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β^{**} GENERALIZED CLOSED SETS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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ABSTRACT. In this paper, we have introduced the notion of intuitionistic fuzzy β^{**} generalized closed sets, and investigated some of their properties and produced some characterization theorems.

1. INTRODUCTION

A generalization of fuzzy set was introduced by Atanassov [1] in 1986 as intuitionistic fuzzy set which incorporated the degree of membership and non-membership. Later the concept of intuitionistic fuzzy topological space was coined by Coker [2] in 1997. And here we have introduced a new type of intuitionistic fuzzy closed set called intuitionistic fuzzy β^{**} generalized closed sets. In this paper we have analyzed some of their properties and obtained some fascinating theorems.

2. PRELIMINARIES

Definition 2.1. *[1] An intuitionistic fuzzy set (IFS) D is of the form*

$$D = \{\langle s, \mu_D(s), \nu_D(s)\rangle : s \in S\},$$

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where the functions $\mu_D : S \to [0,1]$ *and* $\nu_D : S \to [0,1]$ *denote the degree of membership and the degree of non-membership of each element* $s \in S$ *to the set D, respectively, and* $0 \leq \mu_D(s) + \nu_D(s) \leq 1$ *for each* $s \in S$.

An intuitionistic fuzzy set D in S is simply denoted by $D = \langle s, \mu_D, \nu_D \rangle$ *instead of denoting* $D = \{\langle s, \mu_D(s), \nu_D(s)\rangle : s \in S\}$.

Definition 2.2. *[1] Let D and E be two IFSs of the form*

$$D = \{\langle s, \mu_D(s), \nu_D(s)\rangle : s \in S\}$$

and $E = \{\langle t, \mu_E(t), \nu_E(t)\rangle : t \in T\}$. *Then,*

(a) $D \subseteq E$ *if and only if* $\mu_D(s) \leq \mu_E(t)$ *and* $\nu_D(s) \geq \nu_E(t)$ *for all* $s \in S, t \in T$;

(b) $D = E$ *if and only if* $D \subseteq E$ *and* $D \supseteq E$;

(c) $D^c = \{\langle s, \nu_D(s), \mu_D(s)\rangle : s \in S\}$;

(d) $D \cup E = \{\langle s, \mu_D(s) \vee \mu_E(t), \nu_D(s) \wedge \nu_E(t)\rangle : s \in S, t \in T\}$;

(e) $D \cap E = \{\langle s, \mu_D(s) \wedge \mu_E(t), \nu_D(s) \vee \nu_E(t)\rangle : s \in S, t \in T\}$.

The intuitionistic fuzzy sets $0_\sim = \langle s, 0, 1\rangle$ *and* $1_\sim = \langle s, 1, 0\rangle$ *are resp the empty set and the whole set of S.*

Definition 2.3. *[3] An intuitionistic fuzzy topology on S is a family* τ *of IFSs in S satisfying the below conditions:*

(i) $0_\sim, 1_\sim \in \tau$;

(ii) $Q_1 \cap Q_2 \in \tau$ *for any* $Q_1, Q_2 \in \tau$;

(iii) $\cup Q_i \in \tau$ *for any family* $\{Q_k : k \in K\} \in \tau$.

In this case the pair (S, τ) *is called an intuitionistic fuzzy topological space and any IFS in* τ *is said to be an intuitionistic fuzzy open set in S. The complement* D^c *of an IFOS D in an IFTS* (S, τ) *is called an intuitionistic fuzzy closed set in S.*

Definition 2.4. *[7] Two IFSs D and E are said to be q-coincident* $(D_q E)$ *iff there exits an element* $s \in S$ *such that* $\mu_D(s) > \nu_E(s)$ *or* $\nu_D(s) < \mu_E(s)$.

Definition 2.5. *[7] Two IFSs A and E are said to be not q-coincident* $(A_{\bar{q}} E)$ *if and only if* $A \subseteq E^c$.

