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INTUITIONISTIC FUZZY TRANSLATION ON INK-ALGEBRA

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ABSTRACT. In this article, We introduced intuitionistic fuzzy translation and intuitionistic fuzzy multiplication on INK-algebras and derived some results to get the structure of intuitionistic fuzzy INK-subalgebra.

1. INTRODUCTION

The idea of fuzzy translations trendy fuzzy subalgebras in addition ideals in BCK/BCI-algebras has been deliberated respectively by Lee et al. and Jun. They studied relatives' mid fuzzy translations, fuzzy extensions and fuzzy multiplications. Inspired by this, Senapati, T and M. Bhowmik, M. Pal presented fuzzy translations of fuzzy H-ideals in BCK/BCI-algebras. They likewise outspread this learning from fuzzy translations to intuitionistic fuzzy translations in BCK/BCI-algebras. In this paper,intuitionistic fuzzy translations, intuitionistic fuzzy extensions and intuitionistic fuzzy multiplications of intuitionistic fuzzy INK-subalgebra in INK-algebras be situated conversed. Relatives between intuitionistic fuzzy translations,intuitionistic fuzzy extensions too intuitionistic fuzzy multiplications of intuitionistic fuzzy INK-algebra in INK-algebras are as well studied in [2–7] and [1].

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2. PRELIMINARIES

Definition 2.1. *An algebra $(E, \odot, 0)$ is called a INK-algebra if you meet the ensuing conditions for every $p, q, r \in E$.*
INK-1: $((p \odot q) \odot (p \odot r)) \odot (r \odot q) = 0$
INK-2: $((p \odot r) \odot (q \odot r)) \odot (p \odot q) = 0$
INK-3: $p \odot 0 = p$
INK-4: $p \odot q = 0$ *and* $q \odot p = 0$ *imply* $p = q$.

Definition 2.2. *Let $Y \subseteq E$ is called a INK-subalgebra of E. If $p \odot q \in Y$.*

Definition 2.3. *Let E be a INK-algebra. Then a fuzzy set A is defined as $A = \{(p, v_A(p))|p \in E\}, 0 \leq v_A(p) \leq 1,$ for all $p \in E$.*

Definition 2.4. *A FS v in a INK-algebra E is called a fuzzy INK ideal of E, if:*

i) $v(0) \geq v(p)$
ii) $v(p) \geq min\{(v(q \odot p) \odot (q \odot r)), v(q)\}$ *for all* $p, q, r \in E$.

Definition 2.5. *A fuzzy set v in a INK-algebra E is named a fuzzy INK-subalgebra of E if $v_A(p \odot q) \geq min\{v_A(p), v_A(q)\}$, for all $p, q \in E$.*

Definition 2.6. *An intuitionistic fuzzy set(IFS) A in a non-empty set E is an object having the form $A = \{(p, v_A(p), w_A(p))|p \in E\}$, where the function:*
$v_A : E \to [0, 1]$ and $w_A : E \to [0, 1]$, denote the degree of membership and the degree of non-membership of each element $p \in E$ to the set A respectively, and $0 \leq v_A(p) + w_A(p) \leq 1,$ for all $p \in E$. Then denoted by $A = (p, v_A, w_A)$ for the intuitionistic fuzzy set $A = \{(p, v_A(p), w_A(p))|p \in E\}$.

Definition 2.7. *An IFS $A = (p, v_A, w_A)$ is named an IF-subalgebra of E if it satisfies:*

i) $v_A(p \odot q) \geq min\{v_A(p), v_A(q)\}$,
ii) $w_A(p \odot q) \leq max\{w_A(p), w_A(q)\}$, *for all* $p, q \in E$.

3. TRANSLATION OF INTUITIONISTIC FUZZY SUBALGEBRA

For the sake of straightforwardness, we mean to usage the symbol
$A = (p, v_A, w_A)$ for the IFS $A = \{(p, v_A(p), w_A(p)|p \in E)\}$.
We consider $T = 1 - inf\{w_A(p)|p \in E\}$ for any $A = (p, v_A, w_A)$ of E.

