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A NOTE ON NANO $g^{\#}\alpha$-CLOSED MAPS IN NANO TOPOLOGICAL SPACES

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ABSTRACT. The point of this article is to show separation axioms of Nano $g^{\#}\alpha$ closed sets in nano topological space. We moreover present and explore nano $g^{\#}\alpha$-closed maps and additionally consider their principal properties.

1. INTRODUCTION AND PRELIMINARIES

In [6], the specialists introduced a nano topological space with regard to a subset X of an universe which is characterized in terms of lower approximation, upper approximation and boundary region. He has also presented nano closed sets (in brief N-CS) and nano open sets (in brief N-OS). In [8], the experts presented the concept of $g^{\#}\alpha$-closed sets to explore a few topological properties. V. Kokilavani et al [5] introduced $Ng^{\#}\alpha$-closed sets in nano topological space (in brief nts). The essential expected of this paper is to present separation axioms of nano $g^{\#}\alpha$ closed sets. We likewise present the concept of N$g^{\#}\alpha$-closed maps and study their properties in nts.

Definition 1.1. *A subset H of a nts $(U, \tau_R(G))$ is called*

1. *$N\alpha$-CS [6] if $Nint(Ncl(Nint(H))) \subseteq H$.*
2. *Ng-CS [1] if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is N-OS.*
3. *Ngs-CS [2] if $Nscl(H) \subseteq G$ whenever $H \subseteq G$, G is N-OS.*

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(4) *$N\alpha g$-CS [12] if $N\alpha cl(H) \subseteq G$ whenever $H \subseteq G$ and G is N-OS.*
(5) *Ng^*-CS [10] if $Ncl(H) \subseteq G$ whenever $H \subseteq G$ and G is Ng-OS.*
(6) *$Ng\alpha g$-CS [9] if $Ncl(H) \subseteq G$ whenever $H \subseteq G$ and G is $N\alpha g$-OS.*

The complements of the above sets are called their particular N-OS.

Definition 1.2. *A subset H of $(U, \tau_R(G))$ is called nano $g^{\#}\alpha$-closed set[5] (in brief $Ng^{\#}\alpha$-CS) if $N\alpha cl(H) \subseteq V$ whenever $H \subseteq V$ and V is Ng-OS in $(U, \tau_R(G))$. The complements of $Ng^{\#}\alpha$-CS is $Ng^{\#}\alpha$-OS in $(U, \tau_R(G))$.*

Definition 1.3. *Let $(U, \tau_R(G))$ and $(V, \sigma_R(H))$ be nts. Then the map*

$$f : (U, \tau_R(G)) \to (V, \sigma_R(H))$$

is called:

(1) *nano continuous (in brief N-continuous) [13] if $f^{-1}(j)$ is a N-OS (resp N-CS) in $(U, \tau_R(G))$, for each N-OS (resp N-CS) j in $(V, \sigma_R(H))$.*
(2) *$N\alpha$-continuous (in brief $N\alpha$-continuous) [7] if $f^{-1}(j)$ is a $N\alpha$-OS (resp $N\alpha$-CS) in $(U, \tau_R(G))$, for each N-OS (resp N-CS) j in $(V, \sigma_R(H))$.*
(3) *Ng-continuous (in brief Ng-continuous) [4] if $f^{-1}(j)$ is a Ng-OS (resp Ng-CS) in $(U, \tau_R(G))$, for each N-OS (resp N-CS) j in $(V, \sigma_R(H))$.*
(4) *$N\alpha g$-continuous (in brief $N\alpha g$-continuous) [7] if $f^{-1}(j)$ is a $N\alpha g$-OS (resp $N\alpha g$-CS) in $(U, \tau_R(G))$, for each N-OS (resp N-CS) j in $(V, \sigma_R(H))$.*
(5) *Ngs-continuous (in brief Ngs-continuous) [3] if $f^{-1}(j)$ is a Ngs-OS (resp Ngs-CS) in $(U, \tau_R(G))$, for each N-OS (resp N-CS) j in $(V, \sigma_R(H))$.*
(6) *$Ng\alpha g$-continuous (in brief $Ng\alpha g$-continuous) [9] if $f^{-1}(j)$ is a $Ng\alpha g$-OS (resp $Ng\alpha g$-CS) in $(U, \tau_R(G))$, for each N-OS (resp N-CS) j in $(V, \sigma_R(H))$.*
(7) *Ng^*-continuous (in brief Ng^*-continuous) [11] if $f^{-1}(j)$ is a Ng^*-OS (resp Ng^*-CS) in $(U, \tau_R(G))$, for each N-OS (resp N-CS) j in $(V, \sigma_R(H))$.*

