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A STUDY ON INTERSECTION OF TWO INTUITIONISTIC Q-FUZZY SOFT SUBHEMIRING OF A HEMIRING USING HOMOMORPHISM AND ANTI-HOMOMORPHISM

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ABSTRACT. We present a short diagram on intuitionistic RQ-fuzzy sets which cuts over a few definitions. We introduce a new kind of hemiring called soft subhemiring with homomorphism and anti - homomorphism and obtain some related properties. Finally we describe intersection of two intuitionistic Q-fuzzy soft subhemiring of a hemiring using homomorphism and anti-homomorphism.

1. INTRODUCTION

Hemirings which are regarded as a generalization of rings have been found useful in solving problems in different areas of applied mathematics and computer sciences. So many researchers have studied different aspects of hemirings. The notion of a fuzzy set was introduced L.A. Zadeh [5] and since then this has been applied to various algebraic structures. The idea of an intuitionistic fuzzy set was introduced by K.T. Atanassov [3] as a generalization of the notion of fuzzy set. J. S. Golan quoted some special classes of hemirings such as idempotent hemiring, zero sum free hemiring, and regular hemiring and so on. After the introduction of fuzzy sets by

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L.A.Zadeh, [5], several researchers explored on the generalization of the concept of fuzzy sets. The notion of anti-fuzzy left h-ideals in hemiring was introduced by Akram. Mand K.H.Dar [1, 2]. The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N.Palaniappan and K.Arjunan [4] In this paper, we introduce the some theorems in intersection of two intuitionistic q-fuzzy soft subhemiring of a hemiring using homomorphism and anti-homomorphism.

2. Preliminaries

Definition 2.1. *A non-void set $\bar{R}$ together with two binary tasks represented by $+$ and $\cdot$ be addition and multiplication which fulfill the accompanying aphorisms are known as a hemiring.*

(i) $(\bar{R}, +)$ *is a semigroup and commutative with zero,*
(ii) $(\bar{R}, \cdot)$ *is a semigroup,*
(iii) $(a+b)\cdot c = a\cdot c + b\cdot c$ *and* $a\cdot(b+c) = a\cdot b + a\cdot c$, $a, b, c \in \bar{R}$.

Definition 2.2. *A non-void subset S of a hemiring $(\bar{R}, +, \cdot)$ is known as a subhemiring if S itself is a hemiring under the same operation as in R.*

Definition 2.3. *Let $(\bar{R}, +, \cdot)$ and $(\bar{R}', +, \cdot)$ be two hemirings. Then h: $\bar{R} \to \bar{R}'$ is called a hemiring homomorphism if it satisfies the following conditions:*

(i) $h(m+n) = h(m) + h(n)$,
(ii) $h(mn) = h(m)\cdot h(n)$, *for all m and n in $\bar{R}$.*

Definition 2.4. *Let $(\bar{R}, +, \cdot)$ and $(\bar{R}', +, \cdot)$ be two hemirings. Then the function f: $\bar{R} \to \bar{R}'$ is called a hemiring anti-homomorphism if it satisfies the following conditions:*

(i) $h(m+n) = h(m) + h(n)$,
(ii) $h(mn) = h(m)\cdot h(n)$, *for all m and n in $\bar{R}'$.*

Definition 2.5. *Let $(\bar{R}, +, \cdot)$ be a hemiring. A fuzzy soft subset (F, A) of $\bar{R}$, is said to be a fuzzy soft subhemiring $\bar{R}$ of if it satisfies the following conditions:*

(i) $\eta_{(F,A)}(m_{(F,A)} + n_{(F,A)}) \geq$ *min* $\left\{\eta_{(F,A)}(m_{(F,A)}), \eta_{(F,A)}(n_{(F,A)})\right\}$,
(ii) $\eta_{(F,A)}(m_{(F,A)} n_{(F,A)}) \geq min \left\{\eta_{(F,A)}(m_{(F,A)}), \eta_{(F,A)}(n_{(F,A)})\right\}$,
for all $m_{(F,A)}$ and $n_{(F,A)}$ in $\bar{R}$.

Definition 2.6. *Let $\bar{R}$ be a hemiring. An intuitionistic fuzzy soft subset (F,A) of $\bar{R}$ is said to be an intuitionistic fuzzy soft subhemiring (IFSSHR) of $\bar{R}$ on the off chance that it fulfills the accompanying conditions:*

(i) $\eta_{(F,A)}(m_{(F,A)} + n_{(F,A)}) \geq min\left\{\eta_{(F,A)}(m_{(F,A)}), \eta_{(F,A)}(n_{(F,A)})\right\}$,
(ii) $\eta_{(F,A)}(m_{(F,A)}n_{(F,A)}) \geq min\left\{\eta_{(F,A)}(m_{(F,A)}), \eta_{(F,A)}(n_{(F,A)})\right\}$,
(iii) $\upsilon_{(F,A)}(m_{(F,A)} + n_{(F,A)}) \leq max\left\{\upsilon_{(F,A)}(m_{(F,A)}), \upsilon_{(F,A)}(n_{(F,A)})\right\}$,
(iv) $\upsilon_{(F,A)}(m_{(F,A)}n_{(F,A)}) \leq max\left\{\upsilon_{(F,A)}(m_{(F,A)}), \upsilon_{(F,A)}(n_{(F,A)})\right\}$,
for all $m_{(F,A)}$ *and* $n_{(F,A)}$ *in* $\bar{R}$.

