Advances in Mathematics: Scientific Journal **9** (2020), no.1, 417–435
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.1.33

APPLICATION OF ADOMIAN DECOMPOSITION METHOD ON A MATHEMATICAL MODEL OF MALARIA

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ABSTRACT. In this paper, we consider a deterministic model of malaria transmission. Adomian decomposition method (ADM) is used to calculate an approximation to the solution of the non-linear couple of differential equations governing the model. Classical fourth-order Runge-Kutta method implemented in Maple18 confirms the validity of the ADM in solving the problem. Graphical results show that ADM agrees with R-K 4. In order words, these produced the same behaviour, validating ADM's efficiency and accuracy of ADM in finding the malaria model solution.

1. INTRODUCTION

Malaria is a deadly disease instigated by an insect and it is spread in humans through the bites of female Anopheles mosquitoes. Malaria is also the second-largest leading cause of mortality in Africa after HIV/ AIDS according to CDC report [1]. Every year, over 430,000 child mortality in Africa caused by malaria. Roll-Back Malaria (RBM) programme was initiated with the hope of two key areas, prevention and treatment [2]. This initiative of eradication is progressing to reduce disease transmission which characterized a high mortality rate in children and pregnant women. Malaria is one of the major concerned in Africa, it has led to the death of millions of people in the world

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2010 *Mathematics Subject Classification.* 43A12, 34A04.
Key words and phrases. Mathematical model, Relapse rate, Nonlinear forces of infection, Adomian Decomposition Method, Runge-Kutta Method.

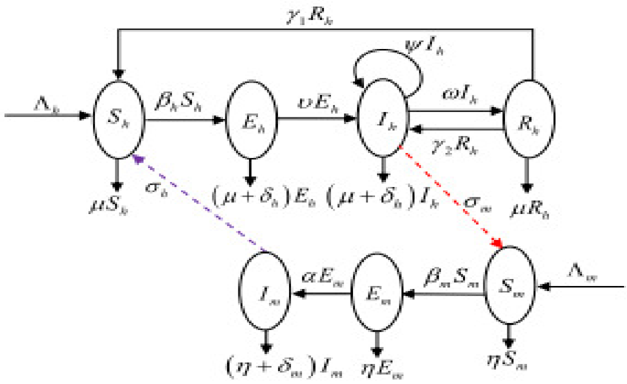
to date. Many mathematical models have been developed on malaria models [3,4,5,6,7,8,9,10,19,20]. Presently, no study has considered disease-induced death rate on exposed class, Nonlinear of forces of infections and newborn birth rate of the newborns birth with infection of humans with the relevance of Adomian decomposition method while solving this deterministic model of malaria.

American Mathematician, George Adomian [11] was the first to develop Adomian decomposition method. It is a semi-analytical method which can be used in solving both nonlinear and linear order of partial and ordinary differential equations. Also, it can be used in solving nonlinear differential equations of higher order. [12,13] considered deterministic models but not on the malaria model. In fluid dynamics, [14,15,16] and in numerical analysis, [17,18]. This work focus on the important role that the ADM plays on proposed malaria model.

2. Materials and Methods

This model built is the modification of SIR system of Ordinary Differential Equation (ODE) to include nonlinear forces of infections, exposed compartment for human and mosquito populations, disease-induced death and relapse. There are four stages for the human (host) and three stages for the mosquito (vector) in this model and these are Vulnerable humans (S_h), Exposed (Bare) humans (E_h), Infected (Malaria carrier) humans (I_h), Recovered (Recuperated) humans (R_h) and Vulnerable mosquitoes (S_m), Exposed (Bare) mosquitoes (E_m), Infected (Malaria carrier) mosquitoes (I_m) respectively. This shows the movement of human and mosquito from one stage to another at different rates where Λ_h is the recruitment rate of human that is, the rate at which humans enter into the Vulnerable population through birth, β_h denote the force of infection in humans, ε is the rate of bare humans that is, the rate at which humans move from bare class to malaria carrier human class, ω is the recuperation rate of human from the disease, γ_2 is the relapse rate of humans that is, the rate at which humans with low immunity return from recuperated class back to malaria carrier human class, ψ is the rate of newborn birth with infection of humans, μ is the mortality rate of humans, δ_h is the disease-induced death rate of humans, γ_1 is the loss of immunity rate of humans, Λ_m is the recruitment rate of mosquitoes, β_m is the force of infection in mosquitoes, α_h is the rate of bare mosquitoes that is, the rate at which mosquitoes move from bare class to malaria carrier mosquito class, η is the natural death rate of mosquitoes, δ_m is the disease-induced death rate of mosquitoes, σ_m is the rate of interaction between human and mosquito in mosquitoes and σ_h is the rate of interaction between human and mosquito in humans. The flow chart is shown in figure 1.