Definition 3.1. *Let $A = (p, v_A, w_A)$ be an IFS of E and let $\alpha \in [0, T]$. An object having the form $A^T_\alpha = ((v_A)^T_\alpha, (w_A)^T_\alpha)$ is named an IF-α -translation of A if $(v_A)^T_\alpha(p) = v_A(p) + \alpha$ and $(w_A)^T_\alpha(p) = v_A(p) - \alpha$ for all $p \in E$.*

Theorem 3.1. *let A be an IF-INK subalgebra of E and $\alpha \in [0, T]$. Then the IF-α-translation A^T_α of A is an IF-INK subalgebra of E.*

Proof. Let $p, q \in E$. Then $v(p \odot q) \geq min\{v(p), v(q)\}$. Now

$$\begin{aligned} v^T_\alpha(p \odot q) &= v(p \odot q) + \alpha \\ &\geq min\{v(p), v(q)\} + \alpha \\ &= min\{v(p) + \alpha, v(q) + \alpha\} \\ &= min\left\{v^T_\alpha(p), v^T_\alpha(q)\right\}, \end{aligned}$$

and

$$\begin{aligned} w^T_\alpha(p \odot q) &= w(p \odot q) - \alpha \\ &\leq max\{w(p), w(q)\} - \alpha \\ &= max\{w(p) - \alpha, w(q) - \alpha\} \\ &= max\left\{w^T_\alpha(p), w^T_\alpha(q)\right\}. \end{aligned}$$

□

Theorem 3.2. *Let A be an IFS of E such that the IF- α-translation A^T_α of A is an IF-INK subalgebra of E for some $\alpha \in [0, T]$. Then A is an IF-INK subalgebra of E.*

Proof. Let A^T_α is an IF-subalgebra of E for some $\alpha \in [0, T]$. Then

$$\begin{aligned} v(p \odot q) + \alpha &= v^T_\alpha(p \odot q) \\ &\geq min\left\{v^T_\alpha(p), v^T_\alpha(q)\right\} \\ &= min\{v(p) + \alpha, v(q) + \alpha\} \\ &= min\{v(p), v(q)\} + \alpha \\ v(p \odot q) &\geq min\{v(p), v(q)\}, \end{aligned}$$

and

$$\begin{aligned} w(p \odot q) - \alpha &= w_\alpha^T(p \odot q) \\ &\leq max\left\{w_\alpha^T(p), w_\alpha^T(q)\right\} \\ &= max\left\{w(p) - \alpha, w(q) - \alpha\right\} \\ &= max\left\{w(p), w(q)\right\} - \alpha \\ w(p \odot q) &\leq max\left\{w(p), w(q)\right\} . \end{aligned}$$

This implies that $v(p \odot q) \geq min\left\{v(p), v(q)\right\}$ and $w(p \odot q) \leq max\left\{w(p), w(q)\right\}$. Hence A is an IF-INK subalgebra of E. □

Definition 3.2. *Let A be an IFS of E and $e \in [0,1]$. An object having the form $A_e^M = ((v_A)_e^M, (w_A)_e^M)$ is called an IF e-multiplication of A if $(v_A)_e^M(p) = v_A(p) \cdot e$ and $(w_A)_e^M(p) = w_A(p) \cdot e$ for all $p \in E$.*

Theorem 3.3. *If $A = (v_A, w_A)$ be an IF-INK subalgebra of E, then the IF e-multiplication of A is an IF-INK subalgebra of E for all $e \in [0,1]$.*

Proof. Assume that $A = (v_A, w_A)$ is a IF-INK subalgebra of E.
Then $e \in [0,1]$.

$$\begin{aligned} (v_A)_e^M(p \odot q) &= e \cdot v_A(p \odot q) \\ &\geq e \cdot min\left\{v_A(p), v_A(q)\right\} \\ &\geq min\left\{(v_A)_e^M(p), (v_A)_e^M(p)\right\} , \end{aligned}$$

and

$$\begin{aligned} (w_A)_e^M(p \odot q) &= e \cdot w_A(p \odot q) \\ &\leq e \cdot max\left\{w_A(p), w_A(q)\right\} \\ &\leq max\left\{(w_A)_e^M(p), (w_A)_e^M(p)\right\} . \end{aligned}$$

Hence $(v_A)_e^M$ and $(w_A)_e^M$ is an IF-INK subalgebra of E. □

Theorem 3.4. *If $A = (v_A, w_A)$ be an IF subset of E, then the following assertions are equivalent.*

i) v_A*is an IF-INK subalgebra of* E.
ii) $(v_A)_e^M$ *is an IF-INK-subalgebra of* E.