3. Properties of intuitionistic Q-fuzzy soft subhemiring of a hemiring

Theorem 3.1. *Intersection of any two intuitionistic fuzzy soft subhemirings of a hemiring $\bar{R}'$ is a intuitionistic fuzzy soft subhemiring of $\bar{R}$.*

Proof. Let $\{(G,V)_i : i \in I\}$ be a group of intuitionistic fuzzy soft sub hemirings of a hemiring $\bar{R}'$ and let $(F,A) = \bigcap_{(G,V)}$ Let $m_{(F,A)}$ and $n_{(F,A)}$ in $\bar{R}$. At that point,

$$\begin{aligned}
\eta_{(F,A)}(m_{(F,A)} + n_{(F,A)}) &= \inf_{i\in I} \eta_{(G,V)i}(m_{(G,V)} + n_{(G,V)}) \geq \\
&\geq \inf_{i\in I} min\left\{\eta_{(G,V)i}(m_{(G,V)}), \eta_{(G,V)i}(n_{(G,V)})\right\} \\
&= min\left\{\inf_{i\in I} \eta_{(G,V)i}(m_{(G,V)}), \inf_{i\in I} \eta_{(G,V)i}(n_{(G,V)})\right\} \\
&= min\left\{\eta_{(F,A)}(m_{(F,A)}), \eta_{(F,A)}(n_{(F,A)})\right\}
\end{aligned}$$

Subsequently,

$$\eta_{(F,A)}(m_{(F,A)} + n_{(F,A)}) \geq min\left\{\eta_{(F,A)}(m_{(F,A)}), \eta_{(F,A)}(n_{(F,A)})\right\} ,$$

for all $m_{(F,A)}$ and $n_{(F,A)}$ in $\bar{R}$. Furthermore,

$$\begin{aligned}
\eta_{(F,A)}(m_{(F,A)}n_{(F,A)}) &= \inf_{i\in I} \eta_{(G,V)i}(m_{(G,V)}n_{(G,V)}) \geq \\
&\geq \inf_{i\in I} min\left\{\eta_{(G,V)i}(m_{(G,V)}), \eta_{(G,V)i}(n_{(G,V)})\right\} \\
&= min\left\{\inf_{i\in I} \eta_{(G,V)i}(m_{(G,V)}), \inf_{i\in I} \eta_{(G,V)i}(n_{(G,V)})\right\} \\
&= min\left\{\eta_{(F,A)}(m_{(F,A)}), \eta_{(F,A)}(n_{(F,A)})\right\}
\end{aligned}$$

Subsequently,

$$\eta_{(F,A)}(m_{(F,A)}n_{(F,A)}) \geq min\left\{\eta_{(F,A)}(m_{(F,A)}), \eta_{(F,A)}(n_{(F,A)})\right\} ,$$

for all $m_{(F,A)}$ and $n_{(F,A)}$ in $\bar{R}$. Next, we have:

$$\begin{aligned}
\upsilon_{(F,A)}(m_{(F,A)} + n_{(F,A)}) &= \sup_{i\in I} \upsilon_{(G,V)i}(m_{(G,V)} + n_{(G,V)}) \\
&\leq \sup_{i\in I} max\left\{\upsilon_{(G,V)i}(m_{(G,V)}), \upsilon_{(G,V)i}(n_{(G,V)})\right\} \\
&= max\left\{\sup_{i\in I} \upsilon_{(G,V)i}(m_{(G,V)}), \sup_{i\in I} \upsilon_{(G,V)i}(n_{(G,V)})\right\} \\
&= max\left\{\upsilon_{(F,A)}(m_{(F,A)}), \upsilon_{(F,A)}(n_{(F,A)})\right\} .
\end{aligned}$$

Subsequently,

$$\upsilon_{(F,A)}(m_{(F,A)} + n_{(F,A)}) \leq max\left\{\upsilon_{(F,A)}(m_{(F,A)}), \upsilon_{(F,A)}(n_{(F,A)})\right\} ,$$

for all $m_{(F,A)}$ and $n_{(F,A)}$ in $\bar{R}'$. In this manner:

$$\begin{aligned}
\upsilon_{(F,A)}(m_{(F,A)}n_{(F,A)}) &= \sup_{i\in I} \upsilon_{(G,V)i}(m_{(G,V)}n_{(G,V)}) \\
&\leq \sup_{i\in I} max\left\{\upsilon_{(G,V)i}(m_{(G,V)}), \upsilon_{(G,V)i}(n_{(G,V)})\right\} \\
&= max\left\{\sup_{i\in I} \upsilon_{(G,V)i}(m_{(G,V)}), \sup_{i\in I} \upsilon_{(G,V)i}(n_{(G,V)})\right\} \\
&= max\left\{\upsilon_{(F,A)}(m_{(F,A)}), \upsilon_{(F,A)}(n_{(F,A)})\right\} .
\end{aligned}$$

