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ON THE CENTROIDS OF INTUITIONISTIC FUZZY NUMBER AND ITS APPLICATION TO IFLP PROBLEMS

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Abstract This paper proposes a new method of Ranking Intuitionistic Fuzzy Number. A Centroid based distance method for ranking Intuitionistic Fuzzy Numbers (IFN) is introduced, where the IFNs are ranked in terms of Euclidean distances from the Centroid point to the Origin. Definition and arithmetic operations of Genralized Trapezoidal Intuitionistic Fuzzy Number are dicussed. Based on this new approach, we solve Intuitionistic Fuzzy Linear Programming Problem (IFLPP) with a numerical example.

Keywords : Fuzzy Number, Intuitionistic Fuzzy Number, Intuitionistic Fuzzy Linear Programming Problem, centroids of Intuitionistic Fuzzy Numbers.

1. Introduction

In operations research, linear programming is a one of the most important technique. In our daily life moments, we frequently deal with vague or imprecise information. Sometimes information available is inexact or insufficient. Vagueness are usually expressed by linguistic terms fuzzy numbers or intuitionistic fuzzy numbers (IFN). Prof. Zadeh proposed [14] the fuzzy set theory in1965. After the introduction of the concept of fuzzy sets, Atanassov [1] proposed a generalization of the notion of fuzzy set which was composed the membership degree, non-membership degree and Hesitation degree of a element in a set. In practice, it is realized that human expressions like perception, knowledge and behavior are represented by Intuitionistic fuzzy sets rather than fuzzy sets. Intuitionistic fuzzy set is applied in many fields such as medical diagnosis, decision making and logic programming etc. Banarjee and Roy [2] generalized the application of the intuitionistic fuzzy optimization in the constrained multi objective stochastic inventory model. Nehi [5] proposed a new ranking method in which the values are defined by the integral of the inverse fuzzy membership and non-membership functions. Arithmetic operations of Triangular Intuitionistic fuzzy number are introduced by D.F.Li [6] and he proposed a ratio ranking method to rank the Triangular Intuitionistic fuzzy numbers. Dipti Dubey [4] solved the Linear programming with Triangular Intuitionistic fuzzy numbers in which Triangular Intuitionistic fuzzy numbers are converted to crisp set and solved. Basic arithmetic operations of Intuitionistic fuzzy number such as addition and multiplication are defined by G.S.Mahapatra [7]. V.L.G.Nayagam [11] proposed modified ranking of IFN which has been applied for clustering problems.Wang [12] presented a correct Centroid formula for fuzzy numbers. Our aim is to propose a new ranking based on centroids of IFN.

The organization of the paper is as follows: in section 2 the concept of IFN is discussed and we propose a definition for Generalized Trapezoidal Intuitionistic fuzzy number, Subtraction and Division of Trapezoidal Intuitionistic fuzzy number are discussed. In section 3 we propose a new ranking method for IFN (Both TrIFN and TIFN). A Centroid based distance method is introduced. Section 4 defines the concepts of IFLPP and the solution procedure is explained through the Algorithm. Based on this new approach a Numerical example is illustrated in section 5. Section 6 concludes the paper.

2. Definitions and Operations

Definition 2.1 The fuzzy number $\tilde{A}$ is a convex fuzzy subset in R with its membership function $\mu_{\tilde{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x)=\begin{cases} \mu^{L}_{\tilde{A}}(x) \quad , & if\ a_1 \le x \le a_2 \\ w & if\ a_2 \le x \le a_3 \\ \mu^{R}_{\tilde{A}}(x), & if\ a_3 \le x \le a_4 \\ 0 & Otherwise \end{cases}$$

where a_1,a_2,a_3,a_4 are real numbers and $\mu^{L}_{\tilde{A}}$ and $\mu^{R}_{\tilde{A}}$ are continous function from $R \to [0,w], 0 \le w \le 1$.

Now we propose a new definition for Generalized Trapezoidal Intuitionistic Fuzzy Number.