Definition 2.6. *[3] An intuitionistic fuzzy point (IFP), written as* $q_{(\alpha,\beta)}$, *is defined to be an IFS of S given by*

$$q_{(\alpha,\beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p \\ (0, 1) & \text{otherwise} \end{cases}$$

An intuitionistic fuzzy point $q_{(\alpha,\beta)}$ is said to belong to a set D if $\alpha \leq \mu_D$ and $\beta \geq \nu_D$.

Definition 2.7. *[5] An IFS D in (S, τ) is an intuitionistic fuzzy Q-set if $int(cl(D)) = cl(int(D))$.*

Definition 2.8. *[2] Let (S, τ) be an IFTS and $D = \langle x, \mu_D, \nu_D \rangle$ be an IFS in S. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by*

$$int(D) = \cup\{Q \,/\, Q \text{ is an IFOS in S and } Q \subseteq D\},$$
$$cl(D) = \cap\{R \,/\, R \text{ is an IFCS in S and } D \subseteq R\}.$$

It is to be noted that for any IFS D in (S, τ), we have $cl(D^c) = (int(D))^c$ and $int(D^c) = (cl(D))^c$.

Definition 2.9. *[4] An IFS $D = \langle s, \mu_D, \nu_D \rangle$ in an IFTS (S, τ) is said to be an*

- (i) *intuitionistic fuzzy γ closed set $(IF\gamma CS)$ if $cl(int(D)) \cap int(cl(D)) \subseteq D$,*
- (ii) *intuitionistic fuzzy γ open set $(IF\gamma OS)$ if $D \subseteq int(cl(D)) \cup cl(int(D))$.*

Definition 2.10. *[3] Let (S, τ) be an IFTS and $D = \langle s, \mu_D, \nu_D \rangle$ be an IFS in S. Then intuitionistic fuzzy kernel of D is the intersection of all IFOSs containing D.*

Definition 2.11. *[6] An IFS D in (S, τ) is an intuitionistic fuzzy nowhere dense set if there exists no IFOS O such that $O \subseteq cl(D)$. That is $int(cl(D)) = 0_\sim$.*

3. Intuitionistic fuzzy β^{**} generalized closed sets

Here we have defined intuitionistic fuzzy β^{**} generalized closed set and investigated some of its properties.

Definition 3.1. *An IFS A of an IFTS (S, τ) is said to be an intuitionistic fuzzy β^{**} generalized closed set (IFβ^{**}GCS) if $cl(int(cl(A))) \cap int(cl(int(A))) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (S, τ).*

Example 1. *Let $S = \{p, q\}$ and $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFT on S, where $G_1 = \langle s, (0.5_p, 0.4_q), (0.5_p, 0.6_q) \rangle$, $G_2 = \langle s, (0.8_p, 0.6_q), (0.2_p, 0.4_q) \rangle$. Then (S, τ) is an IFTS. Here the IFS $D = \langle s, (0.5_p, 0.6_q), (0.3_p, 0.4_q) \rangle$ is an IFβ^{**}GCS in S.*

Theorem 3.1. *Every IFCS is an IFβ^{**}GCS in (S, τ) but the reverse is not true in general.*

Proof. Let D be an IFCS in (S,τ). Let $D \subseteq O$ and O is an IFOS in (S,τ). Since D is an IFCS, cl(D) = D. Now $cl(int(cl(D))) \cap int(cl(int(D))) \subseteq cl(D) \cap cl(D) = D \cap D = D \subseteq O$. Hence D is an IF$\beta^{**}$GCS in (S,τ). □

Example 2. *In example 1, the* $IFSD = \langle s,(0.5_p,0.5_q),(0.5_p,0.5_q)\rangle$ *is an IF*β^{**}*GCS but not an IFCS in* (S,τ), *as* $cl(D) = G_1^c \neq D$.

