \leq n\} \cup \{u_i v_i : 1 \leq i \leq n\} \cup \{v_i u_i : 1 \leq i \leq n\} \cup \{v_i w_{i-1} : 2 \leq i \leq n\}} \text{ be}$$

the vertex set and edge set of the graph $\boldsymbol{L(SL_n)}$.

Assume that $\boldsymbol{n \geq 3}$.

Define $\boldsymbol{f : V(L(SL_n)) \rightarrow \{1, 2, 3, \ldots, 6n-3\}}$ as follows.

$$\boldsymbol{f(u_i) = \begin{cases} 3, & i = 1 \\ 5i - 5, & 2 \leq i \leq 3 \\ 20, & i = 4 \\ 6i - 6, & 5 \leq i \leq n, \end{cases}}$$

$$\boldsymbol{f(v_i) = \begin{cases} 2, & i = 1 \\ 5i - 2, & 2 \leq i \leq 3 \\ 19, & i = 4 \\ 6i - 4, & 5 \leq i \leq n \ \ and \end{cases}}$$

$$\boldsymbol{f(w_i) = \begin{cases} 11i - 10, & 1 \leq i \leq 2 \\ 6i - 3, & 3 \leq i \leq n. \end{cases}}$$

Then the induced edge labeling $\boldsymbol{f^*}$ is obtained as follows.

$$\boldsymbol{f^*(u_i u_{i+1}) = \begin{cases} 3i + 1, & 1 \leq i \leq 2 \\ 7i - 6, & 3 \leq i \leq 4 \\ 6i - 3, & 5 \leq i \leq n - 1, \end{cases}}$$

$$\boldsymbol{f^*(w_i w_{i+1}) = \begin{cases} 5i + 3, & 1 \leq i \leq 2 \\ 6i, & 3 \leq i \leq n - 1, \end{cases}}$$

$$\boldsymbol{f^*(u_i v_{i-1}) = \begin{cases} 6i - 9, & 2 \leq i \leq 3 \\ 5i - 4, & 4 \leq i \leq 5 \\ 6i - 8, & 6 \leq i \leq n, \end{cases}}$$

$$\boldsymbol{f^*(u_i v_i) = \begin{cases} 2, & i = 1 \\ 5i - 4, & 2 \leq i \leq 3 \\ 6i - 5, & 4 \leq i \leq n, \end{cases}}$$

$$\boldsymbol{f^*(v_i w_i) = \begin{cases} 9i - 8, & 1 \leq i \leq 2 \\ 6i - 4, & 3 \leq i \leq n \ \ and \end{cases}}$$

$$\boldsymbol{f^*(v_i w_{i-1}) = \begin{cases} 7i - 9, & 2 \leq i \leq 3 \\ 6i - 7, & 4 \leq i \leq n. \end{cases}}$$

Hence $\boldsymbol{f}$ is an FCM labeling of the graph $\boldsymbol{L(SL_n)}$. Thus the graph $\boldsymbol{L(SL_n)}$ is an FCM graph.

For $\boldsymbol{n = 2}$, an FCM labeling of $\boldsymbol{L(SL_n)}$, is shown in the Figure 1.

3. Conclusion

In this paper we try to analyse that the line graph operation preserves the $\boldsymbol{F}$-centroidal mean property for the graph $\boldsymbol{P_n \circ S_2}$, the graph $\boldsymbol{[P_n; S_1]}$, the graph $\boldsymbol{S(P_n \circ K_1)}$, the ladder graph $\boldsymbol{L_n}$ and the slanting ladder graph $\boldsymbol{SL_n}$. Further investigation can be done to analyse line graph operation preserves the $\boldsymbol{F}$-centroidal mean property for some other class of graphs

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