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EFFICIENT DOMINATION IN OPERATIONS ON INTUITIONISTICFUZZY GRAPHS

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Abstract In this paper, we study efficient domination set using the concept degree of the vertex in fuzzy graph .We define the efficient domination number γ (G) of intuitionistic fuzzy graphs. Also we disuse these efficient domination in various operations on intuitionistic fuzzy graphs like join, direct product, semi product, Cartesian product and composition, further we explain the results with suitable examples.

Keywords : Intuitionistic Fuzzy graph, efficient dominating set, efficient domination number.

2010 Mathematics Subject Classification Number: 05C78

1. Introduction

The first definition of fuzzy graphs was proposed by Kafmann from the fuzzy relations introduced by Zadeh. Although Rosenfeld introduced another elaborated definition, including fuzzy vertex and fuzzy edges, and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness and etc.. The concept of domination in fuzzy graphs was investigated by A. Somasundaram,S. Somasundaram and A. Somasundaram present the concepts of independent domination, total domination, connected domination of fuzzy graphs. C. Natarajan and S.K. Ayyaswamy introduce the strong (weak) domination in fuzzy graph. The first definition of intuitionistic fuzzy graphs was proposed by Atanassov. The concept of domination in intuitionistic fuzzy graphs was investigated by R.parvathi and G.Thamizhendhi .

In this paper, we study efficient domination set using the concept degree of the vertex in intuitionistic fuzzy graph .We define the efficient domination number γ (G) of intuitionistic fuzzy graphs. Also we disuse these efficient domination in various operations on intuitionistic fuzzy graphs like join, direct product, semi product, Cartesian product and composition, further we explain the results with suitable examples**.**

2. Preliminaries

An intuitionistic fuzzy graph (IFG) is of the form G=(V,E) , where $V=\{v_1,v_2,...,v_n\}$ such that $\mu_1:V\to[0,1]$, $\gamma_1:V\to[0,1]$ denote the degree of membership and non-member ship of the element $v_i\in V$ respectively and $0\le\mu_1(v_i)+\gamma_1(v_i)\le1$ for every $v_i\in V,(i=1,2,...n)$. $E\subseteq V\times V$ where $\mu_2:V\times V\to[0,1]$, *and* $\gamma_2:V\times V\to[0,1]$ are such that

$$\mu_2(v_i,v_j)\le\mu_1(v_i)\wedge\mu_1(v_j),\gamma_2(v_i,v_j)\le\gamma_1(v_i)\vee\gamma_1(v_j)$$

and

$$0\le\mu_2(v_i,v_j)+\gamma_2(v_i,v_j)\le1.$$

An arc (v_i,v_j) of an IFG G is called an strong arc if

$$\mu_2(v_i,v_j)=\mu_1(v_i)\wedge\mu_1(v_j),\quad \gamma_2(v_i,v_j)=\gamma_1(v_i)\vee\gamma_1(v_j).$$

Let *G = (V,E)* be a IFG. Then the cardinality of G is defined to be

$$|G|=\left|\sum_{v_i\in V}\left[\frac{(1+\mu_1(v_i)-\gamma_1(v_i))}{2}\right]\right|+\left|\sum_{v_i\in V}\left[\frac{(1+\mu_2(v_i,v_j)-\gamma_1(v_i,v_j))}{2}\right]\right|$$

Let *G = (V,E)* be a IFG. The vertex cardinality of G is defined to be

$$|G|=\left|\sum_{v_i\in V}\left[\frac{(1+\mu_1(v_i)-\gamma_1(v_i))}{2}\right]\right|$$

for all $v_i\in V,(i=1,2,...n)$.

Let G = (V, E) be an IFG. An edge cardinality of G is defined to be

$$|G|=\left|\sum_{v_i\in V}\left[\frac{(1+\mu_2(v_i,v_j)-\gamma_1(v_i,v_j))}{2}\right]\right|$$

for all $(v_i,v_j)\in V\times V$

An IFG, G = (V, E) is said to be strong IFG if $\mu_{2j}=\min(\mu_{1i},\mu_{2j})$ and $\nu_{2j}=\max(\nu_{1i},\nu_{2j})$ for all $(v_i,v_j)\in E$.

An IFG, G = (V, E) is said to be a complete-μ strong IFG if $\mu_{2j}=\min(\mu_{1i},\mu_{2j})$ and $\nu_{2j}<\max(\nu_{1i},\nu_{2j})$ for all *i* and *j*.

An IFG, G = (V, E) is said to be a complete-ν strong IFG if $\mu_{2j}<\min(\mu_{1i},\mu_{2j})$ and $\nu_{2j}=\max(\nu_{1i},\nu_{2j})$ for all *i* and *j*.

An IFG, G = (V, E) is said to be a complete IFG if $\mu_{2j}=\min(\mu_{1i},\mu_{2j})$ and $\nu_{2j}=\max(\nu_{1i},\nu_{2j})$ for every $(v_i,v_j)\in V$.

A set D of V is said to be intuitionistic fuzzy dominating set of G if every $v\in V-D$ there exits $u\in D$ such that u dominates v.A fuzzy dominating set D of a fuzzy graph G is called minimal intuitionistic fuzzy dominating set of G, if every node $v\in D$, $D-\{v\}$ is not aintuitionistic fuzzy dominating set.The fuzzy dominating number $\gamma_f(G)$ of the fuzzy graph G is the minimum cardinality taken over all minimal intuitionistic fuzzy dominating set of G.

Let $G(\sigma,\mu)$ be a fuzzy graph. A set D is subset of V is said to be efficient dominating set of a intuitionistic fuzzy graph G if every $v\in V-D$ there is exactly one $u\in D$ dominates v i.e $N(u)\cap D=\{v\}$.

A efficient dominating set D of a intuitionistic fuzzy graph G is called minimal efficient dominating set of G, if every subset of d is not a efficientintuitionistic fuzzy dominating set. i.e every node $u \in D, D-\{u\}$, is not an efficient intuitionistic fuzzy dominating set.

The efficient intuitionistic fuzzy dominating number $\gamma_e(G)$ of the intuitionistic fuzzy graph G is the minimum cardinality taken over all minimal efficient intuitionistic fuzzy dominating set of G.

3. Main results

Definition 3.1 The join of two intuitionistic fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are denoted by $G_1 + \mathrm{G}_2$ whose vertex and edge set is $(V_1 + \mathrm{V}_2)$ and $(E_1 + \mathrm{E}_2)$ respectively and it is defined by

$$(\mu_{11} + \mu_{21})(\mathrm{u}) = \begin{cases} \mu_{11}(u) \ if \ u \in V_1 \\ \mu_{21}(u) \ if \ u \in V_2 \end{cases} \qquad (\nu_{11} + \nu_{21})(\mathrm{u}) = \begin{cases} \nu_{11}(u) \ if \ u \in V_1 \\ \nu_{21}(u) \ if \ u \in V_2 \end{cases}$$

and

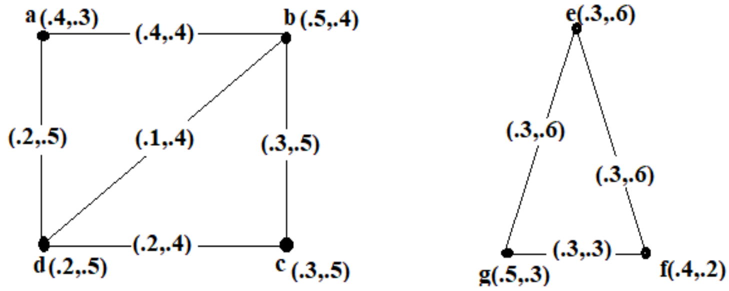
$$(\mu_{12} + \mu_{22})(\mathrm{uv}) = \begin{cases} \mu_{12}(uv) \ if \ uv \in E_1 \\ \mu_{22}(uv) \ if \ uv \in E_2 \\ \mu_{11}(u) \wedge \mu_{21}(\mathrm{v}) \text{ otherwise} \end{cases} \qquad (\nu_{12} + \nu_{22})(\mathrm{uv}) = \begin{cases} \nu_{12}(uv) \ if \ uv \in E_1 \\ \nu_{22}(uv) \ if \ uv \in E_2 \\ \nu_{11}(u) \vee \nu_{21}(\mathrm{v}) \text{ otherwise} \end{cases}$$