Definition 1.4. *A map $c : (U, \tau_R(G)) \to (V, \sigma_R(H))$ is said to be a $Ng^{\#}\alpha$-continuous [5] if $c^{-1}(j)$ is a $Ng^{\#}\alpha$-closed set in $(U, \tau_R(G))$ for each nano closed set j in $(V, \sigma_R(H))$.*

Definition 1.5. *A nts $(U, \tau_R(G))$ is said to be nano $T_{g\alpha g}$-space [9] (in short $NT_{g\alpha g}$-space) if each $Ng\alpha g$-CS in it is N-CS.*

2. SEPERATION AXIOMS IN TERMS $Ng^{\#}\alpha$-CLOSED SET

Definition 2.1. *A nts $(U, \tau_R(G))$ is said to be*

(i) *nano $T^*_{1/2}$-space (in brief $NT^*_{1/2}$-space) if each Ng^*-CS in it is N-CS.*
(ii) *nano $T_{g^{\#}\alpha}$-space (in brief $NT_{g^{\#}\alpha}$-space) if each $Ng^{\#}\alpha$-CS in it is N-CS.*
(iii) *nano $\alpha^{\#}T_{1/2}$-space (in brief $N\alpha^{\#}T_{1/2}$-space) if each $Ng^{\#}\alpha$-CS in it is $N\alpha$-CS.*
(iv) *nano $^*T_{g^{\#}\alpha}$-space (in brief $N^*T_{g^{\#}\alpha}$-space) if each $Ng^{\#}\alpha$-CS in it is Ng^*-CS.*
(v) *nano $^{**}T_{g^{\#}\alpha}$-space (in brief $N^{**}T_{g^{\#}\alpha}$-space) if each $Ng^{\#}\alpha$-CS in it is $Ng\alpha g$-CS.*
(vi) *nano $\alpha^{\#}T_k$-space (in brief $N\alpha^{\#}T_k$-space) if each $N\alpha g$-CS in it is $Ng^{\#}\alpha$-CS.*
(vii) *nano $\alpha^{\#\#}T_k$-space (in brief $N\alpha^{\#\#}T_k$-space) if each Ngs-CS in it is $Ng^{\#}\alpha$-CS.*

Theorem 2.1. *In a nts $(U, \tau_R(G))$, every nano $T_{g^{\#}\alpha}$-space are nano $T^*_{1/2}$-space, $T_{g\alpha g}$-space, nano $^*T_{g^{\#}\alpha}$-space and nano $^{**}T_{g^{\#}_{\alpha}}$-space.*

Proof. Let $(U, \tau_R(G))$ be a nano $T_{g^{\#}\alpha}$-space and let D be a nano g^*-CS (resp $Ng\alpha g$-CS, $Ng^{\#}\alpha$-CS) in $(U, \tau_R(G))$. Since every Ng^*-closed set (resp $Ng\alpha g$-CS) is $Ng^{\#}\alpha$-CS [5] we have D is a nano $g^{\#}\alpha$-CS. Moreover U is a nano $T_{g^{\#}\alpha}$-space, then D is a N-CS in U. Consequently, $(U, \tau_R(G))$ is a nano $T^*_{1/2}$-space (resp $T_{g\alpha g}$-space, nano $^*T_{g^{\#}\alpha}$-space and nano $^{**}T_{g^{\#}\alpha}$-space). □

The above theorem require not be true by the following illustration.

