

ON INTUITIONISTIC SUPRA PRE OPEN SETS

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ABSTRACT. In this paper, a new class of sets called Intuitionistic Supra Pre open sets are defined in intuitionistic supra Topological spaces. Furthermore, the properties of Intuitionistic Supra Pre open sets and Intuitionistic Supra Pre closed sets are investigated in intuitionistic supra topological spaces.

1. INTRODUCTION

In 1996, An Intuitionistic set and Intuitionistic topological space was introduced by Coker [1, 2]. In 1965, Zadeh [8] introduced the concept of fuzzy sets. Later coker [3] introduced intuitionistic fuzzy topological spaces. In 1983, A.S. Mashhour et al. [5] introduced supra topological space and s-continuous functions. In 1982, pre open set was introduced by A.S.Mashhour et al [6]. In 2008, Devi et al [4] introduced the concept of supra α open sets and supra α continuous map in topological space. In 2010, Sayed and Noiri [9] introduced supra b open set and b-continuity in topological space. Supra Pre open set was introduced by O.R.Sayed [10] and studied supra pre continuity on topological spaces.

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2. PRELIMINARIES

Definition 2.1. [1] Let X be a non-empty set, an intuitionistic set (IS in short) A is an object having the form $A = \langle X, A_1, A_2 \rangle$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of members of A , while A_2 is called the set of non-members of A .

Definition 2.2. [2] An Intuitionistic topology on a nonempty set X is a family τ of IS's in X satisfying the following axioms:

- (i) $X, \phi \in \tau$.
- (ii) $A_1 \cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$.
- (iii) $\cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in J\} \subseteq \tau$.

The pair (X, τ) is called intuitionistic topological space (ITS in short) and IS in τ is known as an intuitionistic open set (IOS in short) in X , the complement of IOS is called intuitionistic closed set (ICS in short).

Definition 2.3. [2] Let (X, τ) be an ITS and let $A = \langle X, A_1, A_2 \rangle$ be an IS in X , then the interior and closure of A are defined by:

$$Icl(A) = \bigcap \{K : K \text{ is an ICS in } X \text{ and } A \subseteq K\}.$$

$$Iint(A) = \bigcup \{K : K \text{ is an IOS in } X \text{ and } A \supseteq K\}.$$

Definition 2.4. [5] A subfamily μ of X is said to be Supra topology on X if

- (i) $X, \phi \in \mu$.
- (ii) If $A_i \in \mu, \forall_i \in j$ then $\bigcup A_i \in \mu$.

(X, μ) is called Supra topological space. The element of μ are called Supra open sets in (X, μ) and the complement of Supra open set is called Supra closed sets and it is denoted by μ^c .

Definition 2.5. Let (X, μ) be a Supra topological space. A set A is called

- (i) supra α open [4] if $A \subseteq int^\mu(cl^\mu(int^\mu(A)))$.
- (ii) supra pre open [10] if $A \subseteq int^\mu(cl^\mu(A))$.
- (iii) supra b open [9] if $A \subseteq cl^\mu(int^\mu(A)) \cup int^\mu(cl^\mu(A))$.

Definition 2.6. Let (X, τ) be an ITS. An intuitionistic set A of X is said to be

- (i) Intuitionistic preopen [11] if $A \subseteq Iint(Icl(A))$.
- (ii) Intuitionistic α -open [11] if $A \subseteq Iint(Icl(Iint(A)))$.
- (iii) Intuitionistic Supra α Open [7] if $A \subseteq Iint^\mu(Icl^\mu(Iint^\mu(A)))$.

The family of all intuitionistic pre open set, intuitionistic α -open sets and Intuitionistic Supra α Open of (X, τ) are denoted by IPOS, and $I\alpha OS$, and $IS\alpha OS$ respectively

3. INTUITIONISTIC SUPRA PRE OPEN SETS

In this section, we introduce Intuitionistic supra pre open set and some of its properties.

Definition 3.1. Let (X, μ) be an ISTS. An Intuitionistic set A is called

- (i) Intuitionistic supra pre open set if $A \subseteq Iint^\mu(Icl^\mu(A))$. The complement of Intuitionistic supra pre open Set (ISPOS in short) is called Intuitionistic supra pre closed set (ISPCS in short).
- (ii) Intuitionistic supra b-open set if $A \subseteq Icl^\mu(Iint^\mu(A)) \cup Iint^\mu(Icl^\mu(A))$. The complement of Intuitionistic supra b open Set (ISbOS in short) is called Intuitionistic supra b closed set (ISbCS in short).

Theorem 3.1. Every ISOS is an $IS\alpha OS$.

Proof. Let (X, μ) be an ISTS. Let A be an ISOS in (X, μ) then $A \subseteq Iint^\mu(A)$. We know that $Iint^\mu(A) \subseteq Iint^\mu(Icl^\mu(Iint^\mu(A)))$. Then $A \subseteq Iint^\mu(Icl^\mu(Iint^\mu(A)))$. Therefore, A is an $IS\alpha OS$. \square

The converse of the above theorem need not be true. It is shown by the following example.