Definition 2.2 A Generalized Trapezoidal Intuitionistic Fuzzy Number

$$\tilde{A}^{I}=[(a_1,a_2,a_3,a_4;w)(a_1',a_2,a_3,a_4';u)]$$

in R with membership function and non membership function is defined as follows:

$$\mu_{\tilde{A}}(x)=\begin{cases} \dfrac{w(x-a_1)}{a_2-a_1} & if\ a_1 \le x \le a_2 \\ w & if\ a_2 \le x \le a_3 \\ \dfrac{w(x-a_4)}{a_3-a_4} & if\ a_3 \le x \le a_4 \\ 0 & Otherwise \end{cases}$$

and

$$\nu_{\tilde{A}}(x)=\begin{cases} \dfrac{u(a_2-x)}{a_2-a_1'} & if\ a_1' \le x \le a_2 \\ 0 & if\ a_2 \le x \le a_3 \\ \dfrac{u(x-a_3)}{a_4'-a_3} & if\ a_3 \le x \le a_4' \\ u & Otherwise \end{cases}$$

where $a_1' \le a_1 \le a_2 \le a_3 \le a_4 \le a_1'$. The values w and u represent the maximum degree of membership and minimum degree of non membership function respectively satisfying the conditions $0 \le w \le 1,\ 0 \le u \le 1$ and $0 \le w + u \le 1.$

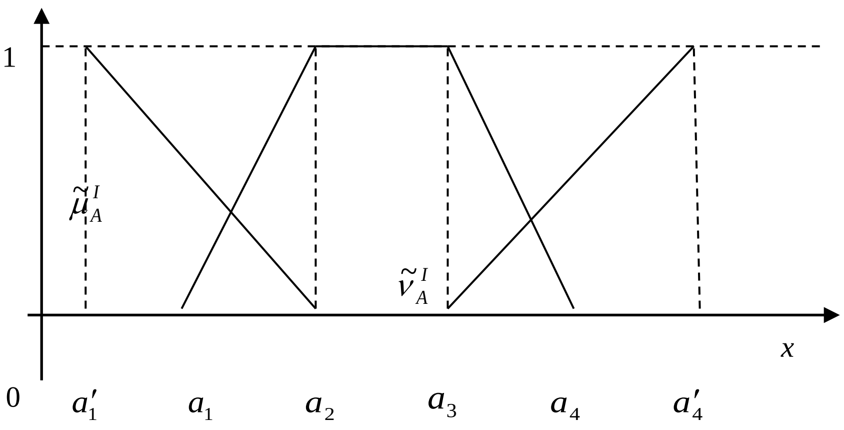

Fig. 1. Membership and non-membership function of Trapezoidal IFN.

Arithmetic Operations on Trapezoidal Intuitionistic Fuzzy Number

If $\tilde{A}^I = (a_1, a_2, a_3, a_4; w_a)(a_1', a_2, a_3, a_4'; u_a)$ and $\tilde{B}^I = (b_1, b_2, b_3, b_4; w_b)(b_1', b_2, b_3, b_4'; u_b)$ are two TrIFN. Then

$$\tilde{A}^I \oplus \tilde{B}^I = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; \min(w_a, w_b))(a_1' + b_1', a_2 + b_2, a_3 + b_3, a_4' + b_4'; \max(u_a, u_b))$$

$$\tilde{A}^I \otimes \tilde{B}^I = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4; \min(w_a, w_b))(a_1' b_1', a_2 b_2, a_3 b_3, a_4' b_4'; \max(u_a, u_b))$$

If $\tilde{A}^I = (a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4')$ is a TrIFN and y=ka with (k>0) then $\tilde{y}^I = k\tilde{A}^I$ is TrIFN

$$(ka_1, ka_2, ka_3, ka_4; \quad ka_1', ka_2, ka_3, ka_4')$$

If y=ka with (k<0) then $\tilde{y}^I = k\tilde{A}^I$ is TrIFN

$$(ka_4, ka_2, ka_3, ka_1; \quad ka_4', ka_2, ka_3, ka_1')$$

V) $$\tilde{A}^I - \tilde{B}^I = \begin{pmatrix} a_1 - b_4, a_2 - b_2, a_3 - b_3, a_4 - b_1; \min(w_a, w_b)) \\ (a_1' - b_4', a_2 - b_2, a_3 - b_3, a_4' - b_1'; \max(u_a, u_b)) \end{pmatrix}$$

$$\tilde{A}^I / \tilde{B}^I = \begin{pmatrix} a_1 / b_4, a_2 / b_2, a_3 / b_3, a_4 / b_1; \min(w_a, w_b)) \\ (a_1' / b_4', a_2 / b_2, a_3 / b_3, a_4' / b_1'; \max(u_a, u_b)) \end{pmatrix}$$