Example 1. *Let $U = \{\alpha, \beta, \gamma, \delta\}$ with $U/R = \{\{\alpha, \beta\}, \{\gamma\}, \{\delta\}\}$ and $G = \{\alpha, \gamma, \delta\}$. Let $\tau_R(G) = \{\emptyset, \{\alpha, \beta\}, \{\gamma, \delta\}, U\}$ be the nts. Then $(U, \tau_R(G))$ is a nano $T^*_{1/2}$-space and nano $T_{g\alpha g}$-space but not nano $T_{g^{\#}\alpha}$-space. Let $U = \{a_1, a_2, a_3, a_4\}$ with $U/R = \{\{a_1\}, \{a_2, a_3\}, \{a_4\}\}$ and $G = \{a_2, a_4\}$. Let $\tau_R(G) = \{\emptyset, \{a_4\}, \{a_2, a_3\}, \{a_2, a_3, a_4\}, U\}$ be the nts. $Ng^{\#}\alpha$-closed sets = Ng^*-closed sets = $\{\emptyset, \{a_1\}, \{a_1, a_2\}, \{a_1, a_4\}, \{a_1, a_3\}, \{a_1, a_2, a_3\}, \{a_1, a_2, a_4\}, \{a_1, a_3, a_4\}\}$. Then $(U, \tau_R(G))$ is a nano $^*T_{g^{\#}\alpha}$-space but not nano $T_{g^{\#}\alpha}$-space. Let $U = \{b, e, d\}$ with $U/R = \{\{b, e\}, \{d\}\}$ and $G = \{b, e\}$. Let $\tau_R(G) = \{\emptyset, \{b, e\}, U\}$ be the nts. Then $(U, \tau_R(G))$ is a nano $^{**}T_{g^{\#}\alpha}$-space but not nano $T_{g^{\#}\alpha}$-space.*

Remark 2.1. *Every $N\alpha g$-closed (resp. $N\alpha g$-open) set is Ngs-closed (resp. Ngs-open) set.*

Theorem 2.2. *Every nano $\alpha^{\#\#}T_k$-space is nano $\alpha^{\#}T_k$-space.*

Proof. The proof is obvious. □

The converse of the above theorem need not be true shown by the following example.

Example 2. *Let $U = \{1, 2, 3, 4\}$ with $U/R = \{\{1\}, \{2, 3\}, \{4\}\}$ and $G = \{2, 4\}$. Let $\tau_R(G) = \{\emptyset, \{4\}, \{2, 3\}, \{2, 3, 4\}, U\}$ be the nts. $N\alpha g$-closed sets = $Ng^{\#}\alpha$-closed sets = $\{\emptyset, \{1\}, \{2, 3\}, \{1, 4\}, \{1, 3\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}\}$. Then $(U, \tau_R(G))$ is a nano $\alpha^{\#}T_k$-space but not nano $\alpha^{\#\#}T_k$-space.*

Theorem 2.3. *If a nts $(U, \tau_R(G))$ is nano $T_{g^{\#}\alpha}$-space, then every singleton of U is either nano g-CS or N-OS. But not conversely.*

Proof. Let $x \in U$ and $(U, \tau_R(G))$ be a nano $T_{g^{\#}\alpha}$-space. Suppose $\{x\}$ is not a nano g-CS of $(U, \tau_R(G))$, then $X\ \{x\}$ is not nano g-OS. Then X is the only nano g-OS containing $X\ \{x\}$. Then $X\ \{x\}$ is a $Ng^{\#}\alpha$-CS of $(U, \tau_R(G))$. Since $(U, \tau_R(G))$ is a nano $T_{g^{\#}\alpha}$-space, $X\ \{x\}$ is N-CS, which implies x is N-OS in $(U, \tau_R(G))$. □

Theorem 2.4. *If a nts $(U, \tau_R(G))$ is nano $\alpha^{\#}T_{1/2}$-space, then every singleton of U is either nano g-CS or nano α-OS. But not conversely.*