FIGURE 1. Flow chart of the model



where $\beta_h = \dfrac{\xi\sigma_h I_m(t)}{1+\varepsilon_h I_m(t)}$ and $\beta_m = \dfrac{\xi\sigma_m I_h(t)}{1+\epsilon_m I_h(t)}$.

2.1. **Assumptions of the Model.** The assumptions of the above model are listed below

(i) All parameters and variables are assumed to be positive.
(ii) E_h and E_m are infected but not infectious.
(iii) Malaria is contacted only from infected mosquitoes.
(iv) Mosquitoes are assumed not to recover from the parasites.
(v) Non-linear forces of infection in form of saturated incidence rates for both humans and mosquito population are considered.
(vi) There is relapse of the infection after treatment.
(vii) The human population has a low or incomplete immunity.
(viii) Newborn births can be affected by malaria.
(ix) Entry and exit into the population is birth and natural death or malaria induced death respectively.

2.2. **Differential Equation of the Model.** The following differential equations give details about figure 1.

$$\begin{aligned}
\frac{dS_h(t)}{dt} &= \Lambda_h - \frac{\xi\sigma_h I_m(t) S_h(t)}{1+\varepsilon_h I_m(t)} - \mu S_h(t) + \gamma_1 R_h(t)\\
\frac{dE_h(t)}{dt} &= \frac{\xi\sigma_h I_m(t) S_h(t)}{1+\varepsilon_h I_m(t)} - (\upsilon + \mu + \delta_h) E_h(t)\\
\frac{dI_h(t)}{dt} &= \upsilon E_h(t) - (\omega + \mu + \delta_h) I_h(t) + \psi I_h(t) + \gamma_2 R_h(t)\\
\frac{dR_h(t)}{dt} &= \omega I_h(t) - (\gamma_1 + \gamma_2 + \mu) R_h(t)\\
\frac{dS_m(t)}{dt} &= \Lambda_m - \frac{\xi\sigma_m I_h(t) S_m(t)}{1+\epsilon_m I_h(t)} - \eta S_m(t)\\
\frac{dE_m(t)}{dt} &= \frac{\xi\sigma_m I_h(t) S_m(t)}{1+\epsilon_m I_h(t)} - (\alpha + \eta) E_m(t)\\
\frac{dI_m(t)}{dt} &= \alpha E_m(t) - (\eta + \delta_m) I_m(t)
\end{aligned} \tag{2.1}$$

2.3. **Adomian Decomposition Method (ADM). Basic Definition of ADM**
Consider the equation

Differential operator $[L(y)]$ + *Remainder of the Differential operator* $[R(y)]$ + *Nonlinear terms* $[N(y)]$ = *Inhomogeneous term* $[g(x)]$.

which is represented mathematically in equation (2.2).

$$L(y) + R(y) + N(y) = g(x) \tag{2.2}$$

Where L may be considered as the highest order of differential equation. Multiplying equation (2.2) through by L^{-1}, we got equation (2.3)

$$L^{-1}L(y) = L^{-1}g(x) - L^{-1}R(y) - L^{-1}N(y) \tag{2.3}$$

Where L^{-1} is an n-fold integration of an nth order L and also called inverse operator, $L^{-1}g(x)$ and $L^{-1}R(y)$ are linear that is, integrable and $L^{-1}N(y)$ is nonlinear. $N(y)$ can be written as equation (2.4)

$$N(y) = \sum_{n=0}^{\infty} A_n(y_0, y_1, ..., y_n) \tag{2.4}$$

where A_n is generated using Adomian polynomials which can be specified by their non-linearity. They only depend on y_0 to y_n components for the series.

The A_n will be written as:

$$\begin{aligned} A_0 &= N(y_0) \\ A_1 &= y_1 \left(\frac{d}{dy_0} \right) N(y_0) \\ A_2 &= y_2 \left(\frac{d}{dy_0} \right) N(y_0) + \left(\frac{y_1^2}{2!} \right) \left(\frac{d^2}{dy_0^2} \right) N(y_0) , \end{aligned}$$

and it goes on till A_n. This can be derived using the formula

$$A_n = \frac{1}{n!} \left\{ \frac{d^n}{d\lambda^n} \left[N \left(\sum_{k=0}^{n} \lambda^k y_k \right) \right] \right\}_{\lambda=0} \quad n = 0, 1, 2, ... \tag{2.5}$$

Therefore, the decomposition series solution is

$$y(t) = \sum_{k=0}^{\infty} y_k = y_0 + y_1 + y_2 + y_3 + ...$$

Some of the Nonlinear terms in Adomian Polynomials

If A_n is the Adomian polynomials of the nonlinear term $N(y)$ which can be generated using equation (2.5) then the following cases can be considered.