Theorem 3.2. *Every IFSCS is an IF*β^{**}*GCS in* (S,τ) *but the reverse is not true in general.*

Proof. Let D be an IFSCS in (S,τ). Let $D \subseteq O$ and O is an IFOS in (S,τ). Since D is an IFSCS, $int(cl(D)) \subseteq D$. Now $cl(int(cl(D))) \cap int(cl(int(D))) \subseteq cl(D) \cap int(cl(D)) \subseteq int(cl(D)) \subseteq D \subseteq O$. Hence D is an IF$\beta^{**}$GCS in (S,τ). □

Example 3. *In example 1, the* $IFSD = \langle s,(0.3_p,0.3_q),(0.7_p,0.7_q)\rangle$ *is an IF*β^{**}*GCS but not an IFSCS in* (S,τ), *as* $int(cl(D)) = G_1 \nsubseteq D$.

Theorem 3.3. *Every IFPCS is an IF*β^{**}*GCS in* (S,τ) *but the reverse is not true in general.*

Proof. Let D be an IFPCS in (S,τ). Let $D \subseteq O$ and O is an IFOS in (S,τ). Now $cl(int(cl(D))) \cap int(cl(int(D))) \subseteq cl(D) \cap cl(int(D)) \subseteq cl(D) \cap D = D \subseteq O$, by hypothesis. Hence D is an IFβ^{**}GCS in (S,τ). □

Example 4. *In example 1, the* $IFSD = \langle s,(0.5_p,0.5_q),(0.5_p,0.5_q)\rangle$ *is an IF*β^{**}*GCS but not an IFPCS in* (S,τ), *as* $cl(int(D)) = cl(G_1) = G_1^c \nsubseteq D$.

Theorem 3.4. *Every IFRCS is an IF*β^{**}*GCS in* (X,τ) *but the reverse is not true in general.*

Proof. Let D be an IFRCS in (S,τ). Since every IFRCS is an IFCS, by theorem 3.1, D is an IFβ^{**}GCS in (S,τ). □

Example 5. *In example 1, the* $IFSD = \langle s,(0.5_p,0.5_q),(0.5_p,0.5_q)\rangle$ *is an IF*β^{**}*GCS but not an IFRCS in* (S,τ), *as* $cl(int(D)) = G_1^c \neq D$.

Theorem 3.5. *Every IF*α*CS is an IF*β^{**}*GCS in* (S,τ) *but the reverse is not true in general.*

Proof. Let D be an IFαCS in (S,τ). Let $D \subseteq O$ and O be an IFOS in (S,τ). Now $cl(int(cl(D))) \cap int(cl(int(D))) \subseteq D \cap int(cl(D)) \subseteq D \cap cl(D) = D \subseteq O$, by hypothesis. Hence D is an IFβ^{**}GCS in (S,τ). □

Example 6. *In example 1, the* $IFSD = \langle s, (0.5_p, 0.5_q), (0.5_p, 0.5_q)\rangle$ *is an IF*β^{**}*GCS but not an IF*α*CS in* (S, τ)*, as* $cl(int(cl(D))) = G_1^c \nsubseteq D$.

Theorem 3.6. *Every IF*β*CS is an IF*β^{**}*GCS in* (S, τ) *but the reverse is not true in general.*

Proof. Let D be an IFβCS in (S, τ). Let $D \subseteq O$ and O be an IFOS in (S, τ). Since D is an IFβCS, $cl(int(cl(D))) \cap int(cl(int(D))) \subseteq cl(D) \cap D = D \subseteq O$, by hypothesis. Hence D is an IFβ^{**}GCS in (S, τ). □

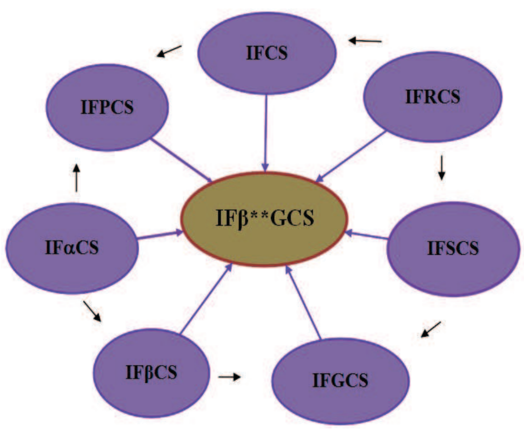
Example 7. *Let* $S = \{p, q\}$ *and* $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ *be an IFT on S, where* $G_1 = \langle s, (0.5_p, 0.6_q), (0.5_p, 0.4_q)\rangle$, $G_2 = \langle s, (0.6_p, 0.7_q), (0.4_p, 0.3_q)\rangle$. *Then* (S, τ) *is an IFTS. Here the* $IFSD = \langle s, (0.7_p, 0.8_q), (0.3_p, 0.2_q)\rangle$ *is an IF*β^{**}*GCS but not an IF*β*CS in* (S, τ)*, as* $int(cl(int(D))) = 1_\sim \nsubseteq D$.