Example 1. Let (X, μ) be an ISTS where $X = \{a, b, c\}$ and

$$\mu = \{\underline{X}, \phi, \langle X, \{a\}, \{c\} \rangle, \langle X, \{a, c\}, \phi \rangle\}.$$

Here $\langle X, \{a, b\}, \phi \rangle$ is an $IS\alpha OS$ but it not ISOS.

Theorem 3.2. Every $IS\alpha OS$ is ISPOS.

Proof. Let (X, μ) be an ISTS. Let A be an $IS\alpha OS$ in (X, τ) , then

$$A \subseteq Iint^\mu(Icl^\mu(Iint^\mu(A))).$$

We know that $Iint^\mu(Icl^\mu(Iint^\mu(A))) \subseteq Iint^\mu(Icl^\mu(A))$. Then $A \subseteq Iint^\mu(Icl^\mu(A))$. Therefore, A is ISPOS. \square

The converse of the above theorem need not be true. It is shown by the following example.

Example 2. Let (X, μ) be an ISTS where $X = \{a, b, c\}$ and

$$\mu = \{\underline{X}, \phi, \langle X, \{a\}, \{b\} \rangle, \langle X, \{b\}, \{c\} \rangle, \langle X, \{a, b\}, \phi \rangle\}.$$

Here, $\langle X, \phi, \phi \rangle$ is ISPOS but it is not IS α OS.

Theorem 3.3. Every ISPOS is ISbOS.

Proof. Let (X, μ) be an ISTS. Let A be an ISPOS in (X, μ) then $A \subseteq \text{Int}^\mu(\text{Icl}^\mu(A))$. We know that $\text{Int}^\mu(\text{Icl}^\mu(A)) \subseteq \text{Icl}^\mu(\text{Int}^\mu(A)) \cup \text{Int}^\mu(\text{Icl}^\mu(A))$. Then $A \subseteq \text{Icl}^\mu(\text{Int}^\mu(A)) \cup \text{Int}^\mu(\text{Icl}^\mu(A))$. Therefore, A is ISbOS. \square

The converse of the above theorem need not be true. It is shown by the following example.

Example 3. Let (X, μ) be an ISTS where $X = \{a, b, c\}$ and

$$\mu = \{\underline{X}, \phi, \langle X, \phi, \{a\} \rangle, \langle X, \{b\}, \phi \rangle, \langle X, \{c\}, \phi \rangle, \langle X, \{b, c\}, \phi \rangle\}.$$

Here, $\langle X, \{a\}, \phi \rangle$ is an ISbOS but it is not ISPOS.

The following diagram show how ISPOS are related to some similar types of ISOS.

$$\text{ISOS} \rightarrow \text{IS}\alpha\text{OS} \rightarrow \text{ISPOS} \rightarrow \text{ISbOS}$$

Theorem 3.4. Let (X, μ) be an ISTS. Then the following statements are true.

- (i) $A = \text{Icl}_p^\mu(A)$ iff A is an ISPCS.
- (ii) $\text{Int}_p^\mu(A) = A$ iff A is an ISPOS.
- (iii) If $A \subseteq B$, then $\text{Icl}_p^\mu(A) \subseteq \text{Icl}_p^\mu(B)$ and $\text{Int}_p^\mu(A) \subseteq \text{Int}_p^\mu(B)$.

Proof.

- (i) Let $A = \text{Icl}_p^\mu(A)$. By definition $\text{Icl}_p^\mu(A)$ is the intersection of all ISPCS containing A. We know that the finite intersection of all ISPCS is ISPCS. Hence $\text{Icl}_p^\mu(A)$ is ISPCS in X. since $A = \text{Icl}_p^\mu(A)$. Then A is an ISPCS.

Conversely, if A is an ISPCS, then A is an intersection of all ISPCS containing A and it is the smallest set, thus the intersection will equal to A. Therefore, $A = \text{Icl}_p^\mu(A)$.

- (ii) Let $Int_p^\mu(A) = A$. Let $x \in A$ then $x \in Int_p^\mu(A)$. Therefore, x is an ISP Interior of A. since x is an arbitrary point of A. So every point of A is an ISP interior point of A. Hence A is an ISPOS.

Conversely, A is an ISPOS. Every point of A is ISP interior point. Then $A \subseteq Int_p^\mu(A)$. We know that $Int_p^\mu(A) \subseteq A$. From the above cases we have, $Int_p^\mu(A) = A$.

- (iii) Let $A \subseteq B$. We know that, $B \subseteq \overline{B}$. Hence $A \subseteq \overline{B}$. Therefore $A \subseteq \overline{A} \subseteq B \subseteq \overline{B}$. Then $Icl_p^\mu(A) \subseteq Icl_p^\mu(B)$.

Let $x \in Int_p^\mu(A)$, there exist an ISOS U such that $x \in U \subseteq A$. since $A \subseteq B$, we have $x \in U \subseteq B$. then $x \in Int_p^\mu(B)$. Hence $Int_p^\mu(A) \subseteq Int_p^\mu(B)$.