Theorem 3.2 The intuitionistic fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ with efficient dominating sets $D_1 \ and \ \mathrm{D}_2$ respectively. Then V_1 or V_2 is the minimum covering of $\mathrm{G}_1 + G_2$.

Proof. Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ be intuitionistic fuzzy graphs with efficient dominating sets $D_1 \ and \ \mathrm{D}_2$ respectively. Therefore every vertex in $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ be dominated by a vertex in $D_1 \ and \ \mathrm{D}_2$ respectively. In $\mathrm{G}_1 + G_2$ the edges of the forms

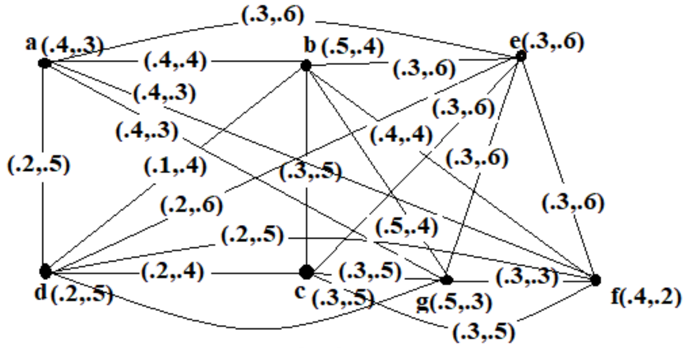
$$(\mu_{12} + \mu_{22})(\mathrm{uv}) = \begin{cases} \mu_{12}(uv) \ if \ uv \in E_1 \\ \mu_{22}(uv) \ if \ uv \in E_2 \\ \mu_{11}(u) \wedge \mu_{21}(\mathrm{v}) \text{ otherwise} \end{cases} \qquad (\nu_{12} + \nu_{22})(\mathrm{uv}) = \begin{cases} \nu_{12}(uv) \ if \ uv \in E_1 \\ \nu_{22}(uv) \ if \ uv \in E_2 \\ \nu_{11}(u) \vee \nu_{21}(\mathrm{v}) \text{ otherwise} \end{cases}$$

The edges of the form $uv \in \mathrm{G}_1 + G_2$ if $\mathrm{u} \in \mathrm{V}_1 \ and \ v \in V_2$ are strong edges in $\mathrm{G}_1 + G_2$ the end vertices are belongs to $\mathrm{V}_1 \ and \ V_2$ respectively. Therefore $\mathrm{V}_1 \ or \ V_2$ efficiently dominates the vertices in $\mathrm{G}_1 + G_2$. Hence proved.



$G_1(V_1, E_1)$ $G_2(V_2, E_2)$

Example 3.3



$G_1 + G_2$

From the above example the efficient dominating set of the intuitionistic fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are $\{a, c\}$ and $\{e\}$ respectively. The efficient dominating number of the intuitionistic fuzzygraphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are 0.95 and 0.35. The covering sets of the graph $G_1 + G_2$ is $\{a, b, c, d\}$ *or* $\{e, f, g\}$ and the efficient dominating number is 1.55.

Definition 3.4 The direct product of two intuitionistic fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are denoted by $G_1 \bullet G_2$ whose vertex is $V_1 \times V_2$ and edge set is defined by

$$(\mu_{11} \bullet \mu_{21})(uv) = \mu_{11}(u) \wedge \mu_{21}(v)$$

$$(\nu_{11} \bullet \nu_{21})(uv) = \nu_{11}(u) \vee \nu_{21}(v)$$

and

$$(\mu_{12} \bullet \mu_{22})((u_1u_2)(v_1 v_2)) = \mu_{12}(u_1v_1) \wedge \mu_{22}(u_2v_2) \, if \; u_1v_1 \in E_1 \; and \; u_2v_2 \in E_2$$

$$(\nu_{12} \bullet \nu_{22})((u_1u_2)(v_1 v_2)) = \nu_{12}(u_1v_1) \vee \nu_{22}(u_2v_2) \, if \; u_1v_1 \in E_1 \; and \; u_2v_2 \in E_2$$

Theorem 3.5 The intuitionistic fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ with efficient dominating sets D_1 and D_2 respectively. Then $V_1 \times D_2$ or $D_1 \times V_2$ is the minimum efficient dominating set of $G_1 \bullet G_2$.

Proof. Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ be intuitionistic fuzzy graphs with efficient dominating sets D_1 and D_2 respectively. Therefore every vertex in $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ be dominated by a vertex in D_1 and D_2 respectively. In $G_1 \bullet G_2$ the edges of the forms

$$(\mu_{12} \bullet \mu_{22})((u_1u_2)(v_1 v_2)) = \mu_{12}(u_1v_1) \wedge \mu_{22}(u_2v_2)$$

if $u_1v_1 \in E_1$ and $u_2v_2 \in E_2$. If the edges $(u_1v_1) \in E_1$ and $(u_2v_2) \in E_2$ are both strong edges in $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ therefore we get

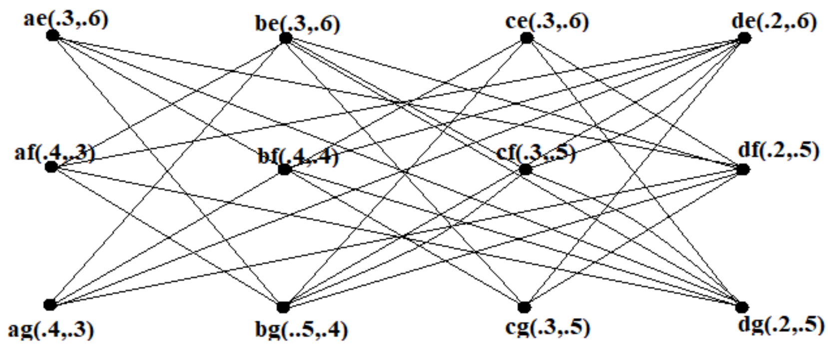
$$(\mu_1 \bullet \mu_2)((u_1u_2)(v_1v_2)) = \mu_1(u_1v_1) \wedge \mu_2(u_2v_2)$$
$$= \sigma_1(u_1) \wedge \sigma_1(\mathrm{v}_1) \wedge \sigma_2(u_2) \wedge \sigma_2(\mathrm{v}_2)$$

since $(u_1v_1) \in E_1$ and $(u_2v_2) \in E_2$ are both strong edges in $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$