Case 1: Nonlinear power function.

(i) $N(y) = y^2$, the polynomials can be written as:

$$\begin{aligned} A_0 &= y_0^2, \\ A_1 &= 2y_0y_1, \\ A_2 &= 2y_0y_2 + y_1^2, \\ A_3 &= 2y_0y_3 + 2y_1y_2, \\ &\vdots \end{aligned}$$

(ii) $N(y) = y^3$, the polynomials can be written as:

$$\begin{aligned} A_0 &= y_0^3, \\ A_1 &= 3{y_0}^2y_1, \\ A_2 &= 3{y_0}^2y_2 + 3y_0y_1^2, \\ A_3 &= 3y_0^2y_3 + 6y_0y_1y_2 + y_1^3, \\ &\vdots \end{aligned}$$

Case 2: Nonlinear Sine function $N(y) = \sin y$. The polynomials can be written as:

$$\begin{aligned}
A_0 &= \sin(y_0),\\
A_1 &= y_1 \cos(y_0),\\
A_2 &= y_2\ \cos(y_0) - \left(\frac{1}{2!}\right) y_1^2 \sin(y_0),\\
A_3 &= y_3\ \cos(y_0) - y_1 y_2 \sin(y_0) - \left(\frac{1}{3!}\right) y_1^3 \cos(y_0)\\
&\vdots
\end{aligned}$$

Case 3: Nonlinear Cosine function $N(y) = \cos y$. The polynomials can be written as:

$$\begin{aligned}
A_0 &= \cos(y_0),\\
A_1 &= -y_1 \sin(y_0),\\
A_2 &= -y_2 \sin(y_0) - \left(\frac{1}{2!}\right) y_1^2 \cos(y_0),\\
A_3 &= -y_3 \sin(y_0) - y_1 y_2 \cos(y_0) + \left(\frac{1}{3!}\right) y_1^3 \sin(y_0),\\
&\vdots
\end{aligned}$$

Case 4: Nonlinear Exponential function, $N(y) = e^y$, the polynomials can be written as:

$$\begin{aligned}
A_0 &= e^{(y_0)},\\
A_1 &= y_1\ e^{(y_0)},\\
A_2 &= \left(y_2 + \frac{1}{2!} y_1^2\right)\ e^{(y_0)},\\
A_3 &= \left(y_3 + u_1 u_2 + \frac{1}{3!} y_1^3\right)\ e^{(y_0)},\\
&\vdots
\end{aligned}$$

Case 5: Nonlinear negative Exponential function $N(y) = e^{-y}$. The polynomials can be written as:

$$\begin{aligned} A_0 &= e^{-(y_0)}, \\ A_1 &= -y_1\, e^{-(y_0)}, \\ A_2 &= \left(-y_2 + \frac{1}{2!}y_1^2\right)\, e^{-(y_0)}, \\ A_3 &= \left(-y_3 + y_1y_2 - \frac{1}{3!}y_1^3\right)\, e^{-(y_0)}, \\ &\vdots \end{aligned}$$

Case 6: Nonlinear hyperbolic Sine function $N(y) = \sinh y$, the polynomials can be written as:

$$\begin{aligned} A_0 &= \sinh(y_0), \\ A_1 &= y_1 \cosh(y_0), \\ A_2 &= y_2 \cosh(y_0) + \left(\frac{1}{2!}\right) y_1^2 \sinh(y_0), \\ A_3 &= y_3 \cosh(y_0) + y_1y_2 \sinh(y_0) + \left(\frac{1}{3!}\right) y_1^3 \cosh(y_0), \\ &\vdots \end{aligned}$$

Case 7: Nonlinear hyperbolic Cosine function $N(y) = Coshy$, the polynomials can be written as:

$$\begin{aligned} A_0 &= \cosh(y_0), \\ A_1 &= y_1 \sinh(y_0), \\ A_2 &= y_2 \sinh(y_0) + \left(\frac{1}{2!}\right) y_1^2 \cosh(y_0), \\ A_3 &= y_3 \sinh(y_0) + y_1y_2 \cosh(y_0) + \left(\frac{1}{3!}\right) y_1^3 \sinh(y_0), \\ &\vdots \end{aligned}$$

Case 8: Nonlinear Logarithmic function: $N(y) = \ln(y)$ for $y > 0$, the polynomials can be written as:

$$\begin{aligned} A_0 &= \ln(y_0), \\ A_1 &= \frac{y_1}{y_0}, \\ A_2 &= \frac{y_2}{y_0} - \frac{1}{2}\left(\frac{y_1^2}{y_0^2}\right), \\ A_3 &= \frac{y_3}{y_0} - \frac{y_1 y_2}{y_0^2} + \frac{1}{3}\left(\frac{y_1^3}{y_0^3}\right), \\ &\vdots \end{aligned}$$