Example 2.3 Let $\tilde{A}^I = (2, 4, 5, 9; 0.2)(1, 4, 5, 11; 0.3)$ and $\tilde{B}^I = (5, 10, 10.5, 18; 0.4)(3, 10, 10.5, 24; 0.5)$. Then

$$\tilde{A}^I - \tilde{B}^I = (-16, -6, -5.5, 4; 0.2)(-23, -6, -5.5, 8; 0.5)$$

$$\tilde{A}^I / \tilde{B}^I = (0.1, 0.4, 0.47, 1.8; 0.2)\ (0.04, 0.4, 0.47, 3.7; 0.5).$$

3. Proposed Ranking Method

To determine the Centroid of a Trapezoidal Number $\tilde{A}^I = (a_1, a_2, a_3, a_4; w)(a_1', a_2, a_3, a_4'; u)$ geometrically:

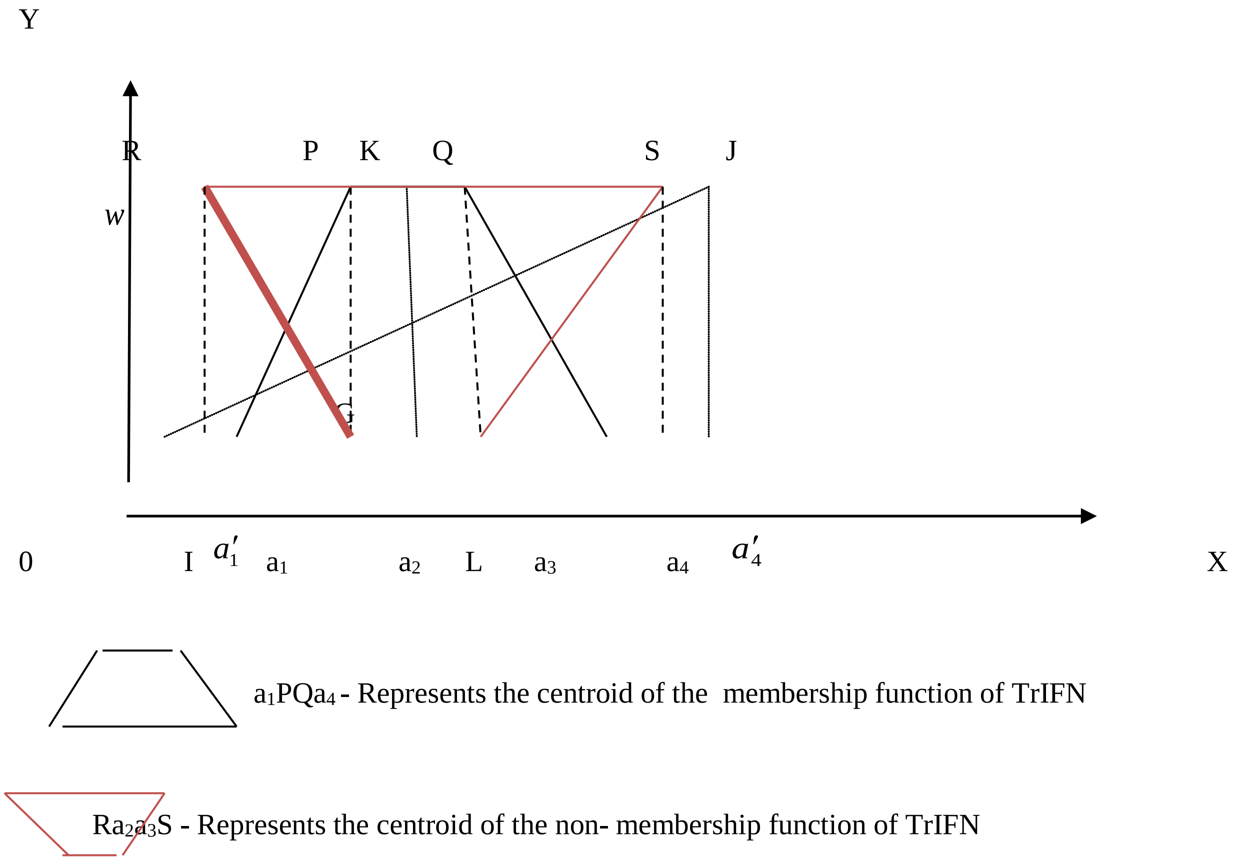

Fig 2: Centroid of a Trapezoid

Figure 2 shows the Geometric representation of Graph $\tilde{A}^I$. In the Trapezoid a_1PQa_4, two lines IJ and KL intersect at a point G, which is the center of gravity of Trapezoid. From these two lines, we can determine the co-ordinates of the point G. The co-ordinate points of I, J, K and L are defined as follows:

$$I\left(a_1+a_2-a_3,0\right),\ \ J\left(a_3+a_4-a_1,w\right),\ \ K\left(\frac{a_2+a_3}{2},w\right) and\ \ L\left(\frac{a_1+a_4}{2},0\right)$$

The equations of IJ and KL are defined as follows [16]:

$$\overrightarrow{IJ}:\ y=\frac{w\left(x-\left(a_1+a_2-a_3\right)\right)}{\left(a_3+a_4-a_1\right)-\left(a_1+a_2-a_3\right)}$$

$$\overrightarrow{KL}:\ y=\frac{w\left(a_1+a_4-2x\right)}{\left(a_1+a_4\right)-\left(a_2+a_3\right)}$$

Let $\frac{w\left(x-\left(a_1+a_2-a_3\right)\right)}{\left(a_3+a_4-a_1\right)-\left(a_1+a_2-a_3\right)}=\frac{w\left(a_1+a_4-2x\right)}{\left(a_1+a_4\right)-\left(a_2+a_3\right)}$. It follows that