Proof. The proof is obvious. □

Theorem 2.5. *In a nts $(U, \tau_R(G))$, the following conditions are equivalent:*

(i) *$(U, \tau_R(G))$ is both $N^*T_{g^{\#}\alpha}$-space and $NT^*_{1/2}$ -space.*
(ii) *$(U, \tau_R(G))$ is $NT_{g^{\#}\alpha}$-space.*

Proof. $(i) \Rightarrow (ii)$: Let C be a $Ng^{\#}\alpha$-CS in $(U, \tau_R(G))$. Since $(U, \tau_R(G))$ is a $N^*T_{g^{\#}\alpha}$-space, C is Ng^*-CS. Moreover since $(U, \tau_R(G))$ is a $NT^*_{1/2}$ space, C is N-CS. Consequently, $(U, \tau_R(G))$ is a $NT_{g^{\#}\alpha}$-space.
$(ii) \Rightarrow (i)$: Let L be a N-CS and M be a $Ng^{\#}\alpha$-CS in $(U, \tau_R(G))$. Moreover each Ng^*-CS is $Ng^{\#}\alpha$-CS in $(U, \tau_R(G))$, we have $L \subseteq M$. Moreover since $(U, \tau_R(G))$ is a $NT_{g^{\#}\alpha}$-space, every $Ng^{\#}\alpha$-CS is N-CS in $(U, \tau_R(G))$, (i,e) $M \subseteq L$. Hence L = M. Let C be a Ng^*-CS in $(U, \tau_R(G))$. Since each N-CS is Ng^*-CS [11] and each Ng^*-CS is $Ng^{\#}\alpha$-CS [5], we have $L \subseteq M \subseteq C$. But $L = M \Longrightarrow M \subseteq C$. Thus L = M = C. (i,e) every $Ng^{\#}\alpha$-CS is N g^*-CS and every Ng^*-CS is N-CS in $(U, \tau_R(G))$. Consequently, $(U, \tau_R(G))$ is both $N^*T_{g^{\#}\alpha}$-space and $NT^*_{1/2}$-space. □

Theorem 2.6. *In a nts $(U, \tau_R(G))$, the taking after conditions are equivalent:*

(i) $(U, \tau_R(G))$ *is both* $N^{**}T_{g^{\#}\alpha}$*-space and* $NT_{g\alpha g}$*-space.*

(ii) $(U, \tau_R(G))$ *is* $NT_{g^{\#}\alpha}$*-space.*

Proof. $(i) \Rightarrow (ii)$: Let C be a $Ng^{\#}\alpha$-CS in $(U, \tau_R(G))$. Since $(U, \tau_R(G))$ is a $N^{**}T_{g^{\#}\alpha}$-space, C is $Ng\alpha g$-CS. Moreover since $(U, \tau_R(G))$ is a N$T_{g\alpha g}$-space, C is N-CS. Consequently, $(U, \tau_R(G))$ is a $NT_{g^{\#}\alpha}$-space.
$(ii) \Rightarrow (i)$: Let L be a N-CS and M be a $Ng^{\#}\alpha$-CS in $(U, \tau_R(G))$. Since each $Ng\alpha g$-CS is $Ng^{\#}\alpha$-CS in $(U, \tau_R(G))$, we have $L \subseteq M$. Moreover since $(U, \tau_R(G))$ is a $NT_{g^{\#}\alpha}$-space, every $Ng^{\#}\alpha$-CS is N-CS in $(U, \tau_R(G))$, (i,e) $M \subseteq L$. Hence L = M. Let C be a $Ng\alpha g$-CS in $(U, \tau_R(G))$. Since each N-CS is $Ng\alpha g$-CS [10] and every $Ng\alpha g$-CS is $Ng^{\#}\alpha$-CS [5], we have $L \subseteq M \subseteq C$. But $L = M \Longrightarrow M \subseteq C$. Thus L = M = C. (i,e) every $Ng^{\#}\alpha$-CS is $Ng\alpha g$-CS and every $Ng\alpha g$-CS is N-CS in $(U, \tau_R(G))$. Consequently, $(U, \tau_R(G))$ is both $N^{**}T_{g^{\#}\alpha}$-space and N$T_{g\alpha g}$-space. □