Subsequently,

$$\upsilon_{(F,A)}(m_{(F,A)}n_{(F,A)}) \leq max\left\{\upsilon_{(F,A)}(m_{(F,A)}), \upsilon_{(F,A)}(n_{(F,A)})\right\} ,$$

for all $m_{(F,A)}$ and $n_{(F,A)}$ in $\bar{R}$. That is, (F, A) is an intuitionistic fuzzy soft subhemiring of a hemiring. Consequently, the intersection of a family of intuitionistic fuzzy soft subhemiring of R is an intuitionistic fuzzy soft subhemiring of $\bar{R}$. □

4. Properties of intuitionistic Q-fuzzy soft subhemiring of a hemiring using homomorphism and anti-homomorphism

Theorem 4.1. *Let ($\bar{R}$, +, .) and ($\bar{R}'$, +, .) be two hemirings. The homomorphic image of an intuitionistic fuzzy soft subhemiring of $\bar{R}$ is an intuitionistic fuzzy soft subhemiring of $\bar{R}^{'}$.*

Proof. Let $(\bar{R}, +, \cdot)$ and $(\bar{R}', +, \cdot)$ be two hemirings. Let $\bar{R} \to \bar{R}'$ be a homomorphism. Then,

(i) $h(m+n) = h(m) + h(n)$,
(ii) $h(mn) = h(m) \cdot h(n)$, for all m and n in $\bar{R}'$.

Let $(G, V) = h((F, A))$, where (F, A) is an intuitionistic fuzzy soft subhemiring of $\bar{R}$. We have to prove that (G, V) is an intuitionistic fuzzy soft subhemiring of R'. Now, for $h(m_{(G,V)})$, $h(n_{(G,V)})$ in $\bar{R}^{'}$ we have:

$$\begin{aligned} \eta_{(G,V)}(h(m_{(G,V)}) + h(n_{(G,V)})) &= \eta_{(G,V)}(h(m_{(G,V)} + n_{(G,V)})) \text{, as } h \text{ is a homomorphism} \\ &\geq \eta_{(F,A)}(m_{(F,A)} + n_{(F,A)}) \\ &\geq min\left\{\eta_{(F,A)}(m_{(F,A)}), \eta_{(F,A)}(n_{(F,A)})\right\} \end{aligned}$$

which suggests that:

$$\eta_{(G,V)}(h(m_{(G,V)}) + h(n_{(G,V)})) \geq min\left\{\eta_{(G,V)}(h(m_{(G,V)})), \eta_{(G,V)}(h(n_{(G,V)}))\right\} .$$

Again:

$$\begin{aligned} \eta_{(G,V)}(h(m_{(G,V)})h(n_{(G,V)})) &= \eta_{(G,V)}(h(m_{(G,V)}n_{(G,V)})) \text{, as } h \text{ is a homomorphism} \\ &\geq \eta_{(F,A)}(m_{(F,A)}n_{(F,A)}) \\ &\geq min\left\{\eta_{(F,A)}(m_{(F,A)}), \eta_{(F,A)}(n_{(F,A)})\right\} . \end{aligned}$$

which suggests that:

$$\eta_{(G,V)}(h(m_{(G,V)})h(n_{(G,V)})) \geq min\left\{\eta_{(G,V)}(h(m_{(G,V)})), \eta_{(G,V)}(h(n_{(G,V)}))\right\} .$$

Now, for $h(m), h(n)$ in $\bar{R}'$

$$\begin{aligned} \upsilon_{(G,V)}(h(m_{(G,V)}) + h(n_{(G,V)})) &= \upsilon_{(G,V)}(h(m_{(G,V)} + n_{(G,V)})) \text{, as } h \text{ is a homomorphism} \\ &\leq \upsilon_{(F,A)}(m_{(F,A)} + n_{(F,A)}) \\ &\leq max\left\{\upsilon_{(F,A)}(m_{(F,A)}), \upsilon_{(F,A)}(n_{(F,A)})\right\} , \end{aligned}$$

which suggests that:

$\upsilon_{(G,V)}(h(m_{(G,V)})+h(n_{(G,V)}))\leq max\left\{\upsilon_{(G,V)}(h(m_{(G,V)})),\upsilon_{(G,V)}(h(n_{(G,V)}))\right\}$.