$$x\left(\tilde{A}_\mu^I\right)=\frac{1}{3}\left[a_1+a_2+a_3+a_4-\frac{a_4a_3-a_1a_2}{\left(a_4+a_3\right)-\left(a_1+a_2\right)}\right]$$

$$y\left(\tilde{A}_\mu^I\right)=w\frac{1}{3}\left[1+\frac{a_3-a_2}{\left(a_4+a_3\right)-\left(a_1+a_2\right)}\right]$$

Definition 3.1 The Ranking for the membership function of Trapezoidal IFN is defined as

$$R\left(\tilde{A}_\mu^I\right)=\sqrt{x\left(\tilde{A}_\mu^I\right)^2+y\left(\tilde{A}_\mu^I\right)^2}$$

If $a_2 = a_3$ in trapezoidal Intuitionistic Fuzzy Number $\tilde{A}^I$, Then it is called as Triangular Intuitionistic Fuzzy Number. Its Centroid of membership function of TIFN is determined by

$$x\left(\tilde{A}_\mu^I\right) = \frac{1}{3}\left[a_1 + a_2 + a_4\right] \text{ and } y\left(\tilde{A}_\mu^I\right) = \frac{1}{3}$$

Similarly for the trapezoid Ra_2a_3S, we can define the co-ordinate points of I, J, K and L are defined as follows:

$$I\left(a_1' + a_2 - a_3, 0\right),\quad J\left(a_3 + a_4' - a_1', w\right),\quad K\left(\frac{a_2 + a_3}{2}, w\right) \text{and } L\left(\frac{a_1' + a_4'}{2}, 0\right)$$

The equations of IJ and KL are defined as follows :

$$\xrightarrow[IJ]{} : \; y = \frac{w\left(x - \left(a_1' + a_2 - a_3\right)\right)}{\left(a_3 + a_4' - a_1'\right) - \left(a_1' + a_2 - a_3\right)}$$

$$\xrightarrow[KL]{} : \; y = \frac{w\left(a_1' + a_4' - 2x\right)}{\left(a_1' + a_4'\right) - \left(a_2 + a_3\right)}$$