For $N(y) = \ln(1+y)$ for $-1 < y \leq 1$, the polynomials can be written as:

$$\begin{aligned} A_0 &= \ln(1+y_0), \\ A_1 &= \frac{y_1}{1+y_0}, \\ A_2 &= \frac{y_2}{1+y_0} - \frac{1}{2}\frac{y_1^2}{(1+y_0)^2}, \\ A_3 &= \frac{y_3}{1+y_0} - \frac{y_1 y_2}{(1+y_0)^2} + \frac{1}{3}\frac{y_1^3}{(1+y_0)^3}, \\ &\vdots \end{aligned}$$

2.4. **Solution of the Model Adomian Decomposition Method.** Differential operator L can be defined as $L = \frac{d(*)}{dt}$ and an inverse operator as L^{-1} can be defined as $L^{-1} = \int_0^t (*)\, \mathrm{d}y$ since equation (2.1) is a set of first-order ordinary differential equations with initial value problem, it will become

$$L^{-1}L(y) = y - y(0) \tag{2.6}$$

Substituting equation (2.6) into model (2.1) to obtain

$$\begin{aligned} S_h(t) - S_h(0) &= L^{-1}[\Lambda_h] - L^{-1}\left[\frac{\xi\sigma_h I_m(t) S_h(t)}{1+\varepsilon_h I_m(t)}\right] - L^{-1}\left[\mu S_h(t)\right] \\ &+ L^{-1}\left[\gamma_1 R_h(t)\right] \end{aligned}$$

$$\begin{aligned}
E_h(t)-E_h(0) &= L^{-1}\left[\frac{\xi\sigma_h I_m(t)S_h(t)}{1+\varepsilon_h I_m(t)}\right]-L^{-1}\left[(\upsilon+\mu+\delta_h)E_h(t)\right]\\
I_h(t)-I_h(0) &= L^{-1}\left[\upsilon E_h(t)\right]-L^{-1}\left[(\omega+\mu+\delta_h)I_h(t)\right]\\
&- L^{-1}\left[\psi I_h(t)\right]+L^{-1}\left[\gamma_2 R_h(t)\right]\\
R_h(t)-R_h(0) &= L^{-1}\left[\omega I_h(t)\right]-L^{-1}\left[(\gamma_1+\gamma_2+\mu)\,R_h(t)\right]\\
S_m(t)-S_m(0) &= L^{-1}[\Lambda_m]-L^{-1}\left[\frac{\xi\sigma_m I_h(t)S_m(t)}{1+\epsilon_m I_h(t)}\right]-L^{-1}\left[\eta S_m(t)\right]\\
E_m(t)-E_m(0) &= L^{-1}\left[\frac{\xi\sigma_m I_h(t)S_m(t)}{1+\epsilon_m I_h(t)}\right]-L^{-1}\left[(\alpha+\eta)E_m(t)\right]\\
I_m(t)-I_m(0) &= L^{-1}\left[\alpha E_m(t)\right]-L^{-1}\left[(\eta+\delta_m)I_m(t)\right] \qquad (2.7)
\end{aligned}$$

Solving equation (2.7) gives

$$\begin{aligned}
S_h(t) &= S_h(0)+\Lambda_h t-\xi\sigma_h\int_0^t\left(\sum_{n=0}^{\infty}A_n\right)\,\mathrm{d}y-\mu\int_0^t\left(\sum_{n=0}^{\infty}S_{hn}\right)\,\mathrm{d}y\\
&+ \gamma_1\int_0^t\left(\sum_{n=0}^{\infty}R_{hn}\right)\,\mathrm{d}y\\
E_h(t) &= E_h(0)+\xi\sigma_h\int_0^t\left(\sum_{n=0}^{\infty}A_n\right)\,\mathrm{d}y-(\upsilon+\mu+\delta_h)\int_0^t\left(\sum_{n=0}^{\infty}E_{hn}\right)\,\mathrm{d}y
\end{aligned}$$