Again

$$\begin{aligned}\upsilon_{(G,V)}(h(m_{(G,V)})h(n_{(G,V)})) &= \upsilon_{(G,V)}(h(m_{(G,V)}n_{(G,V)}))\text{, as } h \text{ is a homomorphism}\\ &\leq \upsilon_{(F,A)}(m_{(F,A)}n_{(F,A)})\\ &\leq max\left\{\upsilon_{(F,A)}(m_{(F,A)}),\upsilon_{(F,A)}(n_{(F,A)})\right\} ,\end{aligned}$$

which suggests that:

$$\upsilon_{(G,V)}(h(m_{(G,V)})h(n_{(G,V)}))\leq max\left\{\upsilon_{(G,V)}(h(m_{(G,V)})),\upsilon_{(G,V)}(h(n_{(G,V)}))\right\} .$$

Hence (G,V) is an intuitionistic fuzzy soft subhemiring of $\bar{R}'$. □

Theorem 4.2. *Let $(\bar{R},+,\cdot)$ and $(\bar{R}',+,\cdot)$ be two hemirings. The homomorphic preimage of an intuitionistic fuzzy soft subhemiring of $\bar{R}'$ is a intuitionistic fuzzy soft subhemiring of $\bar{R}$.*

Proof. Let $(G,V)=h((F,A))$, where (G,V) is an intuitionistic fuzzy soft subhemiring of $\bar{R}'$. We have to prove that (F,A) is an intuitionistic fuzzy soft subhemiring of $\bar{R}$. Let m and n be in $\bar{R}$. Then,

$$\begin{aligned}\eta_{(F,A)}(m_{(F,A)}+n_{(F,A)}) &= \eta_{(G,V)}(h(m_{(F,A)}+n_{(F,A)}))\\ &\quad \text{since } \eta_{(G,V)}(h(m))=\eta_{(F,A)}(m_{(F,A)}=\eta_{(G,V)}(h(m)h(n))\\ &\quad \text{as } h \text{ is a homomorphism}\\ &\geq min\left\{\eta_{(G,V)}(h(m)),\eta_{(G,V)}(h(n))\right\}\\ &= min\left\{\eta_{(F,A)}(m_{(F,A)}),\eta_{(F,A)}(n_{(F,A)})\right\} ,\end{aligned}$$

since $\eta_{(G,V)}(h(m))=\eta_{(F,A)}(m_{(F,A)})$, we have that:

$$\eta_{(F,A)}(m_{(F,A)}+n_{(F,A)})\geq min\left\{\eta_{(F,A)}(m_{(F,A)}),\eta_{(F,A)}(n_{(F,A)})\right\} .$$

Again,

$$\begin{aligned}\eta_{(F,A)}(m_{(F,A)}n_{(F,A)}) &= \eta_{(G,V)}(h(m_{(F,A)}n_{(F,A)}))\\ &\quad \text{since } \eta_{(G,V)}(h(m))=\eta_{(F,A)}(m_{(F,A)}=\eta_{(G,V)}(h(m)h(n))\\ &\quad \text{as } h \text{ is a homomorphism}\\ &\geq min\left\{\eta_{(G,V)}(h(m)),\eta_{(G,V)}(h(n))\right\}\\ &= min\left\{\eta_{(F,A)}(m_{(F,A)}),\eta_{(F,A)}(n_{(F,A)})\right\} ,\end{aligned}$$

since $\eta_{(G,V)}(h(m)) = \eta_{(F,A)}(m_{(F,A)})$, we have that:

$$\eta_{(F,A)}(m_{(F,A)}n_{(F,A)}) \geq min\left\{\eta_{(F,A)}(m_{(F,A)}), \eta_{(F,A)}(n_{(F,A)})\right\} .$$

Let $m_{(F,A)}$ and $n_{(F,A)}$ in $\bar{R}$. Then, $n_{(F,A)}(m_{(F,A)} + n_{(F,A)}) = n_{(G,V)}(h_{(m+n)})$, since $n_{(G,V)}(h(m)) = n_{(F,A)}(m_{(F,A)}) = n_{(G,V)}(h(m) + h(n))$, as h is a homomorphism

$$\begin{aligned} n_{(F,A)}(m_{(F,A)} + n_{(F,A)}) &= n_{(G,V)}(h_{(m+n)}) \leq max\left\{n_{(G,V)}(h(m)), n_{(G,V)}(h(n))\right\} \\ &= max\left\{n_{(F,A)}(m_{(F,A)}), n_{(F,A)}(n_{(F,A)})\right\} , \end{aligned}$$

since $n_{(G,V)}(h(m)) = n_{(F,A)}(m_{(F,A)})$, which suggests that:

$$n_{(F,A)}(m_{(F,A)} + n_{(F,A)}) \leq max\left\{n_{(F,A)}(m_{(F,A)}), n_{(F,A)}(n_{(F,A)})\right\} .$$

Again, $n_{(F,A)}(m_{(F,A)}n_{(F,A)}) = n_{(G,V)}(h_{(mn)})$, since $n_{(G,V)}(h(m)) = n_{(F,A)}(m_{(F,A)}) = n_{(G,V)}(h(m)h(n))$, as f is a homomorphism:

$$\begin{aligned} n_{(F,A)}(m_{(F,A)}n_{(F,A)}) &= n_{(G,V)}(h_{(mn)}) \leq max\left\{n_{(G,V)}(h(m)), n_{(G,V)}(h(n))\right\} \\ &= max\left\{n_{(F,A)}(m_{(F,A)}), n_{(F,A)}(n_{(F,A)})\right\} \end{aligned}$$

since $n_{(G,V)}(h(m)) = n_{(F,A)}(m_{(F,A)})$, which suggests that:

$$n_{(F,A)}(m_{(F,A)}n_{(F,A)}) \leq max\left\{n_{(F,A)}(m_{(F,A)}), n_{(F,A)}(n_{(F,A)})\right\} .$$