Proof. Necessity follows from Theorem 3.3. For the sufficiency part, let $e \in [0,1]$ be such that A_e^M is an IF-INK subalgebra of A. Then for all $p, q \in E$ we have:

$$\begin{aligned} v_A(p \odot q) \cdot e &= (v_A)_e^M(p \odot q) \\ &\geq min\left\{(v_A)_e^M(p), (v_A)_e^M(q)\right\} \\ &\geq min\left\{v_A(p) \cdot e, v_A(q) \cdot e\right\} \\ v_A(p \odot q) \cdot e &= min\left\{v_A(p), v_A(q)\right\} \cdot e\,, \end{aligned}$$

$$\begin{aligned} w_A(p \odot q) \cdot e &= (w_A)_e^M(p \odot q) \\ &\leq max\left\{(w_A)_e^M(p), (w_A)_e^M(q)\right\} \\ &\leq max\left\{w_A(p) \cdot e, w_A(q) \cdot e\right\} \\ w_A(p \odot q) \cdot e &= max\left\{w_A(p), w_A(q)\right\} \cdot e\,. \end{aligned}$$

Hence, A is an IF-INK subalgebra of E. □

4. Intuitionistic fuzzy extension on INK-subalgebra

Definition 4.1. *Let $A = (p, v_A, w_A)$ and $B = (p, v_B, w_B)$ be IFS of E. If $A \geq B,\ v_A(p) \leq v_B(p)$ and $w_A(p) \geq w_B(p)$ for all $p \in E$. Then we call B is an IF-extension of A.*

Definition 4.2. *Let $A = (p, v_A, w_A)$ and $B = (p, v_B, w_B)$ be IFS of E. Then B is named an IF-S-extension of A. It the ensuing assertions are valid:*

i) *B is an IF-extension of A.*
ii) *If A is an IF-INK subalgebra of E, then B is an IF-INK subalgebra of E.*

Theorem 4.1. *Let A be an IF-INK subalgebra of E and $\alpha \in [0, T]$. Then the IF α-translation v_α^T of A is an IF-S-extension of A.*

The converse of Theorem 4.1 is not true in general as seen in the following example.

Example 1. *Consider a INK-algebra $E = \{0, 1, 2, 3, 4\}$ with the following Cayley table.*

$\odot$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	2	0
3	3	1	3	0	1
4	4	4	4	4	0

Define $A = (p, v_A, w_A)$ be an IF subset of E by

E	0	1	2	3	4
v_A	0.8	0.5	0.3	0.6	0.2
w_A	0.36	0.45	0.55	0.65	0.69

Then A is an IF-INK subalgebra of E. Let $B = (p, v_B, w_B)$ be an IF subset of E given by

E	0	1	2	3	4
v_B	0.84	0.56	0.38	0.67	0.21
w_B	0.10	0.35	0.40	0.32	0.25

Then B is an IF-S-extension of A. But it is not an IF-α-translation of A_α^T of A, for all $\alpha \in [0, T]$. clearly, the intersection of IF S-extension of an IF-INK subalgebra A of E is an IF S-extension of A. But the union of IF S-extensions of an IF-INK subalgebra A of E is not an IF S-extension of A as seen in the following example.

Example 2. *Consider a INK-algebra $A = \{0, 1, 2, 3, 4\}$ with the following Cayley table:*

$\odot$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	0	0	0
2	2	1	0	0	0
3	3	1	1	0	0
4	4	3	3	1	0

Define $A = (p, v_A, w_A)$ be an IF subset of E by

E	0	1	2	3	4
v_A	0.7	0.4	0.6	0.3	0.3
w_A	0.2	0.3	0.4	0.4	0.6

Then A is an IF-INK subalgebra of E. Let $B = (p, v_B, w_B)$ be an IF subset of E given by

E	0	1	2	3	4
v_B	0.8	0.6	0.8	0.4	0.4
w_B	0.2	0.3	0.2	0.3	0.2

and

E	0	1	2	3	4
v_C	0.9	0.6	0.6	0.6	0.7
w_C	0.1	0.2	0.2	0.2	0.2

Then B and C are IF- S-extensions of A. But the union BUC is an IF- S-extension of A since, but it is not an IF S-extension of A, since,

$$v_{BUC}(4 \odot 2) = 0.6 \neq 0.7 = min\{v_{BUC}(4), v_{BUC}(2)\}$$

$$w_{BUC}(4 \odot 2) = 0.3 \neq 0.2 = min\{w_{BUC}(4), w_{BUC}(2)\}$$

For an IF-subset $A = (v_A, w_A)$ of $E, \alpha \in [0, T]$ and $t, s \in [0, 1]$ by $t \geq \alpha$. Let $U_\alpha(v_A; t) = \{p \in E | v_A(p) \geq t - \alpha\}$ and $L_\alpha(w_A; s) = \{p \in E | w_A(p) \geq s + \alpha\}$. If A is an IF-INK subalgebra of E, then it is clear that $U_\alpha(v_A; t)$ and $L_\alpha(w_A; s)$ are subalgebras of E, for all $t \in Im(v_A)$ and $s \in Im(w_A)$ with $t \geq \alpha$. But, if we do not give a condition that A is an IF-INK subalgebra of E,then $U_\alpha(v_A; t)$ and $L_\alpha(w_A; s)$ are not INK-subalgebras of E as seen in the following example.