Theorem 3.7. *Every IFGCS is an IF*β^{**}*GCS in* (S, τ) *but the reverse is not true in general.*

Proof. Let D be an IFGCS in (S, τ). Let $D \subseteq O$ and O be an IFOS in (S, τ). Now $cl(int(cl(D))) \cap int(cl(int(D))) \subseteq cl(D) \cap cl(int(D)) = cl(D) \cap cl(D) = cl(D) \subseteq O$, by hypothesis. Hence D is an IFβ^{**}GCS in (S, τ). □

Example 8. *Let* $S = \{p, q\}$ *and* $\tau = \{0_\sim, G, 1_\sim\}$ *be an IFT on S, where* $G = \langle s, (0.5_p, 0.4_q), (0.5_p, 0.6_q)\rangle$. *Then the IFS D* $= \langle s, (0.4_p, 0.4_q), (0.5_p, 0.6_q)\rangle$ *is an IF*β^{**}*GCS in S but not an IFGCS in* (S, τ)*, as* $cl(D) = G^c \nsubseteq G$*, where* $D \subseteq G$.

The interrelation of IFβ^{**}GCS with other existing closed sets is given below.



Remark 3.1. *The union of any two IFβ^{**}GCSs need not be an IFβ^{**}GCS in (S, τ) in general.*

Example 9. *Let*

$$S = \{p, q\}, G_1 = \langle s, (0.6_p, 0.8_q), (0.4_p, 0.2_q) \rangle$$

and

$$G_2 = \langle s, (0.5_p, 0.5_q), (0.4_p, 0.4_q) \rangle \, .$$

Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ is an IFT on S. Here

$$IFSsD = \langle s, (0.5_p, 0.4_q), (0.4_p, 0.5_q) \rangle$$

and $E = \langle s, (0.4_p, 0.6_q), (0.5_p, 0.2_q) \rangle$ are IFβ^{**}GCSs in (S, τ) but

$$D \cup E = \langle s, (0.5_p, 0.6_q), (0.4_p, 0.2_q) \rangle$$

is not an IFβ^{**}GCS in (S, τ), as $int(cl(int(D \cup E))) \cap cl(int(cl(D \cup E))) = 1_\sim \nsubseteq G_1$ whereas $D \cup E \subseteq G_1$.

Remark 3.2. *The intersection of any two IFβ^{**}GCSs need not be an IFβ^{**}GCS in (S, τ) in general.*

Example 10. *Let*

$$S = \{p, q\}, G_1 = \langle s, (0.5_p, 0.7_q), (0.5_p, 0.3_q) \rangle$$

and

$$G_2 = \langle s, (0.5_p, 0.6_q), (0.5_p, 0.4_q) \rangle \, .$$

Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ is an IFT on S. Here

$$IFSsD = \langle s, (0.5_p, 0.6_q), (0.5_p, 0.2_q) \rangle$$

and $E = \langle s, (0.5_p, 0.6_q), (0.4_p, 0.4_q) \rangle$ are IFβ^{**}GCSs in (S, τ) but

$$D \cap E = \langle s, (0.5_p, 0.6_q), (0.5_p, 0.4_q) \rangle$$

is not an IFβ^{**}GCS in (S, τ), as $int(cl(int(D \cap E))) \cap cl(int(cl(D \cap E))) = 1_\sim \nsubseteq G_1, G_2$ whereas $D \cap E \subseteq G_1, G_2$.