□

4. INTUITIONISTIC SUPRA PRE CONTINUOUS FUNCTION

In this section, we introduce a Continuous functions called Intuitionistic supra pre continuous, Intuitionistic supra b continuous and obtain some of their properties.

Definition 4.1. Let (X, μ) and (Y, σ) be two ISTS. A map $f: (X, \mu) \rightarrow (Y, \sigma)$ is called:

- (i) Intuitionistic supra pre continuous function (ISPCF in short) if the inverse image of each ISOS in Y is an ISPOS in X.
- (ii) Intuitionistic supra b continuous function (ISbCF in short) if the inverse image of each ISOS in Y is an ISbOS in X.

Here, Intuitionistic supra continuous function and Intuitionistic supra α continuous function are denoted as ISCF and IS α CF.

Theorem 4.1. Every ISCF is IS α CF.

Proof. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a ISCF and A is ISOS in Y. Then $f^{-1}(A)$ is an ISOS in X. since every ISOS is an IS α OS. Then $f^{-1}(A)$ is a IS α OS in X. Hence f is a IS α CF. □

The converse of the above theorem need not be true as shown in the following example.

Example 4. $X = Y = \{a, b, c\}$ and

$$\tau = \{\underline{X}, \underline{\phi}, \langle X, \{a\}, \{b\} \rangle, \langle X, \{b\}, \{c\} \rangle, \langle X, \{a, b\}, \phi \rangle\}$$

be an ISTS on X . $\sigma = \{\underline{Y}, \underline{\phi}, \langle Y, \{a\}, \{b\} \rangle, \langle Y, \{b\}, \phi \rangle, \langle Y, \{a, b\}, \phi \rangle\}$ be an ISTS on Y . Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a map defined as $f(a) = a, f(b) = b, f(c) = c$. Here, $V = \langle Y, \{b\}, \phi \rangle$ is ISOS in Y and inverse image $f^{-1}(V) = \langle X, \{b\}, \phi \rangle$ is $IS_{\alpha}OS$ but not ISOS. Then f is $IS_{\alpha}CF$ but not ISCF.

Theorem 4.2. Every $IS_{\alpha}CF$ is ISPCF.

Proof. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a $IS_{\alpha}CF$. Let A be ISOS in Y . Then $f^{-1}(A)$ is an $IS_{\alpha}OS$ in X . since every $IS_{\alpha}OS$ is an ISPOS. Then $f^{-1}(A)$ is a ISPOS in X . Hence f is a ISPCF. \square

The converse of the above theorem need not be true as shown in the following example.

Example 5. $X = Y = \{a, b, c\}$ and

$$\tau = \{\underline{X}, \underline{\phi}, \langle X, \{a\}, \{b\} \rangle, \langle X, \{b\}, \{c\} \rangle, \langle X, \{a, b\}, \phi \rangle\}$$

be an ISTS on X . $\sigma = \{\underline{Y}, \underline{\phi}, \langle Y, \{b\}, \{c\} \rangle, \langle Y, \{c\}, \{a\} \rangle, \langle Y, \{b, c\}, \phi \rangle\}$ be an ISTS on Y . Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined as follows: $f(a) = c, f(b) = a, f(c) = b$. Here $V = \langle Y, \{b\}, \{c\} \rangle$ is ISOS in Y and its inverse image $f^{-1}(V) = \langle X, \{c\}, \{a\} \rangle$ is an ISPOS but not $IS_{\alpha}OS$. Then f is ISPCF but not $IS_{\alpha}CF$.

Theorem 4.3. Every ISPCF is ISbCF.

Proof. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a ISPCF. Let A be ISOS in Y . Then $f^{-1}(A)$ is an ISPOS in X . since every ISPOS is an ISbOS. Then $f^{-1}(A)$ is a ISbOS in X . Hence f is a ISbCF. \square

The converse of the above theorem need not be true as shown in the following example.

Example 6. $X = Y = \{a, b, c\}$ and

$$\tau = \{\underline{X}, \underline{\phi}, \langle X, \phi, \{a\} \rangle, \langle X, \{b\}, \phi \rangle, \langle X, \{c\}, \phi \rangle, \langle X, \{b, c\}, \phi \rangle\}$$

be an ISTS on X . $\sigma = \{\underline{Y}, \underline{\phi}, \langle Y, \phi, \{a\} \rangle, \langle Y, \phi, \phi \rangle\}$ be an ISTS on Y . Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined as follows: $f(a) = a, f(b) = b, f(c) = c$. Here $V = \langle Y, \phi, \phi \rangle$ ISOS in Y and its inverse image $f^{-1}(V) = \langle Y, \phi, \phi \rangle$ which is ISbOS but not ISPO. Then f is not ISPCF.

Remark 4.1. *From the above discussion we have the following diagram in which the converses of the implication need not be true.*

$$ISCF \rightarrow IS_{\alpha}CF \rightarrow ISPCF \rightarrow ISbCF.$$

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