$$(\mu_{12} \bullet \mu_{22})((u_1u_2)(v_1v_2)) = \mu_{12}(u_1v_1) \wedge \mu_{22}(u_2v_2)$$
$$= \mu_{11}(u_1) \wedge \mu_{11}(\mathrm{v}_1) \wedge \mu_{21}(u_2) \wedge \mu_{21}(\mathrm{v}_2)$$
$$= \mu_{11}(u_1) \wedge \mu_{21}(u_2) \wedge \mu_{11}(\mathrm{v}_1) \wedge \mu_{21}(\mathrm{v}_2)$$
$$= (\mu_{11} \bullet \mu_{21})(u_1u_2) \wedge (\mu_{11} \bullet \mu_{21})(\mathrm{v}_1\mathrm{v}_2)$$
$$(\nu_{12} \bullet \nu_{22})((u_1u_2)(v_1v_2)) = \nu_{12}(u_1v_1) \vee \nu_{22}(u_2v_2)$$
$$= \nu_{11}(u_1) \vee \nu_{11}(\mathrm{v}_1) \vee \nu_{21}(u_2) \vee \nu_{21}(\mathrm{v}_2)$$
$$= \nu_{11}(u_1) \vee \nu_{21}(u_2) \vee \nu_{11}(\mathrm{v}_1) \vee \nu_{21}(\mathrm{v}_2)$$
$$= (\nu_{11} \bullet \nu_{21})(u_1u_2) \vee (\nu_{11} \bullet \nu_{21})(\mathrm{v}_1\mathrm{v}_2)$$

This implies $(u_1u_2)(v_1v_2) \in G_1 \bullet G_2$ are strong edges whose end vertices belong to $\mathrm{V}_1 \times D_2$ or $D_1 \times V_2$. Therefore the edges of the form are efficiently dominated by the vertices in the sets $\mathrm{V}_1 \times D_2$ or $D_1 \times V_2$. Hence proved.

Example 3.6 The direct product of intuitionistic fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ in Figure 3.1



$G_1 \bullet G_2$

From the above example the efficient dominating set of the intuitionistic fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are $\{\mathrm{a}, c\}$ and $\{e\}$ respectively. The efficient dominating number of the intuitionistic fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are 0.95 and 0.35. The covering sets of the graph $G_1 \bullet G_2$ is $\{ae, be, ce, de\}$ or $\{\mathrm{ae, af, ag, ce, cf, cg}\}$ and the efficient dominating number is 1.35.

Definition 3.7 The semi product of two intuitionistic fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are denoted by $G_1 \odot \mathrm{G}_2$ whose vertex is $V_1 \times V_2$ and edge set is defined by

$$(\mu_{11} \odot \mu_{21})(\mathrm{uv}) = \mu_{11}(u) \wedge \mu_{21}(\mathrm{v})$$
$$(\nu_{11} \odot \nu_{21})(\mathrm{uv}) = \nu_{11}(u) \vee \nu_{21}(\mathrm{v})$$

and

$$(\mu_{12}\odot\mu_{22})((\mathrm{u}_1\mathrm{u}_2)(\mathrm{v}_1 v_2))=\begin{cases}\mu_{11}(u_1)\wedge\mu_{22}(u_2v_2)\, if\ u_1=v_1\\ \mu_{12}(u_1v_1)\wedge\mu_{22}(u_2v_2)\, if\ u_1v_1\in E_1\ and\ u_2v_2\in E_2\end{cases}$$

$$(\nu_{12}\odot\nu_{22})((\mathrm{u}_1\mathrm{u}_2)(\mathrm{v}_1 v_2))=\begin{cases}\nu_{11}(u_1)\vee\nu_{22}(u_2v_2)\, if\ u_1=v_1\\ \nu_{12}(u_1v_1)\vee\nu_{22}(u_2v_2)\, if\ u_1v_1\in E_1\ and\ u_2v_2\in E_2\end{cases}$$

Theorem 3.8 The intuitionistic fuzzy graphs $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ with efficient dominating sets D_1 and D_2 respectively. Then $\mathrm{V}_1\times D_2$ is the minimum efficient dominating set of $G_1\odot G_2$.

Proof. Let $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ be intuitionistic fuzzy graphs with efficient dominating sets D_1 and D_2 respectively. Therefore every vertex in $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ be dominated by a vertex in D_1 and D_2 respectively. In $G_1\odot G_2$ the edges of the forms

$$(\mu_{12}\odot\mu_{22})((\mathrm{u}_1\mathrm{u}_2)(\mathrm{v}_1 v_2))=\begin{cases}\mu_{11}(u_1)\wedge\mu_{22}(u_2v_2)\, if\ u_1=v_1\\ \mu_{12}(u_1v_1)\wedge\mu_{22}(u_2v_2)\, if\ u_1v_1\in E_1\ \text{and}\ u_2v_2\in E_2\end{cases}$$

$$(\nu_{12}\odot\nu_{22})((\mathrm{u}_1\mathrm{u}_2)(\mathrm{v}_1 v_2))=\begin{cases}\nu_{11}(u_1)\vee\nu_{22}(u_2v_2)\, if\ u_1=v_1\\ \nu_{12}(u_1v_1)\vee\nu_{22}(u_2v_2)\, if\ u_1v_1\in E_1\ \text{and}\ u_2v_2\in E_2\end{cases}$$

Case (i): $(\mathrm{u}_1 u_2)(v_1v_2)$ if $u_1=v_1$ and $(u_2v_2)\in E_2$ If the edge $(u_2v_2)\in E_2$ is strong edges in $G_2(V_2,\sigma_2,\mu_2)$ therefore we get

$$(\mu_{12}\odot\mu_{22})((\mathrm{u}_1 u_2)(v_1v_2))=\mu_{11}(u_1)\wedge\mu_{22}(u_2v_2)$$
$$=\mu_{11}(u_1)\wedge\mu_{21}(u_2)\wedge\mu_{21}(v_2)$$

since $(u_2v_2)\in E_2$ is a strong edges in $G_2(V_2,E_2)$.

$$(\mu_{12}\odot\mu_{22})((\mathrm{u}_1 u_2)(v_1v_2))=\mu_{11}(u_1)\wedge\mu_{22}(u_2v_2)$$
$$=\mu_{11}(u_1)\wedge\mu_{21}(u_2)\wedge\mu_{21}(v_2)$$
$$=\mu_{11}(u_1)\wedge\mu_{21}(u_2)\wedge\mu_{11}(u_1)\wedge\mu_{21}(v_2)$$
$$=\mu_{11}(u_1)\wedge\mu_{21}(u_2)\wedge\mu_{11}(\mathrm{v}_1)\wedge\mu_{21}(v_2)$$
$$\therefore u_1=v_1$$
$$=(\mu_{11}\odot\mu_{21})(u_1u_2)\wedge(\mu_{11}\odot\mu_{21})(\mathrm{v}_1 v_2)$$

$$(\nu_{12}\odot\nu_{22})((\mathrm{u}_1 u_2)(v_1v_2))=\nu_{11}(u_1)\vee\nu_{22}(u_2v_2)$$
$$=\nu_{11}(u_1)\vee\nu_{21}(u_2)\vee\nu_{21}(v_2)$$
$$=\nu_{11}(u_1)\vee\nu_{21}(u_2)\vee\nu_{11}(u_1)\vee\nu_{21}(v_2)$$
$$=\nu_{11}(u_1)\vee\nu_{21}(u_2)\vee\nu_{11}(\mathrm{v}_1)\vee\nu_{21}(v_2)$$
$$\therefore u_1=v_1$$
$$=(\nu_{11}\odot\nu_{21})(u_1u_2)\vee(\nu_{11}\odot\nu_{21})(\mathrm{v}_1 v_2)$$

This implies $(u_1u_2)(v_1v_2)\in G_1\odot G_2$ if $u_1=v_1$ and $(u_2v_2)\in E_2$ are strong edges whose end vertices belong to $\mathrm{V}_1\times D_2$. Since $(u_2v_2)\in E_2$ is a strong edge such that $u_2\in D_2$ or $v_2\in D_2$. Therefore the set $\mathrm{V}_1\times D_2$ is efficiently dominated the all other vertices in $G_1\odot G_2$.