Theorem 2.7. *Let $f : (U, \tau_R(G)) \to (V, \sigma_R(H))$ be a $Ng^{\#}\alpha$-continuous function. If $(U, \tau_R(G))$ is a nano $\alpha^{\#}T_{1/2}$-space, then f is $N\alpha$-continuous.*

Proof. Let C be a N-CS in $(V, \sigma_R(H))$. Since f is $Ng^{\#}\alpha$-continuous, $f^{-1}(C)$ could be a $Ng^{\#}\alpha$-CS. Since $(U, \tau_R(G))$ is a nano $\alpha^{\#}T_{1/2}$-space, we have $f^{-1}(C)$ is a $N\alpha$-CS in $(U, \tau_R(G))$. Consequently, f is $N\alpha$-continuous. □

Theorem 2.8. *Let $f : (U, \tau_R(G)) \to (V, \sigma_R(H))$ be a $Ng^{\#}\alpha$-continuous function. If $(U, \tau_R(G))$ is a $NT_{g^{\#}\alpha}$-space, then f is nano continuous.*

Proof. The proof is similar. □

Theorem 2.9. *Let $(U, \tau_R(G))$ and $(V, \sigma_R(H))$ be a nts and Let $j : (U, \tau_R(G)) \to (V, \sigma_R(H))$ be a map then the following statements hold:*

(i) *If j is $N\alpha g$-continuous function and $N\alpha^{\#}T_k$-space, then j is $Ng^{\#}\alpha$-continuous.*

(ii) *If j is Ngs-continuous function and $N\alpha^{\#\#}T_k$-space, then j is $Ng^{\#}\alpha$-continuous.*

Proof. (i) Let D be a N-CS in $(V, \sigma_R(H))$. Since j is $N\alpha g$-continuous, $j^{-1}(D)$ is $N\alpha g$-CS. Since $(U, \tau_R(G))$ is a $N\alpha^{\#}T_k$-space, we have $j^{-1}(D)$ is a $Ng^{\#}\alpha$-CS in $(U, \tau_R(G))$. Consequently, j is $Ng^{\#}\alpha$-continuous.
(ii) The proof is similar. □

Theorem 2.10. *Let $(U, \tau_R(G))$ and $(V, \sigma_R(H))$ be a nts and Let $f : (U, \tau_R(G)) \to (V, \sigma_R(H))$ be a map then the following statements hold:*

(i) *If f is $Ng^{\#}\alpha$-continuous and $(U, \tau_R(G))$ be $N^{*}T_{g^{\#}\alpha}$-space, then f is Ng^{*}-continuous.*

(ii) *If f is $Ng^{\#}\alpha$-continuous and $(U, \tau_R(G))$ be $N^{**}T_{g^{\#}\alpha}$-space, then f is $Ng\alpha g$-continuous.*

Proof. (i) Let C be a N-CS in $(V, \sigma_R(H))$. Since f is $Ng^{\#}\alpha$-continuous, $f^{-1}(C)$ is $Ng^{\#}\alpha$-CS. Since $(U, \tau_R(G))$ is a $N^{*}T_{g^{\#}\alpha}$-space, we have $f^{-1}(C)$ is a Ng^{*}-CS in $(U, \tau_R(G))$. Consequently, f is Ng^{*}-continuous.
(ii) The proof is similar. □

3. $Ng^{\#}\alpha$-CLOSED MAPS

Definition 3.1. *A function $j : (U, \tau_R(G)) \to (V, \sigma_R(H))$ is called $Ng^{\#}\alpha$-open maps if for each nano open set E of $(U, \tau_R(G))$, its image j(E) is $Ng^{\#}\alpha$-open maps in $(V, \sigma_R(H))$.*