Let $\frac{w\left(x - \left(a_1' + a_2 - a_3\right)\right)}{\left(a_3 + a_4' - a_1'\right) - \left(a_1' + a_2 - a_3\right)} = \frac{w\left(a_1' + a_4' - 2x\right)}{\left(a_1' + a_4'\right) - \left(a_2 + a_3\right)}$. It follows that

$$x\left(\tilde{A}_\nu^I\right) = \frac{1}{3}\left[a_1' + a_2 + a_3 + a_4' - \frac{a_4'a_3 - a_1'a_2}{\left(a_4' + a_3\right) - \left(a_1' + a_2\right)}\right]$$

$$y\left(\tilde{A}_\nu^I\right) = (1-u)\frac{1}{3}\left[1 + \frac{a_3 - a_2}{\left(a_4' + a_3\right) - \left(a_1' + a_2\right)}\right]$$

Definition 3.2 The Ranking for the non- membership function of Trapezoidal IFN is defined as

$$R\left(\tilde{A}_\nu^I\right) = \sqrt{x\left(\tilde{A}_\nu^I\right)^2 + y\left(\tilde{A}_\nu^I\right)^2}$$

If $a_2 = a_3$ in trapezoidal Intuitionistic Fuzzy Number $\tilde{A}^I$, Then it is called as Triangular Intuitionistic Fuzzy Number. Its Centroid of non-membership function of TIFN is determined by

$$x\left(\tilde{A}_\nu^I\right) = \frac{1}{3}\left[a_1' + a_2 + a_4'\right] \text{ and } y\left(\tilde{A}_\nu^I\right) = \frac{1}{3}$$

Definition 3.3 Proposed Ranking function of the Intuitionistic Fuzzy Number is defined as

$$R\left(\tilde{A}\right) = \frac{R\left(\tilde{A}_\mu^I\right)}{1 + R\left(\tilde{A}_\nu^I\right)}$$

Let $\tilde{A}_1^I$ and $\tilde{A}_2^I$ are two Intuitionistic fuzzy numbers, then

If $R\left(\tilde{A}_1^I\right) > R\left(\tilde{A}_2^I\right)$, then $\tilde{A}_1^I > \tilde{A}_2^I$

If $R\left(\tilde{A}_1^I\right) < R\left(\tilde{A}_2^I\right)$, then $\tilde{A}_1^I < \tilde{A}_2^I$.

4. Intuitionistic Fuzzy Linear Programming

LPP: Consider the crisp LPP

Max $\sum_{j=1}^{n} c_j x_j$

Subject to the constraints

$$\sum_{j=1}^{n} a_{ij} x_j = b_i , \qquad i = 1,2,...,m$$
$$x_j \geq 0 \tag{1}$$

where c_j is an $m \times n$ real matrix.

FLPP: Fuzzy Linear Programming Problem can be defined as

Max $\sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j$

Subject to the constraints

$$\sum_{j=i}^{n} \tilde{a}_{ij} \tilde{x}_j = \tilde{b}_i \qquad i = 1,2,...,m$$
$$\tilde{x}_j \geq 0 \tag{2}$$

where $\tilde{c}_j , \tilde{a}_{ij}$ are fuzzy numbers

IFLPP: Corresponding IFLPP is defined as

Max $\sum_{j=1}^{n} \tilde{c}_j^I \tilde{x}_j^I$

Subject to the constraints

$$\sum_{j=i}^{n} \tilde{a}_{ij}^I \tilde{x}_j^I = \tilde{b}_i^I \qquad i = 1,2,...,m$$
$$\tilde{x}_j^I \geq 0 \tag{3}$$

where $\tilde{c}_j^I = \left\{ \left(c_{1j}, c_{2j}, c_{3j}, c_{4j};\ c'_{1j}, c_{2j}, c_{3j}, c'_{4j} \right) \right\}$, $\tilde{b}_i^I = \left(b_{1i}, b_{2i}, b_{3i}, b_{4i};\ b'_{1i}, b_{2i}, b_{3i}, b'_{4i} \right)$ and

$\tilde{a}_{ij}^I = \left(a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij};\ a'_{1ij}, a_{2ij}, a_{3ij}, a'_{4ij} \right)$, i=1………… m, j=1-------n are Trapezoidal Intuitionistic Fuzzy Numbers.