Case (ii): If the edges $(u_1v_1)\in E_1$ and $(u_2v_2)\in E_2$ are both strong edges in $G_1(V_1,\sigma_1,\mu_1)$ and $G_2(V_2,\sigma_2,\mu_2)$ therefore we get

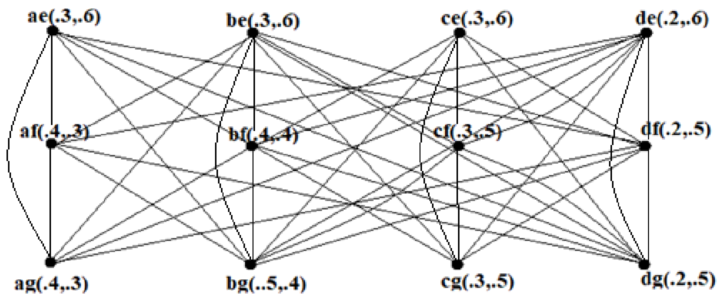
$$(\mu_{12}\odot\mu_{22})((u_1u_2)(v_1v_2)) = \mu_{12}(u_1v_1)\wedge\mu_{21}(u_2v_2)$$
$$= \mu_{11}(u_1)\wedge\mu_{11}(\mathrm{v}_1)\wedge\mu_{21}(u_2)\wedge\mu_{21}(\mathrm{v}_2)$$
$$(\nu_{12}\odot\nu_{22})((u_1u_2)(v_1v_2)) = \nu_{12}(u_1v_1)\vee\nu_{21}(u_2v_2)$$
$$= \nu_{11}(u_1)\vee\nu_{11}(\mathrm{v}_1)\vee\nu_{21}(u_2)\vee\nu_{21}(\mathrm{v}_2)$$

since $(u_1v_1)\in E_1$ and $(u_2v_2)\in E_2$ are both strong edges in $G_1(V_1,\sigma_1,\mu_1)$ and $G_2(V_2,\sigma_2,\mu_2)$

$$(\mu_{12}\odot\mu_{22})((u_1u_2)(v_1v_2)) = \mu_{12}(u_1v_1)\wedge\mu_{22}(u_2v_2)$$
$$= \mu_{11}(u_1)\wedge\mu_{11}(\mathrm{v}_1)\wedge\mu_{21}(u_2)\wedge\mu_{21}(\mathrm{v}_2)$$
$$= \mu_{11}(u_1)\wedge\mu_{21}(u_2)\wedge\mu_{11}(\mathrm{v}_1)\wedge\mu_{21}(\mathrm{v}_2)$$
$$= (\mu_{11}\odot\mu_{11})(u_1u_2)\wedge(\mu_{11}\odot\mu_{11})(\mathrm{v}_1\mathrm{v}_2)$$
$$(\nu_{12}\odot\nu_{22})((u_1u_2)(v_1v_2)) = \nu_{12}(u_1v_1)\vee\nu_{22}(u_2v_2)$$
$$= \nu_{11}(u_1)\vee\nu_{11}(\mathrm{v}_1)\vee\nu_{21}(u_2)\vee\nu_{21}(\mathrm{v}_2)$$
$$= \nu_{11}(u_1)\vee\nu_{21}(u_2)\vee\nu_{11}(\mathrm{v}_1)\vee\nu_{21}(\mathrm{v}_2)$$
$$= (\nu_{11}\odot\nu_{11})(u_1u_2)\vee(\nu_{11}\odot\nu_{11})(\mathrm{v}_1\mathrm{v}_2)$$

This implies $(u_1u_2)(v_1v_2)\in G_1\odot G_2$ are strong edges whose one end vertices belong to $\mathrm{V}_1\times D_2$. Therefore the edges of the form are efficiently dominated by the vertices in the sets $\mathrm{V}_1\times D_2$. From case (i) and (ii) $\mathrm{V}_1\times D_2$ is the minimum efficient dominating set of $G_1\odot G_2$. Hence proved.

Example 3.9 The semi product of intuitionistic fuzzy graphs $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ in Figure 3.1



$G_1\odot G_2$

Edges	(ae)(bf)	(ae)(bg)	(af)(be)	(af)(bg)	(ag)(be)	(ag)(bf)
$(\mu_{12}\odot\mu_{22})$	.3	.3	.3	.3	.3	.3
$(\nu_{12}\odot\nu_{22})$	.6	.6	.6	.4	.6	.4
Edges	(ae)(bf)	(ae)(bg)	(af)(be)	(af)(bg)	(ag)(be)	(ag)(bf)
$(\mu_{12}\odot\mu_{22})$	.3	.3	.3	.3	.3	.3
$(\mu_{12}\odot\mu_{22})$	.6	.6	.6	.4	.6	.4
Edges	(be)(cf)	(be)(cg)	(bf)(ce)	(bf)(cg)	(bg)(ce)	(bg)(cf)
$(\mu_{12}\odot\mu_{22})$	.3	.3	.3	.3	.3	.3
$(\mu_{12}\odot\mu_{22})$	.6	.6	.5	.5	.6	.5
Edges	(be)(df)	(be)(dg)	(bf)(de)	(bf)(dg)	(bg)(de)	(bg)(df)

$(\mu_{12} \odot \mu_{22})$	.1	.1	.1	.1	.1	.1
$(\mu_{12} \odot \mu_{22})$	.6	.6	.4	.4	.6	.4
Edges	(ce)(df)	(ce)(dg)	(cf)(de)	(cf)(dg)	(cg)(de)	(cg)(df)
$(\mu_{12} \odot \mu_{22})$	.2	.2	.2	.2	.2	.2
$(\mu_{12} \odot \mu_{22})$	.6	.6	.4	.4	.6	.4
Edges	(ae)(af)	(af)(ag)	(ae)(ag)	(be)(bf)	(bf)(bg)	(be)(bg)
$(\mu_{12} \odot \mu_{22})$	.3	.3	.3	.3	.3	.3
$(\mu_{12} \odot \mu_{22})$	.6	.3	.6	.6	.4	.6
Edges	(ce)(cf)	(cf)(cg)	(ce)(cg)	(de)(df)	(df)(dg)	(de)(dg)
$(\mu_{12} \odot \mu_{22})$	.3	.3	.3	.3	.3	.3
$(\mu_{12} \odot \mu_{22})$	.6	.5	.6	.6	.4	.6

From the above example the efficient dominating set of the intuitionistic fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are $\{a, c\}$ and $\{e\}$ respectively. The efficient dominating number of the intuitionistic fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are 0.95 and 0.35. The covering sets of the graph $G_1 \odot G_2$ is $\{ae, be, ce, de\}$ and the efficient dominating number is 1.35.