Example 3. *Let $A = \{0, 1, 2, 3, 4\}$ be a INK-algebra which is given in Example 4.1. Define an IF-subset v of E by*

E	0	1	2	3	4
v_A	0.7	0.4	0.6	0.3	0.5
w_A	0.2	0.3	0.4	0.7	0.6

Then v is not an IF-INK subalgebra of E. Then

$$v_A(4 \odot 2) = 0.3 \neq 0.5 = min\{v_A(4), v_A(2)\},$$

and

$$w_A(4 \odot 1) = 0.7 \neq 0.6 = max\{w_A(4), w_A(1)\}$$

$A = (v_A, w_A)$ *is not an IF-INK subalgebra of* E. *For* $\alpha = 0.1$ *and* $t = 0.5$, *we obtain* $U_\alpha(v_A; t) = \{0, 1, 2, 4\}$ *, which is not a INK-subalgebra of* E. *For* $\alpha = 0.1$ *and* $s = 0.6$, *we obtain* $L_\alpha(w_A; s) = \{0, 1, 2, 4\}$ *which is not a INK-subalgebra of* E.

Theorem 4.2. *Let* $A = (v_A, w_A)$ *be an IF-INK subalgebra of* A *and let* $\alpha, \beta \in [0, T]$. *If* $\alpha \geq \beta$ *then the IF translation* $A^T_\alpha = ((v_A)^T_\alpha, (w_A)^T_\alpha)$ *of* A *is an IF S-extension of the IF* β*-translation* $A^T_\beta = ((v_A)^T_\beta, (w_A)^T_\beta)$ *of* A.

For every IF-INK subalgebra A of E and $\beta \in [0, T]$, the IF β -translation A^T_β of A is an IF-INK subalgebra of E . If IF S-extension of A^T_β, then there exists $\alpha \in [0, T]$ such that $\alpha \geq \beta$ and $B \geq A^T_\beta$, that is $v_A(p) \geq (v_A)^T_\alpha$ and $w_A(p) \leq (w_A)^T_\alpha$, for all $\alpha \in E$. Hence, we have the following theorem.

Theorem 4.3. *Let* A *be an IFINK-subalgebra of* E *and let* $\beta \in [0, T]$ *for every IF S-extension* $B = (v_B, w_B)$ *of the IF* β*-translation* A^T_β *of* A*, there exists* $\alpha \in [0, T]$ *such that* $\alpha \geq \beta$ *and* B *is an IF S-extension of the IF* α*-translation* $(v_A)^T_\alpha$ *of* A.

Let us illustrate the Theorem 4.3 using the following example.

Example 4. *Let* $A = \{0, 1, 2, 3, 4\}$ *be a INK-algebra and* $A = (v_A, w_A)$ *be an IFS of* E. *Take* $T = 0.3$. *If we take* $\beta = 0.15$, *then the IF* β *-translation* A^T_β *of* A *is given by*

E	0	1	2	3	4
$(v_A)^T_\beta$	0.85	0.55	0.75	0.45	0.65
$(w_A)^T_\beta$	0.05	0.15	0.25	0.25	0.45

Let $B = (v_B, w_B)$ *be an IFS of* E *defined by*

E	0	1	2	3	4
v_B	0.90	0.68	0.88	0.58	0.78
w_B	0.12	0.22	0.32	0.32	0.52

Then B *is clearly an IF INK subalgebra of* E *which is an IF* S*-extension of the IF* β*-translation* A^T_β *of* A. *But* B *is not an IF translation of* A*, for all* $\alpha \in [0, T]$. *If* *we take $\alpha = 0.18$ then $\alpha = 0.18 > 0.15 = \beta$ and the IF translation $A^T_\alpha = ((v_A)^T_\alpha, (w_A)^T_\alpha)$, of A is given as follows,*

E	0	1	2	3	4
$(v_A)^T_\alpha$	0.90	0.68	0.88	0.58	0.78
$(w_A)^T_\alpha$	0.12	0.22	0.32	0.32	0.52

Note that $B(p) \geq (v_A)^T_\alpha(p)$ that is $v_B(p) \geq (v_A)^T_\alpha$ and $w_B(p) \leq (w_A)^T_\alpha$, for all $p \in E$, and hence,B is an IF S-extension of the IF α-translation $(v_A)^T_\alpha$ of A.

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