Theorem 3.8. *If D is both an IFOS and an IFβ^{**}GCS in (S, τ), then D is an IFROS in (S, τ).*

Proof. Let D be an IFOS and an IFβ^{**}GCS in S. Then

$$int(cl(int(D))) \cap cl(int(cl(D))) \subseteq D,$$

as $D \subseteq D$. Now

$$int(cl(D)) = int(cl(D)) \cap cl(D) \subseteq int(cl(int(D))) \cap cl(int(cl(D))) \subseteq D.$$

Hence $int(cl(D)) \subseteq D$. Since D is an IFOS, it is an IFPOS. Hence $D \subseteq int(cl(D))$. Therefore $D = int(cl(D))$ and Hence D is an IFROS in (S, τ). □

Theorem 3.9. *If D is both an IFOS and an IFβ^{**}GCS in (S, τ) then D is an IFβCS in (S, τ).*

Proof. Let D be an IFOS and an IFβ^{**}GCS in S. Then

$$int(cl(int(D))) \cap cl(int(cl(D))) \subseteq D,$$

as $D \subseteq D$. Now

$$int(cl(int(D))) = int(cl(int(D))) \cap cl(int(D)) \subseteq int(cl(int(D))) \cap cl(int(cl(D))) \subseteq D,$$

by hypothesis. Therefore $int(cl(int(D))) \subseteq D$ and hence D is an IFβCS in (S, τ). □

Theorem 3.10. *An IFS A of an IFTS (S, τ) is an IFβ^{**}GCS if and only if $A_{\bar{q}}E \Rightarrow (int(cl(int(A))) \cap cl(int(cl(A)))_{\bar{q}}E$ for every IFCS E of S.*

Proof. Necessity: Let E be an IFCS in S and $A_{\bar{q}}$E, then $A \subseteq E^c$, where E^c is an IFOS, Then $int(cl(int(A))) \cap cl(int(cl(A))) \subseteq E^c$, by hypothesis. Hence by Definition 2.5, $(int(cl(int(A))) \cap cl(int(cl(A)))_{\bar{q}}E$.
Sufficiency: Let O be an IFOS such that $A \subseteq O$. Then O^c is an IFCS and $A \subseteq (O^c)^c$. By hypothesis, $A_{\bar{q}}O^c \Rightarrow (int(cl(int(A))) \cap cl(int(cl(A)))_{\bar{q}}O^c$. Hence $int(cl(int(A)) \cap cl(int(cl(A))) \subseteq (O^c)^c = O$. Therefore

$$int(cl(int(A))) \cap cl(int(cl(A))) \subseteq O$$

and A is an IFβ^{**}GCS in S. □

Theorem 3.11. *For an IFOS D in (S, τ), the following conditions are equivalent:*

(i) *D is an IFCS,*
(ii) *D is an IFβ^{**}GCS and an IFQ-set.*

Proof. $(i) \Rightarrow (ii)$ Since D is an IFCS, it is an IFβ^{**}GCS, by theorem 3.1, Now $int(cl(D)) = int(D) = D = cl(D) = cl(int(D))$, by hypothesis. Hence D is an IFQ-set.
$(ii) \Rightarrow (i)$ Since D is both an IFOS and an IFβ^{**}GCS, by theorem 3.8, D is an IFROS. Therefore $D = int(cl(D)) = cl(int(D)) = cl(D)$, by hypothesis. Hence D is an IFCS in S. □

Theorem 3.12. *An IFS D of S is an IFβ^{**}GCS if $int(cl(int(D))) \cap cl(int(cl(D))) \subseteq ker(D)$.*

Proof. Let O be any IFOS such that $D \subseteq O$. By hypothesis, $int(cl(int(D))) \cap cl(int(cl(D))) \subseteq ker(D)$ and since $D \subseteq O$, $ker(D) \subseteq O$. Therefore $int(cl(int(D))) \cap cl(int(cl(D))) \subseteq O$ and hence D is an IFβ^{**}GCS in S. □