Example 3. *Let $U = \{b_1, b_2, b_3, b_4\}$ be the universe with $U/R = \{\{b_1\}, \{b_3\}, \{b_2, b_4\}\}$ and let $G = \{b_1, b_2\}$. Then the N-OS are $\{\emptyset, \{b_1\}, \{b_2, b_4\}, \{b_1, b_2, b_4\}, U\}$. The $Ng^{\#}\alpha$-CS are $\{\emptyset, \{b_3\}, \{b_3, b_4\}, \{b_1, b_3\}, \{b_2, b_4\}, \{b_2, b_3, b_4\}, \{b_1, b_3, b_4\}, \{b_1, b_2, b_4\}, U\}$. Let $V = \{a, b, c, d\}$ be the another universe with $V/R = \{\{a\}, \{b, c\}, \{d\}\}$ and let $H = \{b, d\}$. Then the N-OS are $\{\emptyset, \{d\}, \{b, c\}, \{b, c, d\}, V\}$. The $Ng^{\#}\alpha$-CS are $\{\emptyset, \{a\}, \{a, b\}, \{a, d\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}, V\}$. Define the function $j : (U, \tau_R(G)) \to (V, \sigma_R(H))$ as j(e) = b, j(f) = d, j(g) = a, j(h) = c. Now the images of N-OS of U which are $Ng^{\#}\alpha$-OS in V. Thus the function j is $Ng^{\#}\alpha$-open map.*

Definition 3.2. *A function $j : (U, \tau_R(G)) \to (V, \sigma_R(H))$ is called $Ng^{\#}\alpha$-closed maps if for every N-CS E of $(U, \tau_R(G))$, its image $j(E)$ is $Ng^{\#}\alpha$-closed maps in $(V, \sigma_R(H))$.*

Example 4. *In example 3, $j(U) = V$, $j(\emptyset) = \emptyset$, $j(\{g\}) = \{a\}$, $j(\{e, g\}) = \{a, c\}$, $j(\{f, g, h\}) = \{a, c, d\}$ are the images of N-CS of U which are $Ng^{\#}\alpha$-closed maps in V. Thus the function j is $Ng^{\#}\alpha$-closed map.*

Theorem 3.1. *A function $j : (U, \tau_R(G)) \to (V, \sigma_R(H))$ is $Ng^{\#}\alpha$-closed function if and only if $Ng^{\#}\alpha cl(f(C)) \subseteq f(Ncl(C))$ for each subset C of $(U, \tau_R(G))$.*

Proof. Let $j : (U, \tau_R(G)) \to (V, \sigma_R(H))$ be a $Ng^{\#}\alpha$-closed function and $C \subseteq U$. Then Ncl(C) is N-CS in $(U, \tau_R(G))$ and hence j(Ncl(C)) is $Ng^{\#}\alpha$-closed function in $(V, \sigma_R(H))$. Since $C \subseteq Ncl(C)$, it implies that $j(C) \subseteq j(Ncl(C))$. As $Ng^{\#}\alpha$cl(j(Ncl(C))) is the $Ng^{\#}\alpha$-CS containing j(C), it follows that

$$Ng^{\#}\alpha cl(j(C)) \subseteq Ng^{\#}\alpha cl(j(Ncl(C))) \subseteq j(Ncl(C)).$$

Conversely, let C be any N-CS in $(U, \tau_R(G))$. Then C = Ncl(C) and so $j(C) = j(Ncl(C)) \supseteq Ng^{\#}\alpha cl(j(C))$ by the given hypothesis. Also, it follows that $j(C) \subseteq Ng^{\#}\alpha cl(j(C))$. Hence $j(C) = Ng^{\#}\alpha cl(j(C))$. i.e., j(C) is $Ng^{\#}\alpha$-CS and hence $j : (U, \tau_R(G)) \to (V, \sigma_R(H))$ is $Ng^{\#}\alpha$-closed function. □

Theorem 3.2. *A function $f : (U, \tau_R(G)) \to (V, \sigma_R(H))$ is $Ng^{\#}\alpha$-CS if and only if for each subset Y of $(V, \sigma_R(H))$ and for each N-OS A of $(U, \tau_R(G))$ containing $f^{-1}(Y)$, there is a $Ng^{\#}\alpha$-OS B of $(V, \sigma_R(H))$ such that $Y \subseteq B$ and $f^{-1}(B) \subseteq A$.*