Hence (F, A) is an intuitionistic fuzzy soft subhemiring of $\bar{R}$. □

Theorem 4.3. *Let $(\bar{R}, +, \cdot)$ and $(\bar{R}', +, \cdot)$ be two hemirings. The anti-homomorphic image of an intuitionistic fuzzy soft subhemiring of $\bar{R}$ is an intuitionistic fuzzy soft subhemiring of $\bar{R}'$.*

Proof. Let $(G, V) = h((F, A))$, where (F, A) is an intuitionistic fuzzy soft subhemiring of $\bar{R}$. We have to prove that (G, V) is an intuitionistic fuzzy soft subhemiring of $\bar{R}'$.

Now, for $h(m_{(G,V)}), h(n_{(G,V)})$ in $\bar{R}'$,

$$\eta_{(G,V)}(h(m_{(G,V)}) + h(n_{(G,V)})) = \eta_{(G,V)}(h(n_{(G,V)} + m_{(G,V)})) ,$$

as h is an anti-homomorphism:

$$\begin{aligned} &\geq \eta_{(F,A)}(n_{(F,A)} + m_{(F,A)}) \\ &\geq min\left\{\eta_{(F,A)}(n_{(F,A)}), \eta_{(F,A)}(m_{(F,A)})\right\} \\ &= min\left\{\eta_{(F,A)}(m_{(F,A)}), \eta_{(F,A)}(n_{(F,A)})\right\} \end{aligned}$$

which suggests that:

$$\eta_{(G,V)}(h(m_{(G,V)})+h(n_{(G,V)})) \geq min\left\{\eta_{(G,V)}(h(m_{(G,V)})), \eta_{(G,V)}(h(n_{(G,V)}))\right\} .$$

Again:

$$\eta_{(G,V)}(h(m_{(G,V)})h(n_{(G,V)})) = \eta_{(G,V)}(h(n_{(G,V)}m_{(G,V)})) ,$$

as h is an anti-homomorphism:

$$\begin{aligned} &\geq \eta_{(F,A)}(n_{(F,A)}m_{(F,A)}) \\ &\geq min\left\{\eta_{(F,A)}(n_{(F,A)}), \eta_{(F,A)}(m_{(F,A)})\right\} \\ &= min\left\{\eta_{(F,A)}(m_{(F,A)}), \eta_{(F,A)}(n_{(F,A)})\right\} , \end{aligned}$$

which suggests that:

$$\eta_{(G,V)}(h(m_{(G,V)})h(n_{(G,V)})) \geq min\left\{\eta_{(G,V)}(h(m_{(G,V)})), \eta_{(G,V)}(h(n_{(G,V)}))\right\} .$$

Now, for $h(m_{(G,V)}), h(n_{(G,V)})$ in $\bar{R}'$,

$$\upsilon_{(G,V)}(h(m_{(G,V)})+h(n_{(G,V)})) = \upsilon_{(G,V)}(h(n_{(G,V)}+m_{(G,V)})) ,$$

as h is an anti-homomorphism

$$\begin{aligned} &\leq \upsilon_{(F,A)}(n_{(F,A)}+m_{(F,A)}) \\ &\leq max\left\{\upsilon_{(F,A)}(n_{(F,A)}), \upsilon_{(F,A)}(m_{(F,A)})\right\} \\ &= max\left\{\upsilon_{(F,A)}(m_{(F,A)}), \upsilon_{(F,A)}(n_{(F,A)})\right\} , \end{aligned}$$

which suggests that:

$$\upsilon_{(G,V)}(h(m_{(G,V)})+h(n_{(G,V)})) \leq max\left\{\upsilon_{(G,V)}(h(m_{(G,V)})), \upsilon_{(G,V)}(h(n_{(G,V)}))\right\} .$$

Again,

$$\upsilon_{(G,V)}(h(m_{(G,V)})h(n_{(G,V)})) = \upsilon_{(G,V)}(h(n_{(G,V)}m_{(G,V)})) ,$$

as h is an anti-homomorphism:

$$\begin{aligned} &\leq \upsilon_{(F,A)}(n_{(F,A)}m_{(F,A)}) \\ &\leq max\left\{\upsilon_{(F,A)}(n_{(F,A)}), \upsilon_{(F,A)}(m_{(F,A)})\right\} \\ &= max\left\{\upsilon_{(F,A)}(m_{(F,A)}), \upsilon_{(F,A)}(n_{(F,A)})\right\} , \end{aligned}$$

which suggests that:

$$\upsilon_{(G,V)}(h(m_{(G,V)})h(n_{(G,V)})) \leq min\left\{\upsilon_{(G,V)}(h(m_{(G,V)})), \upsilon_{(G,V)}(h(n_{(G,V)}))\right\} .$$

Hence (G,V) is an intuitionistic fuzzy soft subhemiring of $\bar{R}'$. □

Theorem 4.4. *Let $(\bar{R},+,\cdot)$ and $(\bar{R}',+,\cdot)$ be two hemirings. The anti-homomorphic preimage of an intuitionistic fuzzy soft subhemiring of $\bar{R}'$ is an intuitionistic fuzzy soft subhemiring of $\bar{R}$.*

Proof. Let $(\bar{R},+,\cdot)$ and $(\bar{R}',+,\cdot)$ be two hemirings. Let $h:\bar{R}\to\bar{R}'$ be an anti-homomorphism.