Theorem 3.13. *For an IFβ^{**}GCS D in an IFTS (S, τ), the following conditions hold:*

(i) *If D is an IFROS then $scl(D)$ is an IFβ^{**}GCS,*
(ii) *If D is an IFRCS then $sint(D)$ is an IFβ^{**}GCS.*

Proof. (i) Let D be an IFROS in (S, τ). Then $int(cl(D)) = D$. By definition, we have $scl(D) = D \cup int(cl(D)) = D$. Since D is an IF$\beta^{**}$GCS in S, $scl(D)$ is an IFβ^{**}GCS in S.
(ii) Let D be an IFRCS in (S, τ). Then $cl(int(D)) = D$. By definition, we have $sint(D) = D \cap cl(int(D)) = D$. Since D is an IF$\beta^{**}$GCS in S, sint(D) is an IFβ^{**}GCS in S. □

Theorem 3.14. *If an IFS D of an IFTS (S, τ) is intuitionistic fuzzy nowhere dense, then D is an IFβ^{**}GCS in S.*

Proof. If D is an intuitionistic fuzzy nowhere dense subset, then by Definition 2.11 we get, $int(cl(D)) = 0_\sim$. Let $D \subseteq O$ where O is an IFOS in S. Then $cl(int(cl(D))) \cap int(cl(int(D))) \subseteq cl(int(cl(D))) \cap int(cl((D)) = cl(int(cl(D))) \cap 0_\sim = 0_\sim \subseteq O$ and hence D is an IFβ^{**}GCS in S. □

Theorem 3.15. *If every IFS in (S, τ) is an IFβ^{**}GCS, then $IFO(S) \subseteq IF\gamma C(S)$.*

Proof. Suppose that every IFS in (S, τ) is an IFβ^{**}GCS. Let $O \in IFO(X)$. Then as $O \subseteq O$, and by hypothesis, $int(cl(O)) \cap cl(int(O)) = int(cl(int(O))) \cap cl(int(cl(O))) \subseteq O$. Therefore $O \in IF\gamma C(S)$ and $IFO(S) \subseteq IF\gamma C(S)$. □

4. Intuitionistic fuzzy β^{**} generalized open sets

Here we have discussed and analyzed some of the properties of intuitionistic fuzzy β^{**} generalized open sets and produced many interesting characterization theorems.

Definition 4.1. *The complement A^c of an IFβ^{**}GCS A in an IFTS (S, τ) is called an intuitionistic fuzzy β^{**} generalized open set (IFβ^{**}GOS) in S.*
The family of all IFβ^{**}GOSs of an IFTS (S, τ) is denoted by IFβ^{**}GO(S).

Example 11. *Let $S = \{p, q\}$ and $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFT on S, where $G_1 = \langle s, (0.5_p, 0.4_q), (0.5_p, 0.6_q)\rangle$, $G_2 = \langle s, (0.8_p, 0.6_q), (0.2_p, 0.4_q)\rangle$. Then (S, τ) is an IFTS. Here the $IFSD = \langle s, (0.3_p, 0.4_q), (0.5_p, 0.6_q)\rangle$ is an IFβ^{**}GOS in S.*

Theorem 4.1. *Every IFOS, IFSOS, IFPOS, IFαOS, IFROS, IFGOS and IFβOS are IFβ^{**}GOS in (S, τ) but the reverse is not true in general.*

Proof. Straightforward. □

Example 12. *In Ex. 1, the IFS D = $\langle s, (0.5_p, 0.5_q), (0.5_p, 0.5_q)\rangle$ is an IFβ^{**}GOS but not an IFOS, IFPOS, IFαOS and IFROS in (S, τ).*

Example 13. *In Ex. 1, the IFS D = $\langle s, (0.7_p, 0.7_q), (0.3_p, 0.3_q)\rangle$ is an IFβ^{**}GOS but not an IFSOS in (S, τ).*

Example 14. *In Ex. 7, the IFS D= $\langle s, (0.3_p, 0.2_q), (0.7_p, 0.8_q)\rangle$ is an IFβ^{**}GOS but not an IFβOS in (S, τ).*