$$\begin{aligned}
I_h(t) =& I_h(0)+\upsilon\int_0^t\left(\sum_{n=0}^{\infty}E_{hn}\right)\,\mathrm{d}y-(\omega+\mu+\delta_h)\int_0^t\left(\sum_{n=0}^{\infty}I_{hn}\right)\,\mathrm{d}y\\
&-\psi\int_0^t\left(\sum_{n=0}^{\infty}I_{hn}\right)\,\mathrm{d}y+\gamma_2\int_0^t\left(\sum_{n=0}^{\infty}R_{hn}\right)\,\mathrm{d}y\\
R_h(t) =& R_h(0)+\omega\int_0^t\left(\sum_{n=0}^{\infty}I_{hn}\right)\,\mathrm{d}y-(\gamma_1+\gamma_2+\mu)\int_0^t\left(\sum_{n=0}^{\infty}R_{hn}\right)\,\mathrm{d}y\\
S_m(t) =& S_m(0)+\Lambda_m t-\xi\sigma_m\int_0^t\left(\sum_{n=0}^{\infty}B_n\right)\,\mathrm{d}y-\eta\int_0^t\left(\sum_{n=0}^{\infty}S_{mn}\right)\,\mathrm{d}y\\
E_m(t) =& E_m(0)+\xi\sigma_m\int_0^t\left(\sum_{n=0}^{\infty}B_n\right)\,\mathrm{d}y-(\alpha+\eta)\int_0^t\left(\sum_{n=0}^{\infty}E_{mn}\right)\,\mathrm{d}y\\
I_m(t) =& I_m(0)+\alpha\int_0^t\left(\sum_{n=0}^{\infty}E_{mn}\right)\,\mathrm{d}y-(\eta+\delta_m)\int_0^t\left(\sum_{n=0}^{\infty}I_{mn}\right)\,\mathrm{d}y
\end{aligned} \qquad (2.8)$$

where

$$A_n = \frac{d^n}{d\lambda^n}\left[\sum_{k=0}^{n}\left(S_{hk}I_{mk}\right)\lambda^k\right]_{\lambda=0},$$

$$B_n = \frac{d^n}{d\lambda^n}\left[\sum_{k=0}^{n}\left(S_{mk}I_{hk}\right)\lambda^k\right]_{\lambda=0}.$$

The initial stage from equation (2.8) gives

$$\begin{aligned} &S_{h0}(t) = S_h(0) + \Lambda_h t\ ;\ E_{h0}(t) = E_h(0)\ ;\ I_{h0}(t) = I_h(0)\ ; \\ &R_{h0}(t) = R_h(0)\ ; S_{m0}(t) = S_m(0) + \Lambda_m t\ ; E_{m0}(t) = E_m(0)\ ; \\ &I_{m0}(t) = I_m(0)\,. \end{aligned}$$

Then recursive formula can now be written as

$$\begin{aligned} S_{h(n+1)}(t) =& -\xi\sigma_h \int_0^t \left(\sum_{n=0}^{\infty} A_n\right) \mathrm{d}y - \mu \int_0^t \left(\sum_{n=0}^{\infty} S_{hn}\right) \mathrm{d}y \\ &+ \gamma_1 \int_0^t \left(\sum_{n=0}^{\infty} R_{hn}\right) \mathrm{d}y \\ E_{h(n+1)}(t) =& +\xi\sigma_h \int_0^t \left(\sum_{n=0}^{\infty} A_n\right) \mathrm{d}y - (\upsilon + \mu + \delta_h) \int_0^t \left(\sum_{n=0}^{\infty} E_{hn}\right) \mathrm{d}y \\ I_{h(n+1)}(t) =& +\upsilon \int_0^t \left(\sum_{n=0}^{\infty} E_{hn}\right) \mathrm{d}y - (\omega + \mu + \delta_h) \int_0^t \left(\sum_{n=0}^{\infty} I_{hn}\right) \mathrm{d}y \\ &+ \gamma_2 \int_0^t \left(\sum_{n=0}^{\infty} R_{hn}\right) \mathrm{d}y \\ R_{h(n+1)}(t) =& +\omega \int_0^t \left(\sum_{n=0}^{\infty} I_{hn}\right) \mathrm{d}y - (\gamma_1 + \gamma_2 + \mu) \int_0^t \left(\sum_{n=0}^{\infty} R_{hn}\right) \mathrm{d}y \\ S_{m(n+1)}(t) =& -\xi\sigma_m \int_0^t \left(\sum_{n=0}^{\infty} B_n\right) \mathrm{d}y - \eta \int_0^t \left(\sum_{n=0}^{\infty} S_{mn}\right) \mathrm{d}y \\ E_{m(n+1)}(t) =& +\xi\sigma_m \int_0^t \left(\sum_{n=0}^{\infty} B_n\right) \mathrm{d}y - (\alpha + \eta) \int_0^t \left(\sum_{n=0}^{\infty} E_{mn}\right) \mathrm{d}y \\ I_{m(n+1)}(t) =& +\alpha \int_0^t \left(\sum_{n=0}^{\infty} E_{mn}\right) \mathrm{d}y - (\eta + \delta_m) \int_0^t \left(\sum_{n=0}^{\infty} I_{mn}\right) \mathrm{d}y \end{aligned}$$

