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NUMERICAL SOLUTION FOR FUZZY DIFFERENTIAL EQUATIONS

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Abstract In this paper, we presents fuzzy number to the numerical solution of fuzzy differential equations. In the first four sections, we recall basic concepts. Solving numerically the fuzzy differential equation by RK method is discussed in section V. An example is given in last section.

Keywords : Fuzzy differential equations, Parallelogram fuzzy number, Runge-Kutta method of order two and three.

1. Introduction

Fuzzy differential equations, which appears in many field. The topics of fuzzy differential equations attracted to make interest for solving problems. Fuzzy differential equations and initial value problems were regularly treated by O. Kaleva in [4], S. Seikkala in [5]

2. Preliminaries

A Parallelogram fuzzy number u is defined by four real numbers a < b < c < d, where the base of the parallelogram is the interval [a, d] and its vertices at x = b, x = c. The membership function is defined as follows:

$$u(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \dfrac{x-d}{c-d}, & c \le x \le d \end{cases} \tag{1}$$

We denote by $[u]_r = [\underline{u}(r), \overline{u}(r)]$.

We will have:

u > 0 if a > 0; u > 0 if b> 0; u > 0 if c > 0; & u > 0 if d > 0.

We denote by $[u]_r = [\underline{u}(r), \overline{u}(r)]$. It is clear that the following statements are true,

1. $\underline{u}$ (r) is a bounded left continuous non decreasing function over [0,1],
2. $\overline{u}$ (r) is a bounded right continuous non increasing function over [0,1],

3. $\underline{u}(r) \le \overline{u}(r)$ for all $r \in (0,1]$,

for more details see [2], [3].

3. Fuzzy initial value problem

Consider a fuzzy differential equation,

$$\begin{cases} y'(t) = f(t, y(t)), & t \in [t_0, T] \\ y(t_0) = y_0 \end{cases} \tag{2}$$

where y is a fuzzy function of t, $f(t, y)$ is a fuzzy function of the crisp variable t and the fuzzy variable y, y' is the fuzzy derivative of y and $y(t_0) = y_0$ is a parallelogram or a parallelogram shaped fuzzy number.

We denote the fuzzy function y by $y = [\underline{y}, \overline{y}]$. It means that the r-level set of $y(t)$ for $t \in [t^0, T]$ is

$$[y(t)]^r = [\underline{y}(t; r), \overline{y}(t; r)],$$

$$[y(t_0)]^r = [\underline{y}(t^0; r), \overline{y}(t^0; r)], \ r \in (0,1]$$

and also

$$\underline{f}(t, y) = F[t, \underline{y}, \overline{y}],$$

$$\overline{f}(t, y) = G[t, \underline{y}, \overline{y}].$$

4. Runge-kutta method

Generally,

$$\begin{cases} y'(t) = f(t, y(t)), & t \in [t_0, T] \\ y(t_0) = y_0 \end{cases} \tag{3}$$

It is known that, the sufficient conditions for the existence of a unique solution to (3) are that f to be continuous function satisfying the Lipchitz condition.

We have an interval,

$$h = \frac{T - t_0}{N}, \ t_i = t_0 + ih, \ i = 0,1,\ldots,N$$

Finally, Runge-Kutta method of order two is defined as,

$$y(t_{n+1}) = y(t_n) + 1/2(K_1 + K_2)$$

where,

$$\begin{aligned} K_1 &= hf(t_n, y(t_n)) \\ K_2 &= hf(t_{n+1}, \underline{y}(t_n) + K_1) \end{aligned} \tag{4}$$

Similarly, we can defined of order three.

5. Results

Solutions are obtained by Runge-Kutta method are plotted.

Example 5.1 Consider,

$$\begin{cases} y'(t) = f(t), & t \in [0,1] \\ y(0) = (0.8 + 0.125r \ , \ 0.95 + 0.125r). \end{cases}$$

The exact solution at $t = 1$,

$$Y(1; r) = \left[(0.8 + 0.125r)e \ , \ (0.95 + 0.125r)e\right] \ , \quad 0 \le r \le 1.$$

Using iterative solution,

$\underline{y}(0;r) = 0.8 + 0.125r$, $\overline{y}(0;r) = 0.95 + 0.125r$, $h = \frac{1}{N}$.

where $$\underline{y}^j(t_{i+1};r) = \underline{y}(t_i;r) + \frac{1}{2}[K_1 + K_2],$$

and $$K_1 = h\underline{y}(t_i;r)$$

$$K_2 = h[\underline{y}(t_i;r) + K_1]$$

where $$\overline{y}^j(t_{i+1};r) = \overline{y}(t_i;r) + \frac{1}{2}[K_1 + K_2],$$

$$K_1 = h\overline{y}(t_i;r)$$

$$K_2 = h[\overline{y}(t_i;r) + K_1]$$

Similarly, we can defined of order three.

r	Exact solution
0	2.174625 , 2.582368
0.2	2.242583 , 2.650325
0.4	2.310540 , 2.718282
0.6	2.378497 , 2.786239
0.8	2.446454 , 2.854196
1	2.514411 , 2.922153