Definition 3.10 The Cartesian product of two intuitionistic fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are denoted by $G_1 \times G_2$ whose vertex is $V_1 \times V_2$ and edge set is defined by

$$(\mu_{11} \times \mu_{21})(uv) = \mu_{11}(u) \wedge \mu_{21}(v)$$
$$(\nu_{11} \times \nu_{21})(uv) = \nu_{11}(u) \vee \nu_{21}(v)$$

and

$$(\mu_{12} \times \mu_{22})((u_1u_2)(v_1 v_2)) = \begin{cases} \mu_{11}(u_1) \wedge \mu_{22}(u_2v_2)\, if\ u_1 = v_1 \\ \mu_{21}(u_2) \wedge \mu_{12}(u_1v_1)\, if\ u_2 = v_2 \\ 0 \quad otherwise \end{cases}$$

$$(\nu_{12} \times \nu_{22})((u_1u_2)(v_1 v_2)) = \begin{cases} \nu_{11}(u_1) \vee \nu_{22}(u_2v_2)\, if\ u_1 = v_1 \\ \nu_{21}(u_2) \vee \nu_{12}(u_1v_1)\, if\ u_2 = v_2 \\ 0 \quad otherwise \end{cases}$$

Theorem 3.11 The intuitionistic fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ with efficient dominating sets D_1 and D_2 respectively. Then $V_1 \times D_2$ or $D_1 \times V_2$ is the minimum efficient dominating set of $G_1 \times G_2$.

Proof. Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ be intuitionistic fuzzy graphs with efficient dominating sets D_1 and D_2 respectively. Therefore every vertex in $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ be dominated by a vertex in D_1 and D_2 respectively. In $G_1 \times G_2$ the edges of the forms

$$(\mu_{12}\times\mu_{22})((\mathrm{u}_1\mathrm{u}_2)(\mathrm{v}_1 v_2)) = \begin{cases} \mu_{11}(u_1)\wedge\mu_{22}(u_2v_2)\, if\ u_1 = v_1 \\ \mu_{21}(u_2)\wedge\mu_{12}(u_1v_1)\, if\ u_2 = v_2 \\ 0 \quad otherwise \end{cases}$$

$$(\nu_{12}\times\nu_{22})((\mathrm{u}_1\mathrm{u}_2)(\mathrm{v}_1 v_2)) = \begin{cases} \nu_{11}(u_1)\vee\nu_{22}(u_2v_2)\, if\ u_1 = v_1 \\ \nu_{21}(u_2)\vee\nu_{12}(u_1v_1)\, if\ u_2 = v_2 \\ 0 \quad otherwise \end{cases}$$

Case (i): $(\mathrm{u}_1 u_2)(v_1v_2)$ $if\ u_1 = v_1$ and $(u_2v_2)\in E_2$ If the edge $(u_2v_2)\in E_2$ is strong edges in $G_2(V_2,\sigma_2,\mu_2)$ therefore we get

$$(\mu_{12}\times\mu_{22})((\mathrm{u}_1 u_2)(v_1v_2)) = \mu_{11}(u_1)\wedge\mu_2(u_2v_2)$$
$$= \mu_{11}(u_1)\wedge\mu_{22}(u_2)\wedge\mu_{22}(v_2)$$
$$(\nu_{12}\times\nu_{22})((\mathrm{u}_1 u_2)(v_1v_2)) = \nu_{11}(u_1)\vee\nu_{22}(u_2v_2)$$
$$= \nu_{11}(u_1)\vee\nu_{22}(u_2)\vee\nu_{22}(v_2)$$

since $(u_2v_2)\in E_2$ is a strong edges in $G_2(V_2,\sigma_2,\mu_2)$.

$$(\mu_{12}\times\mu_{22})((\mathrm{u}_1 u_2)(v_1v_2)) = \mu_{11}(u_1)\wedge\mu_{22}(u_2v_2)$$
$$= \mu_{11}(u_1)\wedge\mu_{21}(u_2)\wedge\mu_{21}(v_2)$$
$$= \mu_{11}(u_1)\wedge\mu_{21}(u_2)\wedge\mu_{11}(u_1)\wedge\mu_{21}(v_2)$$
$$= \mu_{11}(u_1)\wedge\mu_{21}(u_2)\wedge\mu_{11}(\mathrm{v}_1)\wedge\mu_2 1(v_2)$$
$$\therefore u_1 = v_1$$
$$= (\mu_{11}\times\mu_{21})(u_1u_2)\wedge(\mu_{11}\times\mu_{21})(\mathrm{v}_1 v_2)$$
$$(\nu_{12}\times\nu_{22})((\mathrm{u}_1 u_2)(v_1v_2)) = \nu_{11}(u_1)\vee\nu_{22}(u_2v_2)$$
$$= \nu_{11}(u_1)\vee\nu_{21}(u_2)\vee\nu_{21}(v_2)$$
$$= \nu_{11}(u_1)\vee\nu_{21}(u_2)\vee\nu_{11}(u_1)\vee\nu_{21}(v_2)$$
$$= \nu_{11}(u_1)\vee\nu_{21}(u_2)\vee\nu_{11}(\mathrm{v}_1)\vee\nu_2 1(v_2)$$
$$\therefore u_1 = v_1$$
$$= (\nu_{11}\times\nu_{21})(u_1u_2)\vee(\nu_{11}\times\nu_{21})(\mathrm{v}_1 v_2)$$

This implies $(u_1u_2)(v_1v_2)\in G_1\times G_2$ if $u_1 = v_1$ and $(u_2v_2)\in E_2$ are strong edges whose end vertices belong to $\mathrm{V}_1\times D_2$. Since $(u_2v_2)\in E_2$ is a strong edge such that $u_2\in D_2$ or $v_2\in D_2$. Therefore the set $\mathrm{V}_1\times D_2$ is efficiently dominated the all other vertices in $G_1\times G_2$.

Case (ii): $(\mathrm{u}_1 u_2)(v_1v_2)$ if $u_2 = v_2$ and $(u_1v_1)\in E_1$ If the edge $(u_1v_1)\in E_1$ is strong edges in $G_1(V_1,\sigma_1,\mu_1)$ therefore we get

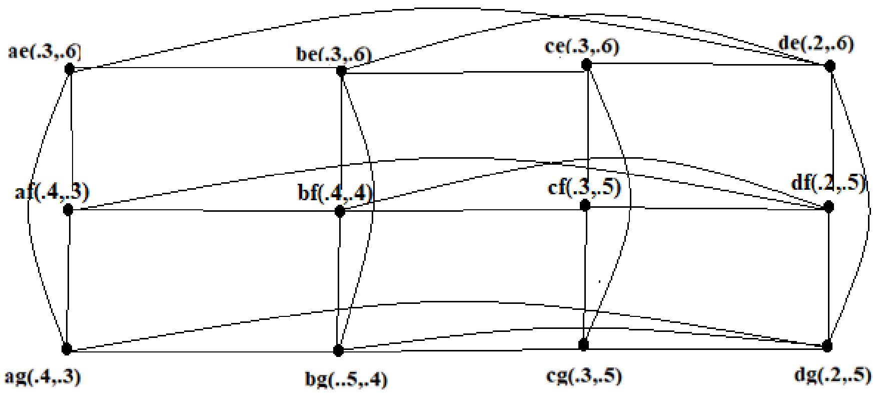
$$(\mu_{12}\times\mu_{22})((\mathrm{u}_1 u_2)(v_1v_2)) = \mu_{21}(u_2)\wedge\mu_{12}(u_1v_1)$$
$$= \mu_{11}(u_1)\wedge\mu_{11}(\mathrm{v}_1)\wedge\mu_{21}(u_2)$$
$$(\nu_{12}\times\nu_{22})((\mathrm{u}_1 u_2)(v_1v_2)) = \nu_{21}(u_2)\vee\nu_{12}(u_1v_1)$$
$$= \nu_{11}(u_1)\vee\nu_{11}(\mathrm{v}_1)\vee\nu_{21}(u_2)$$

since $(u_1v_1)\in E_1$ is a strong edges in $G_1(V_1,\sigma_1,\mu_1)$.