for $n = 0, 1, 2, 3, ...$
Using the initial conditions, $S_h(0) = 100,\ E_h(0) = 20,\ I_h(0) = 10,\ R_h(0) = 0,\ S_m(0) = 1000,\ E_m(0) = 20$ and $I_m(0) = 30$ according to Olaniyi *et al.* [3], with the value of parameters in the table 1 and Adomian decomposition solution series are obtained through Maple 18 as follows:

$$\begin{aligned} S_h(t) &= 100 - 1.107290322580645\,t + 0.01177503262747137\,t^2 - 0.00018859070700 \\ &\quad 22728\,t^3 + 0.000014789970910688 59\,t^4 - 0.0000002249610967180816\,t^5 - 0. \\ &\quad 000000105259045335055 1\,t^6 - 0.0000000001154860352112395\,t^7 - 6.9693396 \\ &\quad 80644870 \times 10^{-13}\,t^8 - 9.809876038253857 \times 10^{-17}\,t^9 + ... \end{aligned}$$

$$\begin{aligned} E_h(t) &= 20 - 1.427909677419355\,t + 0.08700130934027057\,t^2 - 0.0036107820467818 \\ &\quad 10\,t^3 + 0.0001026111215170176\,t^4 - 0.000002465660112703953\,t^5 + 0.00000 \\ &\quad 01588872948410600\,t^6 + 0.0000000009425892863161730\,t^7 + 7.12110150457 \\ &\quad 6005 \times 10^{-13}\,t^8 + 1.047662133195424 \times 10^{-16}\,t^9 + ... \end{aligned}$$

$$\begin{aligned} I_h(t) &= 10 + 0.20040\,t - 0.04363973393548388\,t^2 + 0.002613269097498274\,t^3 - 0.000 \\ &\quad 09742533994843784\,t^4 + 0.000002586126951011056\,t^5 - 0.0000000550622677 \\ &\quad 6521536\,t^6 + 0.0000000001622530571102172\,t^7 - 3.747017000830179 \times \\ &\quad 10^{-15}\,t^8 - 2.825191922459250 \times 10^{-18}\,t^9 + ... \end{aligned}$$

$$\begin{aligned} R_h(t) &= 0.0350\,t + 0.00007717500\,t^2 - 0.000051315104674731 20\,t^3 + 0.000002487124 \\ &\quad 231827501\,t^4 - 0.0000000759724883125992 8\,t^5 + 0.0000000017064823868107 \\ &\quad 71\,t^6 - 3.783581467971381 \times 10^{-12}\,t^7 - 1.377045901526503 \times 10^{-16}\,t^8 + ... \end{aligned}$$

$$\begin{aligned} S_m(t) &= 1000 - 9.948181818181818 2\,t + 0.04402155330578512\,t^2 + 0.001310059120448 \\ &\quad 782\,t^3 - 0.0001094598369358004\,t^4 + 0.000008350670249448175\,t^5 - 0.0000 \\ &\quad 006723413657143677\,t^6 + 0.0000000021796942231284 28\,t^7 - 9.71736298823 \\ &\quad 8751 \times 10^{-13}\,t^8 - 1.491968771903293 \times 10^{-15}\,t^9 + ... \end{aligned}$$

$$E_m(t) = 20 + 8.141581818181818\,t - 0.3811474597603306\,t^2 + 0.009328292100372793\,t^3 - 0.0000863096820252555\,0t^4 - 0.00000688543178768139\,8t^5 + 0.0000007673871474567493\,t^6 - 0.00000000245563871420237\,6t^7 + 9.686865375092404 \times 10^{-13}\,t^8 + 1.580099843478342 \times 10^{-15}\,t^9 + ...$$

$$I_m(t) = 30 + 1.60510\,t + 0.3364069789545454\,t^2 - 0.01075028797719808\,t^3 + 0.0001984803178323036\,t^4 - 0.0000015053845179458\,64t^5 - 0.00000009478933078495254\,t^6 + 0.0000000002764055798617308\,t^7 + 3.261761395797067 \times 10^{-15}\,t^8 - 0.0000000009713501357 \times 10^{-17}\,t^9 + ...$$