Let $(G,V)=h((F,A))$, where (G,V) is an intuitionistic fuzzy soft subhemiring of $\bar{R}'$. We have to prove that (F,A) is an intuitionistic fuzzy soft subhemiring of $\bar{R}$. Let $m_{(F,A)}$ and $n_{(F,A)}$ be in $\bar{R}$. Then

$$\eta_{(F,A)}(m_{(F,A)}+n_{(F,A)})=\eta_{(G,V)}(h(m_{(G,V)}+n_{(G,V)}))\,,$$

since $\eta_{(G,V)}(h(m_{(G,V)}))=\eta_{(F,A)}(m_{(F,A)})=\eta_{(G,V)}(h(n_{(G,V)})+h(m_{(G,V)}))$, as h is an anti-homomorphism

$$\begin{aligned}
&\geq\; min\left\{\eta_{(G,V)}(h(n_{(G,V)})),\eta_{(G,V)}(h(m_{(G,V)}))\right\}\\
&=\; min\left\{\eta_{(G,V)}(h(m_{(G,V)})),\eta_{(G,V)}(h(n_{(G,V)}))\right\}\\
&=\; min\left\{\eta_{(F,A)}(m_{(F,A)}),\eta_{(F,A)}(n_{(F,A)})\right\}\;,
\end{aligned}$$

since $\eta_{(G,V)}(h(m_{(G,V)}))=\eta_{(F,A)}(m_{(F,A)})$, which suggests that:

$$\eta_{(F,A)}(m_{(F,A)}+n_{(F,A)})\geq min\left\{\eta_{(F,A)}(m_{(F,A)}),\eta_{(F,A)}(n_{(F,A)})\right\}\;.$$

Then, $\eta_{(F,A)}(m(F,A)n_{(F,A)})=\eta_{(G,V)}(h(m_{(G,V)}n_{(G,V)}))$, since $\eta_{(G,V)}(h(m_{(G,V)}))=\eta_{(F,A)}(m_{(F,A)})=\eta_{(G,V)}(h(n_{(G,V)})h(m_{(G,V)}))$, as h is an anti-homomorphism

$$\begin{aligned}
&\geq\; min\left\{\eta_{(G,V)}(h(n_{(G,V)})),\eta_{(G,V)}(h(m_{(G,V)}))\right\}\\
&=\; min\left\{\eta_{(G,V)}(h(m_{(G,V)})),\eta_{(G,V)}(h(n_{(G,V)}))\right\}\\
&=\; min\left\{\eta_{(F,A)}(m_{(F,A)}),\eta_{(F,A)}(n_{(F,A)})\right\}\;,
\end{aligned}$$

since $\eta_{(G,V)}(h(m_{(G,V)}))=\eta_{(F,A)}(m_{(F,A)})$, which suggests that:

$$\eta_{(F,A)}(m_{(F,A)}n_{(F,A)})\geq min\left\{\eta_{(F,A)}(m_{(F,A)}),\eta_{(F,A)}(n_{(F,A)})\right\}\;.$$

Next, $\upsilon_{(F,A)}(m_{(F,A)}+n_{(F,A)})=\upsilon_{(G,V)}(f(m_{(G,V)}+n_{(G,V)}))$, since $\upsilon_{(G,V)}(h(m_{(G,V)}))=\upsilon_{(F,A)}(m_{(F,A)})=\upsilon_{(G,V)}(h(n_{(G,V)})+h(m_{(G,V)}))$, as h is an anti-homomorphism

$$\begin{aligned}
&\leq\; max\left\{\upsilon_{(G,V)}(h(n_{(G,V)})),\upsilon_{(G,V)}(h(m_{(G,V)}))\right\}\\
&=\; max\left\{\upsilon_{(G,V)}(h(m_{(G,V)})),\upsilon_{(G,V)}(h(n_{(G,V)}))\right\}\\
&=\; max\left\{\upsilon_{(F,A)}(m_{(F,A)}),\upsilon_{(F,A)}(n_{(F,A)})\right\}\;,
\end{aligned}$$

since $\upsilon_{(G,V)}(g(m_{(G,V)})) = \upsilon_{(F,A)}(m_{(F,A)})$, which suggests that:

$$\upsilon_{(F,A)}(m_{(F,A)} + n_{(F,A)}) \leq max\left\{\upsilon_{(F,A)}(m_{(F,A)}), \upsilon_{(F,A)}(n_{(F,A)})\right\} .$$