$$(\mu_{12} \times \mu_{22})((u_1 u_2)(v_1 v_2)) = \mu_{21}(u_2) \wedge \mu_{12}(u_1 v_1)$$
$$= \mu_{11}(u_1) \wedge \mu_{11}(v_1) \wedge \mu_{21}(u_2)$$
$$= \mu_{11}(u_1) \wedge \mu_{21}(u_2) \wedge \mu_{11}(u_1) \wedge \mu_{21}(u_2)$$
$$= \mu_{11}(u_1) \wedge \mu_{21}(u_2) \wedge \mu_{11}(v_1) \wedge \mu_{21}(v_2)$$
$$\therefore u_2 = v_2$$
$$= (\mu_{11} \times \mu_{21})(u_1 u_2) \wedge (\mu_{11} \times \mu_{21})(v_1 v_2)$$
$$(\nu_{12} \times \nu_{22})((u_1 u_2)(v_1 v_2)) = \nu_{21}(u_2) \vee \nu_{12}(u_1 v_1)$$
$$= \nu_{11}(u_1) \vee \nu_{11}(v_1) \vee \nu_{21}(u_2)$$
$$= \nu_{11}(u_1) \vee \nu_{21}(u_2) \vee \nu_{11}(u_1) \vee \nu_{21}(u_2)$$
$$= \nu_{11}(u_1) \vee \nu_{21}(u_2) \vee \nu_{11}(v_1) \vee \nu_{21}(v_2)$$
$$\therefore u_2 = v_2$$
$$= (\nu_{11} \times \nu_{21})(u_1 u_2) \vee (\nu_{11} \times \nu_{21})(v_1 v_2)$$

This implies $(u_1u_2)(v_1v_2) \in G_1 \times G_2$ if $u_2 = v_2$ and $(u_1v_1) \in E_1$ are strong edges whose end vertices belong to $D_1 \times V_2$. Since $(u_1v_1) \in E_1$ is a strong edge such that $u_1 \in D_1$ or $v_1 \in D_1$. Therefore the set $D_1 \times V_2$ is efficiently dominated the all other vertices in $G_1 \times G_2$.

Example 3.12 The Cartesian product of intuitionistic fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ in Figure.



$G_1 \times G_2$

Edges	(ae)(af)	(af)(ag)	(ae)(ag)	(be)(bf)	(bf)(bg)	(be)(bg)
$(\mu_{12} \times \mu_{22})$	.3	.3	.3	.3	.3	.3
$(\nu_{12} \times \nu_{22})$	.6	.3	.6	.6	.4	.6
Edges	(ce)(cf)	(cf)(cg)	(ce)(cg)	(de)(df)	(df)(dg)	(de)(dg)
$(\mu_{12} \times \mu_{22})$	.3	.3	.3	.3	.3	.3
$(\nu_{12} \times \nu_{22})$	.6	.5	.6	.6	.4	.6
Edges	(ae)(be)	(be)(ce)	(ce)(de)	(ae)(de)	(be)(de)	(af) (bf)
$(\mu_{12} \times \mu_{22})$	.3	.3	.3	.3	.3	.4
$(\nu_{12} \times \nu_{22})$	.6	.6	.6	.6	.6	.4
Edges	(bf)(cf)	(cf)(df)	(af)(df)	(bf)(df)	(ag)(bg)	(bg)(cg)
$(\mu_{12} \times \mu_{22})$	.3	.2	.2	.1	.4	.3

$(\nu_{12} \times \nu_{22})$	.5	.4	.5	.4	.4	.5
Edges	(cg)(dg)	(ag)(dg)	(bg)(dg)			
$(\mu_{12} \times \mu_{22})$	.2	.2	.1			
$(\nu_{12} \times \nu_{22})$	.4	.5	.4			

From the above example the efficient dominating set of the intuitionistic fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are $\{a, c\}$ and $\{e\}$ respectively. The efficient dominating number of the intuitionistic fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are 0.95 and 0.35. The covering sets of the graph $G_1 \times G_2$ is $\{ae, be, ce, de\}$ or $\{\text{ae, af, ag, ce, cf, cg}\}$ and the efficient dominating number is 1.35.

Definition 3.13 The Composition of two intuitionistic fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are denoted by $G_1 \circ G_2$ whose vertex is $V_1 \times V_2$ and edge set is defined by

$$(\mu_{11} \circ \mu_{21})(\text{uv}) = \mu_{11}(u) \wedge \mu_{21}(\text{v})$$

$$(\nu_{11} \circ \nu_{21})(\text{uv}) = \nu_{11}(u) \vee \nu_{21}(\text{v})$$

and

$$(\mu_{12} \circ \mu_{22})((\text{u}_1\text{u}_2)(\text{v}_1 v_2)) = \begin{cases} \mu_{11}(u_1) \wedge \mu_{22}(u_2 v_2) \text{ if } u_1 = v_1 \\ \mu_{21}(u_2) \wedge \mu_{12}(u_1 v_1) \text{ if } u_2 = v_2 \\ \mu_{21}(u_2) \wedge \mu_{21}(\text{v}_2) \wedge \mu_{12}(u_1 v_1) \text{ otherwise} \end{cases}$$

$$(\nu_{12} \circ \nu_{22})((\text{u}_1\text{u}_2)(\text{v}_1 v_2)) = \begin{cases} \nu_{11}(u_1) \vee \nu_{22}(u_2 v_2) \text{ if } u_1 = v_1 \\ \nu_{21}(u_2) \vee \nu_{12}(u_1 v_1) \text{ if } u_2 = v_2 \\ \nu_{21}(u_2) \vee \nu_{21}(\text{v}_2) \vee \nu_{12}(u_1 v_1) \text{ otherwise} \end{cases}$$

Theorem 3.14 The intuitionistic fuzzy graphs $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ with efficient dominating sets D_1 and D_2 respectively. Then $\text{V}_1 \times D_2$ or $D_1 \times V_2$ is the minimum efficient dominating set of $G_1 \circ G_2$.