Definition 4.1 A set of variables $\tilde{x}_i^I = \left\langle x_1, x_2, x_3, x_4;\ x'_1, x_2, x_3, x'_4 \right\rangle$ satisfies the above constraints of a general IFLPP is called a Intuitioistic fuzzy solution(IFS).

Definition 4.2 Any solution to a general FLPP which also satisfies the non negative restriction of the problem, is a called a Intuitioistic Fuzzy Feasible solution(IFFS).

Definition 4.3 An Intuitionistic Fuzzy feasible solution to a IFLPP which is also a basic solution to the problem is called a Intuitioistic Fuzzy Basic Feasible solution(IFBFS) to the IFLPP.

Definition 4.4 An Intuitionistic Basic Fuzzy feasible solution that also optimizes the objective function is called Intuitioistic Fuzzy Optimum Feasible solution(IFOFS).

Example 4.5 Algorithm to solve Intuitionistic Fuzzy Linear Programming Problem:

We propose a new algorithm to solve IFLPP as follows:

Step1: Take all the values as Trapezoidal Intuitionistic Fuzzy numbers. Apply simplex method procedure to obtain initial IFBFS.

Step2: To find most negative of $\tilde{z}_j^I - \tilde{c}_j^I$, we use ranking method based on centroid formula (4.3). Taking the minimum value of $R(\tilde{A}_l^I)$ and which enters the basis $\tilde{y}_b$.

Step 3: Compute $\left\{ \frac{\tilde{x}^I_{Bi}}{\tilde{y}^I_{ir}}, i = 1, 2, ,, , m \right\}$ and choose minimum of them using ranking method. Then the vector $\tilde{y}_k$ will leave the basis . This element is called Trapezoidal Intuitionistic Fuzzy pivotal number or leading element.
Step 4: Convert pivotal into unit Trapezoidal Intuitionistic Fuzzy Number and all other elements in its column to zero Trapezoidal Intuitionistic Fuzzy Number using the arithmetic operations defined in chapter 2.
Step 5: Repeat the procedure until an Intuitioistic Fuzzy Optimum Feasible solution is obtained.

5. Numerical Example

We illustrate this method with a numerical example.
Solve the following IFLPP:

$$\text{Max } Z = (3, 4, 5, 5.5; 0.3)\ (2, 4, 5, 6; 0.6)\ \tilde{x}^I_1 + \left(3.5, 4, 4.5, 6; 0.3\right)\ \left(3, 4, 4.5, 7; 0.6\right) \tilde{x}^I_2$$

Subject to the constraints

$$\left(1, 1.8, 2, 3; 0.4\right)\ \left(0.5, 1.8, 2, 4; 0.6\right) \tilde{x}^I_1 + \left(2, 3, 5, 5.8; 0.4\right)\ \left(1, 3, 5, 6; 0.5\right) \tilde{x}^I_2 \le \left(10, 11, 13, 15; 0.3\right)\ \left(9, 11, 13, 17; 0.6\right)$$

$$\left(4, 4.5, 5, 8; 0.7\right) \left(3, 4.5, 5, 9; 0.2\right) \tilde{x}^I_1 + \left(3.8, 4, 6, 9; 0.7\right) \left(3, 4, 6, 10; 0.2\right) \tilde{x}^I_2 \le \left(11, 12, 15, 17; 0.6\right) \left(8, 12, 15, 19; 0.3\right)$$

$$\tilde{x}^I_1, \tilde{x}^I_2 \quad \ge 0$$

The initial feasible solution is given in table I

Table I

C_B	Y_B	X_B	Y_1	Y_2	S_1	S_2
(0,0,0,0;0.3) (0,0,0,0;0.6)	S_1	(10,11,13,15;0.3) (9,11,13,17; 0.6)	(1,1.8,2,3;0.4) (0.5,1.8,2,4;0.6)	(2,3,5,5.8;0.4) (1,3,5,6;0.5)	(1,1,1,1;0.3) (1,1,1,1;0.6)	(0,0,0,0;0.3) (0,0,0,0;0.6)
(0,0,0,0;0.3) (0,0,0,0;0.6)	S_2	(11,12,15,17;0.6) (8,12,15,19;0.3)	(4,4.5,5,8;0.7) (3,4.5,5,9;0.2)	(3.8,4,6,9;0.7) (3,4,6,10;0.2)	(0,0,0,0;0.3) (0,0,0,0;0.6)	(1,1,1,1;0.3) (1,1,1,1;0.6)
$\tilde{Z}^I_j - \tilde{C}^I_j$			(-5.5,-4,-5,-3;0.3) (-6,-4,-5,-2;0.6)	(-8,-4.5,-5,-4;0.3) (-9,-4.5,-5,-3;0.6)	(0,0,0,0; 0.3) (0,0,0,0;0.6)	(0,0,0,0; 0.3) (0,0,0,0;0.6)