Then $\upsilon_{(F,A)}(m_{(F,A)}n_{(F,A)}) = \upsilon_{(G,V)}(f(m_{(G,V)}n_{(G,V)}))$, since $\upsilon_{(G,V)}(h(m_{(G,V)})) = \upsilon_{(F,A)}(m_{(F,A)}) = \upsilon_{(G,V)}(h(n_{(G,V)})h(m_{(G,V)}))$, as h is an anti-homomorphism

$$\begin{aligned} &\leq max\left\{\upsilon_{(G,V)}(h(n_{(G,V)})), \upsilon_{(G,V)}(h(m_{(G,V)}))\right\} \\ &= max\left\{\upsilon_{(G,V)}(h(m_{(G,V)})), \upsilon_{(G,V)}(h(n_{(G,V)}))\right\} \\ &= max\left\{\upsilon_{(F,A)}(m_{(F,A)}), \upsilon_{(F,A)}(n_{(F,A)})\right\} , \end{aligned}$$

since $\upsilon_{(G,V)}(h(m_{(G,V)})) = \upsilon_{(F,A)}(m_{(F,A)})$, which suggests that:

$$\upsilon_{(F,A)}(m_{(F,A)}n_{(F,A)}) \leq max\left\{\upsilon_{(F,A)}(m_{(F,A)}), \upsilon_{(F,A)}(n_{(F,A)})\right\} .$$

Hence (F, A) is an intuitionistic fuzzy soft subhemiring of $\bar{R}$. □

Theorem 4.5. *Let $(\bar{R}, +, \cdot)$ and $(\bar{R}', +, \cdot)$ be two hemirings. If $h : \bar{R}$ m Q $\rightarrow \bar{R}'$ m Q is a Q-homomorphism, then the Q-homomorphic pre-image of a level subhemiring of a Q- fuzzy soft subhemiring of $\bar{R}'$ m Q is a level subhemiring of a Q- fuzzy soft subhemiring of $\bar{R}$ m Q.*

Proof. Let $(\bar{R}, +, \cdot)$ and $(\bar{R}', +, \cdot)$ be two hemirings and $h : \bar{R}$ m Q $\rightarrow \bar{R}'$ m Q is a Q-homomorphism. That is, $h(m + n, q) = h(m, q) + h(n, q)$, for all m and n in $\bar{R}$ and $h(mn, q) = h(m, q)h(n, q)$, for all m and n in $\bar{R}$.

Let $\eta_{(G,V)} = h(\eta_{(F,A)})$, where $\eta_{(G,V)}$ is a Q- fuzzy soft subhemiring of $\bar{R}'$. Clearly $\eta_{(F,A)}$ is a Q- fuzzy soft subhemiring of $\bar{R}$.

Let $m_{(F,A)}$ and $n_{(F,A)}$ in R. Let $h((_{(F,A)\alpha})$ be a level subhemiring of $\eta_{(G,V)}$. Suppose $h(m_{(F,A)}, q)$ and $h(n_{(F,A)}, q)$ in $h(\eta_{(F,A)\alpha})$, that is:

$$\begin{aligned} \eta_{(G,V)}(h(m_{(G,V)}, q)) &\geq \alpha \text{ and } \eta_{(G,V)}(h(n_{(G,V)}, q)) \geq \alpha \\ \eta_{(G,V)}(h(m_{(G,V)}, q) + h(n_{(G,V)}, q)) &\geq \alpha, \eta_{(G,V)}(h(m_{(G,V)}, q)h(n_{(G,V)}, q)) \geq \alpha . \end{aligned}$$

We have to prove that $\eta_{(F,A)\alpha}$ is a level subhemiring of $\eta_{(F,A)}$.
Now, $\eta_{(F,A)}(m_{(F,A)}, q) = \eta_{(G,V)}(h(m_{(G,V)}, q)) \geq \alpha$, implies that:

$$\eta_{(F,A)}(m_{(F,A)}, q) \geq \alpha; \eta_{(F,A)}(n_{(F,A)}, q) = \eta_{(G,V)}(h(n_{(G,V)}, q)) \geq \alpha ,$$

implies that $\eta_{(F,A)}(n_{(F,A)}, q) \geq \alpha$, and we have:

$$\begin{aligned} \eta_{(F,A)}(m_{(F,A)} + n_{(F,A)}, q) &= \eta_{(G,V)}(h(m_{(G,V)} + n_{(G,V)}, q)) \\ &= \eta_{(G,V)}(h(m_{(G,V)}, q) + h(n_{(G,V)}, q)) \end{aligned}$$

as h is a Q-homomorphism $\geq \alpha$, which implies that

$$\eta_{(F,A)}(m_{(F,A)} + n_{(F,A)}, q) \geq \alpha \,,$$

for all $m_{(F,A)}$ and $n_{(F,A)}$ in $\bar{R}$.