Proof. Let $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ be intuitionistic fuzzy graphs with efficient dominating sets D_1 and D_2 respectively. Therefore every vertex in $G_1(V_1, \sigma_1, \mu_1)$ and $G_2(V_2, \sigma_2, \mu_2)$ be dominated by a vertex in D_1 and D_2 respectively. In $G_1 \circ G_2$ the edges of the forms (i). $(\text{u}_1 u_2)(v_1 v_2)$ *if* $u_1 = v_1$ and $(u_2 v_2) \in E_2$

(ii) $(\text{u}_1 u_2)(v_1 v_2)$ *if* $u_2 = v_2$ and $(u_1 v_1) \in E_1$

(iii) $(u_1 v_1) \in E_1$ and $(u_2 v_2) \in E_1$

Case (i): $(\text{u}_1 u_2)(v_1 v_2)$ if $u_1 = v_1$ and $(u_2 v_2) \in E_2$ If the edge $(u_2 v_2) \in E_2$ is strong edges in $G_2(V_2, \sigma_2, \mu_2)$ therefore we get

$$\begin{aligned}(\mu_{12} \circ \mu_{22})((\text{u}_1 u_2)(v_1 v_2)) &= \mu_{11}(u_1) \wedge \mu_{22}(u_2 v_2) \\ &= \mu_{11}(u_1) \wedge \mu_{21}(u_2) \wedge \mu_{21}(v_2)\end{aligned}$$

$$\begin{aligned}(\nu_{12} \circ \nu_{22})((\text{u}_1 u_2)(v_1 v_2)) &= \nu_{11}(u_1) \vee \nu_{22}(u_2 v_2) \\ &= \nu_{11}(u_1) \vee \nu_{21}(u_2) \vee \nu_{21}(v_2)\end{aligned}$$

since $(u_2 v_2) \in E_2$ is a strong edges in $G_2(V_2, \sigma_2, \mu_2)$.

$$(\mu_{12}\circ\mu_{22})((\mathrm{u}_1 u_2)(v_1v_2)) = \mu_{11}(u_1)\wedge\mu_{22}(u_2v_2)$$
$$= \mu_{11}(u_1)\wedge\mu_{21}(u_2)\wedge\mu_{21}(v_2)$$
$$= \mu_{11}(u_1)\wedge\mu_{21}(u_2)\wedge\mu_{11}(u_1)\wedge\mu_{21}(v_2)$$
$$= \mu_{11}(u_1)\wedge\mu_{21}(u_2)\wedge\mu_{11}(\mathrm{v}_1)\wedge\mu_{21}(v_2)$$
$$\therefore u_1 = v_1$$
$$= (\mu_{12}\circ\mu_{22})(u_1u_2)\wedge(\mu_{12}\circ\mu_{22})(\mathrm{v}_1 v_2)$$
$$(\nu_{12}\circ\nu_{22})((\mathrm{u}_1 u_2)(v_1v_2)) = \nu_{11}(u_1)\vee\nu_{22}(u_2v_2)$$
$$= \nu_{11}(u_1)\vee\nu_{21}(u_2)\vee\nu_{21}(v_2)$$
$$= \nu_{11}(u_1)\vee\nu_{21}(u_2)\vee\nu_{11}(u_1)\vee\nu_{21}(v_2)$$
$$= \nu_{11}(u_1)\vee\nu_{21}(u_2)\vee\nu_{11}(\mathrm{v}_1)\vee\nu_{21}(v_2)$$
$$\therefore u_1 = v_1$$
$$= (\nu_{12}\circ\nu_{22})(u_1u_2)\vee(\nu_{12}\circ\nu_{22})(\mathrm{v}_1 v_2)$$

This implies $(u_1u_2)(v_1v_2)\in G_1\circ G_2$ if $u_1 = v_1$ and $(u_2v_2)\in E_2$ are strong edges whose end vertices belong to $\mathrm{V}_1\times D_2$. Since $(u_2v_2)\in E_2$ is a strong edge such that $u_2\in D_2$ or $v_2\in D_2$. Therefore the set $\mathrm{V}_1\times D_2$ is efficiently dominated the all other vertices in $G_1\circ G_2$.

Case (ii): $(\mathrm{u}_1 u_2)(v_1v_2)$ if $u_2 = v_2$ and $(u_1v_1)\in E_1$ If the edge $(u_1v_1)\in E_1$ is strong edges in $G_1(V_1,\sigma_1,\mu_1)$ therefore we get

$$(\mu_{12}\times\mu_{22})((\mathrm{u}_1 u_2)(v_1v_2)) = \mu_{21}(u_2)\wedge\mu_{12}(u_1v_1)$$
$$= \mu_{11}(u_1)\wedge\mu_{11}(\mathrm{v}_1)\wedge\mu_{21}(u_2)$$
$$(\nu_{12}\times\nu_{22})((\mathrm{u}_1 u_2)(v_1v_2)) = \nu_{21}(u_2)\vee\nu_{12}(u_1v_1)$$
$$= \nu_{11}(u_1)\vee\nu_{11}(\mathrm{v}_1)\vee\nu_{21}(u_2)$$

since $(u_1v_1)\in E_1$ is a strong edges in $G_1(V_1,\sigma_1,\mu_1)$.

$$(\mu_{12}\circ\mu_{21})((\mathrm{u}_1 u_2)(v_1v_2)) = \mu_{21}(u_2)\wedge\mu_{12}(u_1v_1)$$
$$= \mu_{11}(u_1)\wedge\mu_{11}(\mathrm{v}_1)\wedge\mu_{21}(u_2)$$
$$= \mu_{11}(u_1)\wedge\mu_{21}(u_2)\wedge\mu_{11}(u_1)\wedge\sigma_{21}(\mathrm{u}_2)$$
$$= \mu_{11}(u_1)\wedge\mu_{21}(u_2)\wedge\mu_{11}(\mathrm{v}_1)\wedge\mu_{21}(v_2)$$
$$\therefore u_2 = v_2$$
$$= (\mu_{12}\circ\mu_{22})(u_1u_2)\wedge(\mu_{12}\circ\mu_{22})(\mathrm{v}_1 v_2)$$
$$(\nu_{12}\circ\nu_{21})((\mathrm{u}_1 u_2)(v_1v_2)) = \nu_{21}(u_2)\vee\nu_{12}(u_1v_1)$$
$$= \nu_{11}(u_1)\vee\nu_{11}(\mathrm{v}_1)\vee\nu_{21}(u_2)$$
$$= \nu_{11}(u_1)\vee\nu_{21}(u_2)\vee\nu_{11}(u_1)\vee\nu_{21}(\mathrm{u}_2)$$
$$= \nu_{11}(u_1)\vee\nu_{21}(u_2)\vee\nu_{11}(\mathrm{v}_1)\vee\nu_{21}(v_2)$$
$$\therefore u_2 = v_2$$
$$= (\nu_{12}\circ\nu_{22})(u_1u_2)\vee(\nu_{12}\circ\nu_{22})(\mathrm{v}_1 v_2)$$

This implies $(u_1u_2)(v_1v_2)\in G_1\circ G_2$ if $u_2 = v_2$ and $(u_1v_1)\in E_1$ are strong edges whose end vertices belong to $D_1\times V_2$. Since $(u_1v_1)\in E_1$ is a strong edge such that $u_1\in D_1$ or $v_1\in D_1$. Therefore the set $D_1\times V_2$ is efficiently dominated the all other vertices in $G_1\circ G_2$.