TABLE 1. Description of Parameters of the Model

Parameter	Description	Value	Reference
Λ_h	Recruitment rate of humans	1.2	Osman *et al.* [5]
υ	The exposed rate of humans to infectious class	0.05	Osman *et al.* [5]
ω	The recovery rate of humans	0.0035	Osman *et al.* [5]
γ_1	Loss of immunity for humans	0.00017	Osman *et al.* [5]
γ_2	The relapse rate of humans	0.004	Huo and Qiu [4]
μ	The natural Death of humans	0.01146	Osman *et al.* [5]
δ_h	Disease-induced death of humans	0.068	Osman *et al.* [5]
δ_m	Disease-induced death of mosquitoes	0.001	Assumed
Λ_m	Recruitment rate of mosquitoes	0.7	Osman *et al.* [5]
η	The natural death rate of mosquitoes	0.00083	Assumed
α	The exposed rate of mosquitoes to infectious class	0.083	Osman *et al.* [5]
ξ	Biting rate of Mosquitoes	0.12	Olaniyi and Obabiyi [3]

TABLE 2. Description of Parameters of the Model

Parameter	Description	Value	Reference
σ_h	Probability of transmission of infection from infectious mosquitoes to susceptible humans	0.1	Olaniyi and Obabiyi [3]
σ_m	Probability of transmission of infection from infectious humans to susceptible mosquitoes	0.09	Olaniyi and Obabiyi [3]
ψ	Rate of the newborn birth with infection in humans	0.003	Osman *et al.* [5]
ϵ_h	The rate of antibodies produced against antigens of humans	1.0	Assumed
ϵ_m	The rate of antibodies produced against antigens of mosquitoes	1.0	Assumed

3. RESULTS AND DISCUSSION

3.1. **Results of the Model.** In the following Figures $2, 3, 4, 5, 6, 7$ and 8 a comparison graphs between fourth-Order Runge-Kutta and Adomian Decomposition Methods are given.

FIGURE 2. The graph of Susceptible Human against Time for both ADM and R-K 4

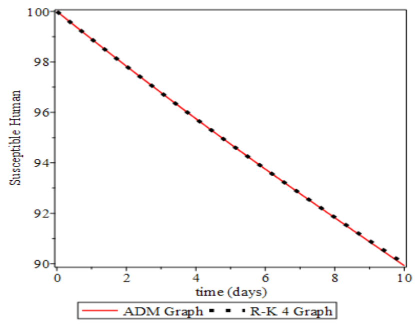


FIGURE 3. The graph of Exposed Human against Time for both ADM and R-K 4

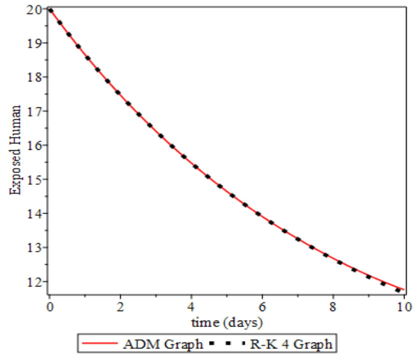


FIGURE 4. The graph of Infectious Human against Time for both ADM and R-K 4

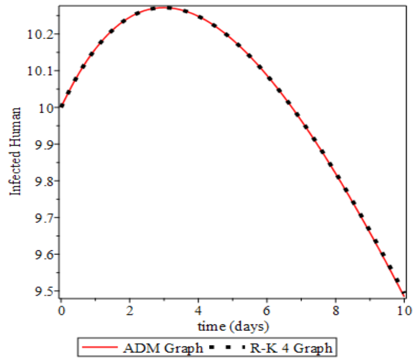


FIGURE 5. The graph of Recovered Human against Time for both ADM and R-K 4

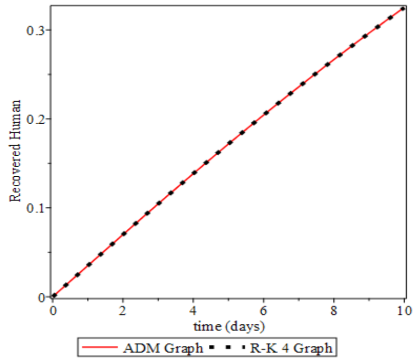


FIGURE 6. The graph of Susceptible Mosquito against Time for both ADM and R-K 4

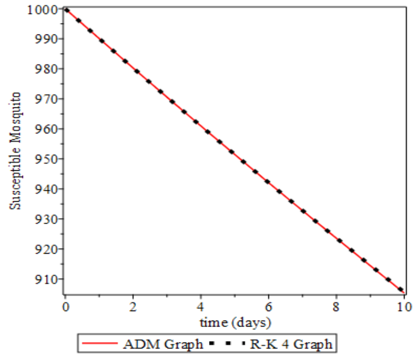


FIGURE 7. The graph of Exposed Mosquito against Time for both ADM and R-K 4

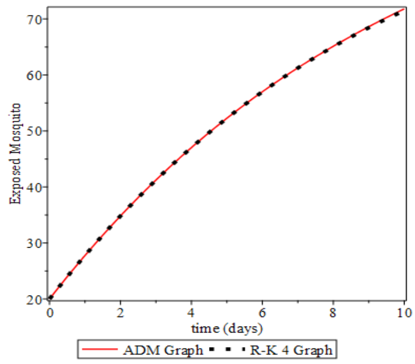
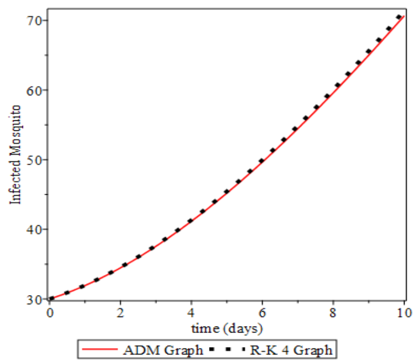