And,

$$\begin{aligned}\eta_{(F,A)}(m_{(F,A)}n_{(F,A)}, q) &= \eta_{(G,V)}(h(m_{(G,V)}n_{(G,V)}, q)) \\ &= \eta_{(G,V)}(h(m_{(G,V)}, q)h(n_{(G,V)}q))\end{aligned}$$

as h is a Q- homomorphism $\geq \alpha$, which implies that

$$\eta_{(F,A)}(m_{(F,A)}n_{(F,A)}, q) \geq \alpha,$$

for all $m_{(F,A)}$ and $n_{(F,A)} \neq 0$ in $\bar{R}$. Hence $\eta_{(F,A)}$ is a level subhemiring of a Q- fuzzy soft subhemiring ,$\eta_{(F,A)}$ of $\bar{R}$ m Q. □

Theorem 4.6. *Let $(\bar{R}, +, \cdot)$ and $(\bar{R}', +, \cdot)$ be two hemirings. If $h : \bar{R}$ m Q $\rightarrow \bar{R}^{'}$ m Q is a Q-homomorphism, then the Q-homomorphic image of a level subhemiring of a Q- fuzzy soft subhemiring of R is a level subhemiring of a Q-fuzzy soft subhemiring of $R^{'}$.*

Proof. Let $(\bar{R}, +, \cdot)$ and $(\bar{R}', +, \cdot)$ be two hemirings and $h : \bar{R}$ m Q $\rightarrow \bar{R}^{'}$ m Q is a Q-homomorphism. That is, $h(m+n, q) = h(m, q) + h(n, q)$, for all m and n in and $\bar{R}$, and $h(mn, q) = h(m, q)h(n, q)$, for all m and n in $\bar{R}$.

Let $\eta_{(G,V)} = h(\eta_{(F,A)})$, where $\eta_{(F,A)}$ is a Q-fuzzy soft subhemiring of $\bar{R}$. Clearly $\eta_{(G,V)}$ is a Q- fuzzy subhemiring of R' m Q. If $m_{(F,A)}$ and $n_{(F,A)}$ in $\bar{R}$, then $h(m_{(G,V)}, q)$ and $h(n_{(G,V)}, q)$ in $\bar{R}'$ m Q. Let $\eta_{(F,A)\alpha}$ be a level subhemiring oh A.

Suppose $m_{(F,A)}$ and $n_{(F,A)}$ in $\eta_{(F,A)\alpha}$, then $m_{(F,A)} + n_{(F,A)}$ and $m_{(F,A)}n_{(F,A)}$ are in $\eta_{(F,A)\alpha}$. That is, $\eta_{(F,A)}(m_{(F,A)}, q) \geq \alpha$ and $\eta_{(F,A)}(n_{(F,A)}, q) \geq \alpha$, $\eta_{(F,A)}(m_{(F,A)} + n_{(F,A)}, q) \geq \alpha$, $\eta_{(F,A)}(m_{(F,A)}n_{(F,A)}, q) \geq \alpha$.

We have to prove that $h(\eta_{(F,A)\alpha})$ is a level subhemiring of $\eta_{(G,V)}$.

$$\eta_{(G,V)}(h(m_{(G,V)}, q)) \geq \alpha; \eta_{(G,V)}(h(n_{(G,V)}, q)) \geq \eta_{(F,A)}(n_{(F,A)}, q) \geq \alpha$$

implies that:

$$\begin{aligned}\eta_{(G,V)}(h(n_{(G,V)}, q)) &\geq \alpha \\ \eta_{(G,V)}(h(m_{(G,V)}, q) + h(n_{(G,V)}, q)) &= \eta_{(G,V)}(h(m_{(G,V)} + n_{(G,V)}, q))\,,\end{aligned}$$

as h is a Q-homomorphism,

$$\begin{aligned} &\geq \eta_{(F,A)}(m_{(F,A)} + n_{(F,A)}, q) \\ &\geq \alpha\,, \end{aligned}$$

which implies that $\eta_{(G,V)}(h(m_{(G,V)}+n_{(G,V)}, q)) \geq \alpha$, for all $h(m_{(G,V)}, q)$ and $h(n_{(G,V)}, q)$ in $\bar{R}$ m Q . On the other hand

$$\begin{aligned} \eta_{(G,V)}(h(m_{(G,V)}, q)h(n_{(G,V)}, q)) &= \eta_{(G,V)}(h(m_{(G,V)}n_{(G,V)}, q)) \\ &\quad \text{as } h \text{ is a } Q-\text{homomorphism} \\ &\geq \eta_{(H,A)}(m_{(G,V)}n_{(G,V)}, q) \\ &\geq \alpha\,, \end{aligned}$$

which implies that $\eta_{(G,V)}(h(m_{(G,V)}, q)h(n_{(G,V)}, q)) \geq \alpha$, for all $h(m_{(G,V)}, q)$ and $h(n_{(G,V)}, q) \neq 0$ in $\bar{R}'$.

Therefore, $\eta_{(G,V)}(h(m_{(G,V)}, q) + h(n_{(G,V)}, q)) \geq \alpha$, for all $h(m_{(G,V)}, q)$ and $h(n_{(G,V)}, q)$ in $\bar{R}'$ and $\eta_{(G,V)}(h(m_{(G,V)}, q)(h(n_{(G,V)}, q))) \geq \alpha$, for all $h(m_{(G,V)}, q)$ and $h(n_{(G,V)}, q) \neq 0$ in $\bar{R}'$ and hence $h((\eta_{(F,A))\alpha})$ is a level subhemiring of a Q- fuzzy soft subhemiring $\eta_{(G,V)}$ of a hemiring $\bar{R}'$. □

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