	RK method of order two
	Approximated solution
h r	0.1
0	1.958172,2.325330
0.2	2.019365,2.386523
0.4	2.080558,2.447715
0.6	2.141751,2.508909
0.8	2.202944,2.570101
1	2.264137,2.631294
h r	0.01
0	2.174514,2.582236
0.2	2.242468,2.650189
0.4	2.310421,2.718143
0.6	2.378375,2.786096
0.8	2.446329,2.854050
1	2.514282,2.922003

RK method of order three	
RK method of order three	
Approximated solution	
hr	0.1
0	1.958468,2.325680
0.2	2.019670,2.386883
0.4	2.080872,2.448085
0.6	2.142074,2.509287
0.8	2.203276,2.570489
1	2.264478,2.631691
h r	0.01
0	2.174515,2.582236
0.2	2.242468,2.650190
0.4	2.310422,2.718143
0.6	2.378375,2.786097
0.8	2.446329,2.854050

1	2.514283,2.922004

Graphical representation of Exact and Approximate Solutions

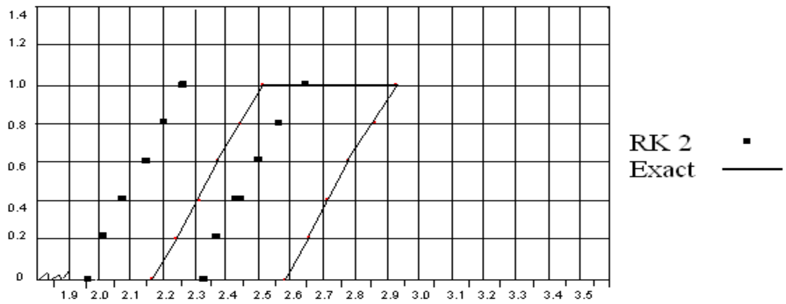

Figure 1 : h = 0.1

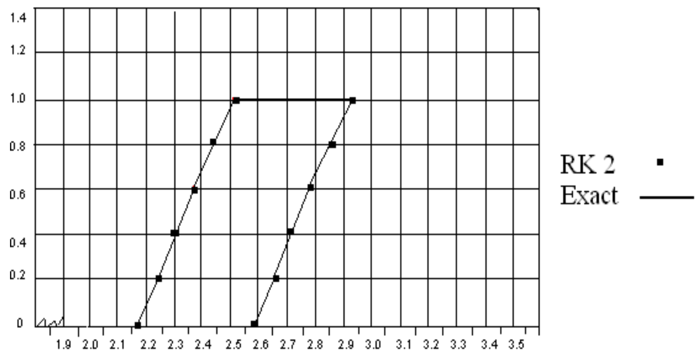

Figure 2 : h = 0.01

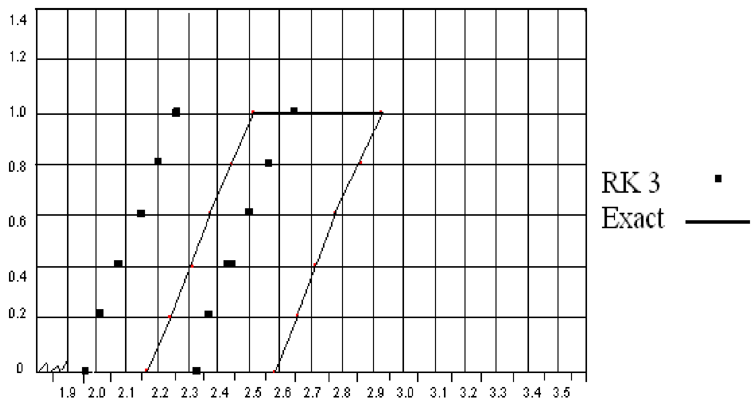

Figure 3 : h = 0.1

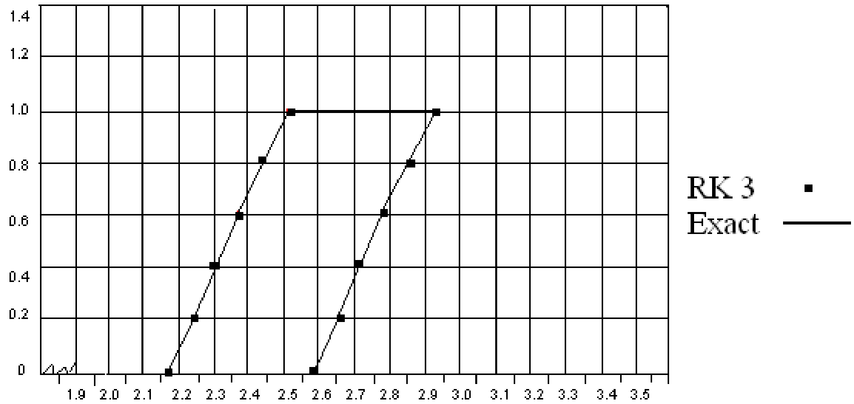

Figure 4: h = 0.01

6. Conclusion

By minimizing the step size *h*, the solution by exact method and Runge-Kutta method almost coincides. The Runge-Kutta method of order three gives better result.

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