Proof. Let Y be the subset of $(V, \sigma_R(H))$ and A be a N-OS of $(U, \tau_R(G))$ such that $f^{-1}(Y) \subset A$. Now V - f(U-A), say B, is a $Ng^{\#}\alpha$-OS containing Y in V such that $f^{-1}(B) \subseteq A$. Conversely, let F be a N-OS of U, then $f^{-1}(V - f(F)) \subset U - F$ and $U - F$ is N-OS. Now, there is a $Ng^{\#}\alpha$-OS B of $(V, \sigma_R(H))$ such that $V - f(F) \subset B$ and $f^{-1}(B) \subset U - F$. Hence $F \subset U - f^{-1}(B)$ and thus $V - B \subset f(F) \subset f(U - f^{-1}(B)) \subset V - B$ which implies $f(F) = V - B$. Since $V - B$ is $Ng^{\#}\alpha$-CS, f(F) is a $Ng^{\#}\alpha$-CS in $(V, \sigma_R(H))$ for each N-CS F in $(U, \tau_R(G))$. Hence $f : (U, \tau_R(G)) \to (V, \sigma_R(H))$ is a $Ng^{\#}\alpha$-closed function. □

Remark 3.1. *The following illustration shows that the composition of two $Ng^{\#}\alpha$-closed function require not be $Ng^{\#}\alpha$-closed.*

Example 5. *Let $U = \{i, j, k, l\}$ be the universe with $U/R = \{\{i\}, \{k\}, \{j, l\}\}$ and let $G = \{i, j\}$. Then the N-OS are $\{\emptyset, \{i\}, \{j, l\}, \{i, j, l\}, U\}$ and the N-CS are $\{\emptyset, \{k\}, \{i, k\}, \{j, k, l\}, U\}$. The $Ng^{\#}\alpha$-CS are $\{\emptyset, \{k\}, \{k, l\}, \{i, k\}, \{j, l\}, \{j, k, l\}, \{i, k, l\}, \{i, j, l\}, U\}$. The $Ng^{\#}\alpha$-OS are $\{\emptyset, \{k\}, \{j\}, \{i\}, \{i, k\}, \{j, l\}, \{i, l\}, \{i, j, l\}, U\}$. Let $V = \{a, b, c, d\}$ be another universe with $V/R = \{\{a\}, \{b, c\}, \{d\}\}$ and let $H = \{b, d\}$. Then the N-OS are $\{\emptyset, \{d\}, \{b, c\}, \{b, c, d\}, V\}$ and the N-CS are $\{\emptyset, \{a\}, \{a, d\}, \{a, b, c\}, V\}$. The $Ng^{\#}\alpha$-CS are $\{\emptyset, \{a\}, \{a, b\}, \{a, d\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}, V\}$and $Ng^{\#}\alpha$-OS are $\{\emptyset, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}, V\}$.*

Also, $W = \{t, u, v\}$ with $W/R = \{\{t, u\}, \{v\}\}$ and $I = \{t, u\}$. Let $\{\emptyset, \{t, u\}, W\}$ be the N-OS and $\{\emptyset, \{v\}, W\}$ be the N-CS. The $Ng^{\#}\alpha$-CS are $\{\emptyset, \{v\}, \{u, v\},$

$\{t,v\}, W\}$ and the $Ng^{\#}\alpha$-OS are $\{\emptyset, \{t\}$, $\{u\}$, $\{t,u\}, W\}$. Define the function $f:(U,\tau_R(G))\to(V,\sigma_R(H))$ as f(i) = b, f(j) = d, f(k) = a, f(l) = c. Also define the map $h:(V,\sigma_R(H))\to(W,\eta_R(I))$ be h(a) = t = h(d), h(b) = u, h(c) = v. Then both f and h are $Ng^{\#}\alpha$-closed functions but their composition $h\circ f:(U,\tau_R(G))\to(W,\eta_R(I))$ is not $Ng^{\#}\alpha$-closed functions since for the N-CS $\{j,k,l\}$ in $(U,\tau_R(G))$, $h\circ f(\{j,k,l\}) = h[f(\{j,k,l\})] = h[\{a,c,d\}] = \{t,v\}$ is not $Ng^{\#}\alpha$-CS in $(W,\eta_R(I))$. Consequently, the composition of two $Ng^{\#}\alpha$-closed functions require not be $Ng^{\#}\alpha$-CS.