Example 15. *In Ex. 8, the $IFSD = \langle s, (0.5_p, 0.6_q), (0.4_p, 0.4_q)\rangle$ is an IFβ^{**}GOS but not an IFGOS in (S, τ).*

Remark 4.1. *The union of two IFβ^{**}GOSs need not be an IFβ^{**}GOS in (S, τ) in general.*

Example 16. *Let*

$$S = \{p, q\}, G_1 = \langle s, (0.5_p, 0.7_q), (0.5_p, 0.3_q)\rangle$$

and

$$G_2 = \langle s, (0.5_p, 0.6_q), (0.5_p, 0.4_q)\rangle .$$

Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ is an IFT on S. Here

$$IFSsD = \langle s, (0.5_p, 0.2_q), (0.5_p, 0.6_q)\rangle$$

and $E = \langle s, (0.4_p, 0.4_q), (0.5_p, 0.6_q)\rangle$ *are IF*β^{**}*GOSs in S but*

$$D \cup E = \langle s, (0.5_p, 0.4_q), (0.5_p, 0.6_q)\rangle$$

is not an IFβ^{**}*GOS in S.*

Remark 4.2. *The intersection of two IF*β^{**}*GOSs need not be an IF*β^{**}*GOS in* (S, τ) *in general.*

Example 17. *Let*

$$S = \{p, q\}, G_1 = \langle s, (0.6_p, 0.8_q), (0.4_p, 0.2_q)\rangle$$

and

$$G_2 = \langle s, (0.5_p, 0.5_q), (0.4_p, 0.4_q)\rangle .$$

Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ *is an IFT on S. Here*

$$IFSsD = \langle s, (0.4_p, 0.5_q), (0.5_p, 0.4_q)\rangle$$

and $E = \langle s, (0.5_p, 0.2_q), (0.4_p, 0.6_q)\rangle$ *are IF*β^{**}*GOSs in S but*

$$D \cap E = \langle s, (0.4_p, 0.2_q), (0.5_p, 0.6_q)\rangle$$

is not an IFβ^{**}*GOS in S.*

Theorem 4.2. *An IFS D of an IFTS* (S, τ) *is an IF*β^{**}*GOS if and only if* $F \subseteq cl(int(cl(D))) \cup int(cl(int(D)))$ *whenever E is an IFCS and* $E \subseteq D$.

Proof. Necessity: Suppose D is an IFβ^{**}GOS in S. Let E be an IFCS, such that $E \subseteq D$. Then E^c is an IFOS and $D^c \subseteq E^c$, by hypothesis D^c is an IFβ^{**}GCS. We have $int(cl(int(D^c))) \cap cl(int(cl(D^c))) \subseteq E^c$. Therefore $E \subseteq cl(int(cl(D))) \cup int(cl(int(D)))$.
Sufficiency: Let O be an IFOS, such that $D^c \subseteq O$, and $O^c \subseteq D$ then $O^c \subseteq cl(int(cl(D))) \cup int(cl(int(D)))$, by hypothesis. Therefore $int(cl(int(D^c))) \cap cl(int(cl(D^c))) \subseteq O$ and D^c is an IFβ^{**}GCS. Hence D is an IFβ^{**}GOS in S. □

Theorem 4.3. *Let* (S, τ) *be an IFTS. Then for every* $D \in IFS(S)$ *and for every* $E \in IFRO(S)$, $E \subseteq D \subseteq cl(int(cl(E))) \cap int(cl(int(E)))$ *implies* $D \in IF\beta^{**}GO(S)$.

Proof. Let E be an IFROS in S. Then $E = int(cl(E))$. By hypothesis, $D \subseteq cl(int(cl(E))) \cap int(cl(int(E))) \subseteq cl(E) \cap int(cl(E)) \subseteq int(cl(E)) \subseteq int(cl(D))$ as $E \subseteq D$. Therefore D is an IFPOS and by Theorem 4.1, A is an IFβ^{**}GOS. Hence $A \in IF\beta^{**}GO(S)$. □

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