The most negative occurs at Y_1. We use centroid formula to calculate the most negative among $\tilde{Z}^I_j - \tilde{C}^I_j$.

(1, 1.8, 2, 3; 0.3) (0.5, 1.8, 2, 4; 0.6) is the leading element. Convert the leading element to unity and the reaming elements in the column to zero using the above defined arithmetic operations.
To find the leaving variable, We use our proposed ranking.

Min $\left\{ \frac{\tilde{x}^I_{B1}}{\tilde{Y}^I_{11}}, \frac{\tilde{x}^I_{B1}}{\tilde{Y}^I_{21}} \right\}$ = Min {6.055, 6.558} = 6.055 occurs at S_1.

Therefore S_1 leaves the basis and Y_2 enters the basis.

Table II

C_B	Y_B	X_B	Y_1	Y_2	S_1	S_2
(3,4,5,5.5;0.3) (2,4,5,6;0.6)	Y_1	(3.3,6.1,6.5,15; 0.3) (2.3,6.1,6.5,34;0.6)	(0.3,1,1,3; 0.3) (0.13,1,1,8;0.6)	(0.7,1.6,2.5,5.8;0.3) (0.3,1.6,2.5,12;0.6)	(0.3,0.5,0.56,1; 0.3) (0.3,0.5,0.56,2;0.6)	(0,0,0,0; 0.3) (0,0,0,0;0.6)
(0,0,0,0; 0.3) (0,0,0,0;0.6)	S_2	(-3.8,15.5,17.5,109; 0.3) (-12.3,15.5,17.5,298;0.6)	(-6.8,0,0,20;0.3) (-8.7,0,0,69;0.6)	(-6.2,2.8,8,42.6; .3) (-9.2,2.8,8,105;0.6)	(1.2,2.3,2.8,8; 0.3) (0.8,2.3,2.8,18;0.6)	(1,1,1,1; 0.3) (1,1,1,1;0.6)
$\tilde{Z}^I_j - \tilde{C}^I_j$			(-4.6,0,0,13.5;0.3) (-5.7,0,0,46;0.6)	(-3.9,2.4,8,28.9;0.3) (-6.5,2.4,8,69;0.6)	(0.9,2,2.8,5.5;0.3) (0.5,2,2.8,12;0.6)	(0,0,0,0;0.3) (0,0,0,0;0.6)

Here all the $\tilde{Z}_j^I - \tilde{C}_j^I \geq 0$ and R($\tilde{Z}_j^I - \tilde{C}_j^I$) $\geq$ 0, the current Intuitionistic Fuzzy Basic Feasible solution is obtained.
The Intuitionistic Fuzzy Optimum Basic Feasible solution is
Maximum Z = (9.9, 24.4, 32.5, 82.5; 4.5, 24.4, 32.5, 204)
At X_1 = (3.3, 6.1, 6.5, 15; 2.25, 6.1, 6.5, 34) and X_2 = (0, 0, 0, 0; 0, 0, 0, 0)

6. Conclusion

This paper proposes a new method of ranking Intuitionistic Fuzzy Numbers which is simple and compact. We have presented the centroid formula for IFN from the view point of analytical geometry. We have proposed the definition for generalized Intuitionistic Fuzzy number, Subtraction and division of Trapezoidal Intuitionistic Fuzzy Number. The solution methodology is illustrated through a numerical example. Thus the method is very useful in the real world problems where the product is uncertain. By increasing the averaged flow ratedecreases.Best pumping can be seen at higher values of the magnetic field.The frictional forces have an opposite behavior as compared to the pressure rise

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