FIGURE 8. The graph of Infectious Mosquito against Time for both ADM and R-K 4



Comparison Tables for both ADM and R-K 4

TABLE 3. Human Population of the Model

T	$S_h(t)$		$E_h(t)$		$I_h(t)$		$R_h(t)$	
	R-K 4	ADM	R-K 4	ADM	R-K 4	ADM	R-K 4	ADM
0	100.0000	100.0000	20.00000	20.00000	10.00000	10.00000	0.00000	0.00000
1	98.90429	98.90431	18.65560	18.65558	10.15928	10.15928	0.03503	0.03503
2	97.83116	97.83123	17.46494	17.46487	10.24567	10.24567	0.06994	0.06994
3	96.77994	96.78008	16.40974	16.40962	10.27171	10.27170	0.10449	0.10449
4	95.75018	95.75028	15.47373	15.47370	10.24813	10.24810	0.13852	0.13852
5	94.74156	94.74116	14.64251	14.64312	10.18413	10.18401	0.17186	0.17186
6	93.75384	93.75161	13.90336	13.90615	10.08752	10.08716	0.20440	0.20441
7	92.78677	92.77965	13.24510	13.25360	9.96494	9.96401	0.23605	0.23607
8	91.84015	91.82190	12.65789	12.67923	9.82204	9.81984	0.26676	0.26680
9	90.91372	90.87294	12.13311	12.18027	9.66358	9.65887	0.29646	0.29656
10	90.00724	89.92454	11.66321	11.75818	9.49355	9.48421	0.32513	0.32535

3.2. **Discussion of results.** Adomian decomposition method is used to solve system of the first-order differential equations of the spread of malaria and the solution is compared with Runge-Kutta Fourth order method inbuilt in Maple 18. Figure 2 vividly shows that the population of susceptible humans decreases with an increase in time, so also in Figure 3, the population of exposed humans decreases with an increase in time. Figure 4 illustrates a slight increase of infected humans between the initial stage (at $t = 0$) and after three (3) days there is a steady decrease of infected humans with the increase in time. Figure 5 shows that as recovered human increases as time increases. Figure 6 represents a steady decrease of susceptible mosquito as time increases while Figures 7 and 8 depict that there is an increase of exposed and infected mosquito respectively as time increases. Tables 2 and 3 are the comparison tables between fourth-order Runge-Kutta and Adomian decomposition Methods for humans and mosquitoes population respectively.

TABLE 4. Mosquito Population of the Model

Time	$S_m(t)$		$E_m(t)$		$I_m(t)$	
(days)	R-K 4	ADM	R-K 4	ADM	R-K 4	ADM
0	1000.0000	1000.0000	20.00000	20.00000	30.00000	30.00000
1	990.09705	990.09705	27.76967	27.76967	31.96906	31.93095
2	980.28868	980.28868	34.83164	34.83165	34.55305	34.47295
3	970.57979	970.57970	41.23806	41.23817	37.69552	37.56835
4	960.97392	960.97327	47.03818	47.03894	41.34423	41.16378
5	951.47363	951.47074	52.27821	52.28160	45.45088	45.20977
6	942.08073	942.07097	57.00137	57.01281	49.97086	49.66037
7	932.79646	932.76936	61.24795	61.27976	54.86302	54.47262
8	923.62161	923.55627	65.05545	65.13219	60.08942	59.60608
9	914.55659	914.41515	68.45868	68.62488	65.61510	65.02219
10	905.60154	905.32022	71.48994	71.82065	71.40791	70.68365

4. CONCLUSION

The deterministic model developed is solved using Adomian Decomposition Method (ADM) and the results obtained were compared with the Fourth order Runge-Kutta (R-K 4) method inbuilt in Maple 18. Tables 2 and 3 clearly show the result between ADM and R-K 4 are almost the same. Furthermore, the solution obtained by ADM is in perfect agreement with R-K 4 for figures $2, 3, 4, 5, 6, 7$ and 8. This validates the efficiency and accuracy of ADM in solving the proposed malaria model.

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