Case (iii): If the edges $(u_1v_1) \in E_1$ and $(u_2v_2) \in E_1$ are both strong edges in $G_1(V_1, \sigma_1, \mu_1)$ therefore we get

$$(\mu_{12} \circ \mu_{22})((u_1u_2)(v_1v_2)) = \mu_{21}(u_2) \wedge \mu_{21}(v_2) \wedge \mu_{12}(u_1v_1)$$
$$= \mu_{11}(u_1) \wedge \mu_{11}(\mathrm{v}_1) \wedge \mu_{21}(u_2) \wedge \mu_{21}(\mathrm{v}_2)$$
$$(\nu_{12} \circ \nu_{22})((u_1u_2)(v_1v_2)) = \nu_{21}(u_2) \vee \nu_{21}(v_2) \vee \nu_{12}(u_1v_1)$$
$$= \mu_{11}(u_1) \vee \mu_{11}(\mathrm{v}_1) \vee \mu_{21}(u_2) \vee \mu_{21}(\mathrm{v}_2)$$

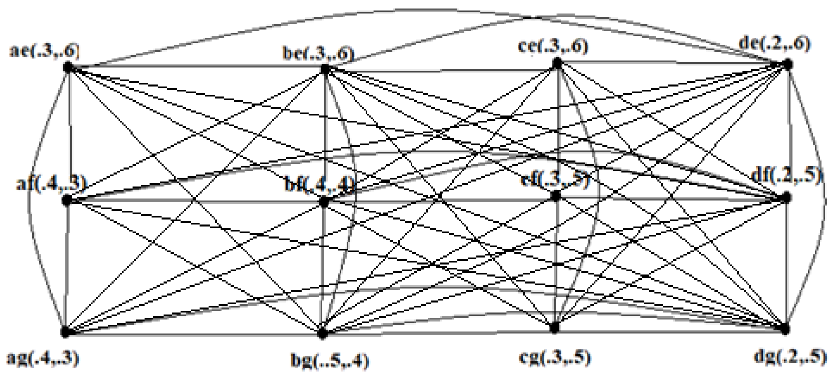
since $(u_1v_1) \in E_1$ are both strong edges in $G_1(V_1, \sigma_1, \mu_1)$.

$$(\mu_{12} \circ \mu_{22})((u_1u_2)(v_1v_2)) = \mu_{21}(u_2) \wedge \mu_{21}(v_2) \wedge \mu_{12}(u_1v_1)$$
$$= \mu_{11}(u_1) \wedge \mu_{11}(\mathrm{v}_1) \wedge \mu_{21}(u_2) \wedge \mu_{21}(\mathrm{v}_2)$$
$$= (\mu_{12} \circ \mu_{22})(u_1u_2) \wedge (\mu_{12} \circ \mu_{22})(\mathrm{v}_1\mathrm{v}_2)$$
$$(\nu_{12} \circ \nu_{22})((u_1u_2)(v_1v_2)) = \nu_{21}(u_2) \vee \nu_{21}(v_2) \vee \nu_{12}(u_1v_1)$$
$$= \nu_{11}(u_1) \vee \nu_{11}(\mathrm{v}_1) \vee \nu_{21}(u_2) \vee \nu_{21}(\mathrm{v}_2)$$
$$= (\nu_{12} \circ \nu_{22})(u_1u_2) \vee (\nu_{12} \circ \nu_{22})(\mathrm{v}_1\mathrm{v}_2)$$

This implies $(u_1u_2)(v_1v_2) \in G_1 \circ G_2$ are strong edges whose one end vertices belong to $D_1 \times V_2$. Therefore the edges of the form are efficiently dominated by the vertices in the sets $D_1 \times V_2$.

From case (i) , (ii) and (ii) $\mathrm{V}_1 \times D_2$ or $D_1 \times V_2$ is the minimum efficient dominating set of $G_1 \circ G_2$. Hence proved.

Example 3.15 The Cartesian product of the intuitionistic fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ in following Figure



$G_1 \circ G_2$

Edges	(ae)(af)	(af)(ag)	(ae)(ag)	(be)(bf)	(bf)(bg)	(be)(bg)
$(\mu_{12} \circ \mu_{22})$	.3	.3	.3	.3	.3	.3
$(\nu_{12} \circ \nu_{22})$	.6	.3	.6	.6	.4	.6
Edges	(ce)(cf)	(cf)(cg)	(ce)(cg)	(de)(df)	(df)(dg)	(de)(dg)
$(\mu_{12} \circ \mu_{22})$	.3	.3	.3	.3	.3	.3
$(\nu_{12} \circ \nu_{22})$	.6	.5	.6	.6	.4	.6
Edges	(ae)(be)	(be)(ce)	(ce)(de)	(ae)(de)	(be)(de)	(af) (bf)
$(\mu_{12} \circ \mu_{22})$	.3	.3	.3	.3	.3	.4
$(\nu_{12} \circ \nu_{22})$	.6	.6	.6	.6	.6	.4
Edges	(bf)(cf)	(cf)(df)	(af)(df)	(bf)(df)	(ag)(bg)	(bg)(cg)

$(\mu_{12} \circ \mu_{22})$	.3	.2	.2	.1	.4	.3
$(\nu_{12} \circ \nu_{22})$	.5	.4	.5	.4	.4	.5
Edges	(cg)(dg)	(ag)(dg)	(bg)(dg)	(ae)(bf)	(ae)(bg)	(ae)(df)
$(\mu_{12} \circ \mu_{22})$	.2	.2	.1	.3	.3	.2
$(\nu_{12} \bullet \nu_{22})$	.4	.5	.4	.6	.6	.6
Edges	(ae)(dg)	(af)(be)	(af)(bg)	(af)(de)	(af)(dg)	(ag)(be)
$(\mu_{12} \circ \mu_{22})$	.2	.3	.4	.2	.2	.3
$(\nu_{12} \circ \nu_{22})$	.6	.6	.4	.6	.5	.6
Edges	(ag)(bf)	(ag)(de)	(ag)(df)	(be)(cf)	(be)(cg)	(be)(df)
$(\mu_{12} \circ \mu_{22})$	.4	.2	.5	.3	.3	.1
$(\nu_{12} \circ \nu_{22})$	.4	.6	.5	.6	.6	.6
Edges	(be)(dg)	(bf)(ce)	(bf)(cg)	(bf)(de)	(bf)(dg)	(bg)(ce)
$(\mu_{12} \circ \mu_{22})$	.3	.3	.3	.1	.1	.3
$(\nu_{12} \circ \nu_{22})$	.6	.6	.5	.6	.4	.6
Edges	(bg)(cf)	(bg)(de)	(bg)(df)	(ce)(df)	(ce)(dg)	(cf)(de)
$(\mu_{12} \circ \mu_{22})$	.3	.1	.1	.2	.2	.2
$(\nu_{12} \circ \nu_{22})$	.5	.6	.4	.6	.6	.6
Edges	(cf)(dg)	(cg)(de)	(cg)(df)			
$(\mu_{12} \circ \mu_{22})$	.2	.2	.2			
$(\nu_{12} \circ \nu_{22})$	.6	.6	.4			

From the above example the efficient dominating set of the intuitionistic fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are $\{a, c\}$ and $\{e\}$ respectively. The efficient dominating number of the intuitionistic fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are 0.95 and 0.35. The covering sets of the graph $G_1 \circ G_2$ is $\{ae, be, ce, de\}$ or $\{ae, af, ag, ce, cf, cg\}$ and the efficient dominating number is 1.35.

4. Conclusion

In this paper, we study efficient domination set using the concept degree of the vertex in fuzzy graph. We define the efficient domination number γ (G) of intuitionistic fuzzy graphs. Also we disuse these efficient domination in various operations on fuzzy graphs like join, direct product, semi product, Cartesian product and composition, Also we explain the results with suitable examples. Further we study the efficient domination in intuitionistic fuzzy graphs.

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