Theorem 3.3. *If $f:(U,\tau_R(G))\to(V,\sigma_R(H))$ be a nano closed map and $h:(V,\sigma_R(H))\to(W,\eta_R(I))$ be $Ng^{\#}\alpha$-closed mapping then their composition $h\circ f:(U,\tau_R(G))\to(W,\eta_R(I))$ is $Ng^{\#}\alpha$-closed mapping.*

Proof. Let C be a N-CS in $(U,\tau_R(G))$. Then f(C) is nano closed in $(V,\sigma_R(H))$. Then $h\circ f(C) = h(f(C))$ is $Ng^{\#}\alpha$-closed since $h:(V,\sigma_R(H))\to(W,\eta_R(I))$ is $Ng^{\#}\alpha$-closed map. Hence their composition is $Ng^{\#}\alpha$-closed mapping. □

Theorem 3.4. *If $f:(U,\tau_R(G))\to(V,\sigma_R(H))$ and $h:(V,\sigma_R(H))\to(W,\eta_R(I))$ be two mappings such that their composition $h\circ f:(U,\tau_R(G))\to(W,\eta_R(I))$ is $Ng^{\#}\alpha$-closed mapping. If f is nano continuous and surjective then h is $Ng^{\#}\alpha$-closed.*

Proof. Let D be a N-CS in $(V,\sigma_R(H))$. Since $f:(U,\tau_R(G))\to(V,\sigma_R(H))$ is nano continuous, it follows that $f^{-1}(D)$ is nano closed in $(U,\tau_R(G))$. Since $h\circ f:(U,\tau_R(G))\to(W,\eta_R(I))$ is $Ng^{\#}\alpha$-closed mapping, $(h\circ f)[f^{-1}(D)]$ is $Ng^{\#}\alpha$-closed in $(W,\eta_R(I))$. i.e., h(D) is $Ng^{\#}\alpha$-closed in $(W,\eta_R(I))$ as the function $f:(U,\tau_R(G))\to(V,\sigma_R(H))$ is surjective. Hence the image of a nano closed set in $(V,\sigma_R(H))$ is $Ng^{\#}\alpha$-closed in $(W,\eta_R(I))$. Thus h: $(V,\sigma_R(H))\to(W,\eta_R(I))$ is $Ng^{\#}\alpha$-closed. □

Theorem 3.5. *Let $(U,\tau_R(G))$ and $(V,\sigma_R(H))$ be any two nts. Let $j:(U,\tau_R(G))\to(V,\sigma_R(H))$ be nano closed map. Then j is $Ng^{\#}\alpha$-closed map but not conversely.*

Proof. Let $j:(U,\tau_R(G))\to(V,\sigma_R(H))$ be a nano closed map. Let E be a N-CS in nts $(U,\tau_R(G))$. Then the image under the map j is nano closed in the nts $(V,\sigma_R(H))$. Since every N-CS is $Ng^{\#}\alpha$-closed set. j(E) is $Ng^{\#}\alpha$-CS. Hence j is $Ng^{\#}\alpha$-closed. □

The subsequent illustration shows that the reverse implication is not true.

Example 6. *In example 3, the set $\{e, g\}$ is $Ng^{\#}\alpha$-closed map but it is not nano closed map.*

4. Conclusion

In this article we examined separation axioms of N$g^{\#}\alpha$-CS and N$g^{\#}\alpha$-homeomorphism. Additionally we analyzed their fundamental properties. In future it makes a difference to apply the concept in mappings and compactness.

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