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SOFT PRE SEPERATION AXIOMS AND SOFT PRE COMPACT SPACES

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ABSTRACT. In this paper soft pre separations are defined and few of their properties are stated and proved. Also soft pre compactness is defined and properties of a such a space are discussed.

1. INTRODUCTION

Separation axioms [2] gives a way of classifying topological spaces according to topological distinguishability of points and subsets in the space. Separation axioms are of various degrees of strengths and they are called $T_0, T_1, T_2, T_3, T_4,$ and T_5 axioms. T_0 is the weakest axiom. The letter T stands for the German word for separation 'Trennung'. In this paper a study is done on soft separation axioms and soft pre separation axioms and soft pre compactness. Compactness [2] help to study about a space just with a finite number of open sets. Soft set theory was proposed by Molodtsov [3] in 1999 to deal with uncertainty. He defined soft set over X as a pair (F, E) where F is a mapping of E- a set of parameters - into the set of all subsets of the set X. He also defined an operation on soft sets as follows: (F, A) $*$(G, B) = (H, A $\times$ B) where $H(\alpha, \beta) = F(\alpha) * G(\beta),\ \alpha \in A,\ \beta \in B$ and $A \times B$ is the Cartesian product of A and B. Soft topological spaces were defined by Shabir [4]. Soft neighbourhoods

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were explained by Nazmul [5]. Soft preopen sets were introduced by Mrudula. R [1].

Definition 1.1. *[3] Let X be the initial universe and E be the set of parameters. Let P(X) denote the power set of X and A be a non-empty subset of E . A pair (F, A) is called a soft set over X, where F is a mapping given by* $F : A \to P(X)$*. such that F(e) =* φ *if e* $\in$ *A. Here F is called approximate function of the soft set (F, E) and the set F(e) is called e approximate value set which consist of related objects of the parameter e*$\in$ *E . In other words, a soft set over X is a parametrized family of subsets of the universe X .*

Definition 1.2. *[3] For two soft sets (F, A) and (G, B) over a common universe X and* $A, B \in E$ *we say that (F, A) is a soft subset of (G, B) if* $A \subset B \; \forall \; e \in A, F(e) \subseteq G(e)$*. We write,* $(F, A) \,\tilde{\subseteq}\, (G, B)$*.*

Definition 1.3. *[3] Union of two soft sets (F, A) and (G,B) over the common universe (X, A) is the soft set (H, C), where* $C = A \cup B$ *and for* $e \in C$*,*

$$H(e) = \begin{cases} F(e) \text{ , if } e \in A - B \\ G(e) \text{ , if } e \in B - A \\ F(e) \cup G(e) \text{ , if } e \in A \cap B \end{cases}$$

and $(F, A) \,\tilde{\cup}\, (G, B) = (H, C)$*.*

Definition 1.4. *[3] Let (F, A) and (G, B) be two soft sets over the universe X with* $A \cap B \neq \varphi$*. Then intersection of two soft sets (F, A) and (G, B) is a soft set (H, C) where C = A* $\cap$ *B and* $\forall e \in C, H(e) = F(e) \cap G(e)$*. We write (F, A)* $\cap$ *(G,B) = (H,C).*

Definition 1.5. *[3] The complement of a soft set (F, A) is denoted by (F, A)*^{c} *and is defined by (F, A)*^{c} *= (*F^c*, A) where* F^c *: A* $\to$ *P(X) is a mapping given by* $F^c(e)$ *=* $[F(e)]^c, \forall e \in A$

Example 1. *Let* $X = \{a, b\}, A = \{e_1, e_2\}$*. Define*

$(F_1, A) = \{(e_1, \Phi), (e_2, \Phi)\}$, $(F_2, A) = \{(e_1, \Phi), (e_2, \{a\})\}$,
$(F_3, A) = \{(e_1, \Phi), (e_2, \{b\})\}$, $(F_4, A) = \{(e_1, \Phi), (e_2, \{a, b\})\}$,
$(F_5, A) = \{(e_1, \{a\}), (e_2, \Phi)\}$, $(F_6, A) = \{(e_1, \{a\}), (e_2, \{a\})\}$,
$(F_7, A) = \{(e_1, \{a\}), (e_2, \{b\})\}$, $(F_8, A) = \{(e_1, \{a\}), (e_2, \{a, b\})\}$,
$(F_9, A) = \{(e_1, \{b\}), (e_2, \Phi)\}$, $(F_{10}, A) = \{(e_1, \{b\}), (e_2, \{a\})\}$,

$(F_{11}, A) = \{(e_1, \{b\}), (e_2, \{b\})\}$, $(F_{12}, A) = \{(e_1, \{b\}), (e_2, \{a, b\})\}$,
$(F_{13}, A) = \{(e_1, \{a, b\}), (e_2, \Phi)\}$, $(F_{14}, A) = \{(e_1, \{a, b\}), (e_2, \{a\})\}$,
$(F_{15}, A) = \{(e_1, \{a, b\}), (e_2, \{b\})\}$, $(F_{16}, A) = \{(e_1, \{a, b\}), (e_2, \{a, b\})\}$
are all soft on universal set X under the parameter set A.

$\tau = \{(F_1, A), (F_5, A), (F_7, A), (F_8, A), (F_{16}, A)\}$ *is a soft topology over X.*
Soft open sets are $(F_1, A), (F_5, A), (F_7, A), (F_8, A), (F_{16}, A)$
Soft closed sets are $(F_1, A), (F_9, A), (F_{10}, A), (F_{12}, A), (F_{16}, A)$.
Soft preopen sets are $(F_1, A), (F_5, A), (F_6, A), (F_7, A), (F_8, A), (F_{13}, A), (F_{14}, A), (F_{15}, A), (F_{16}, A)$.
Soft preclosed sets are $(F_1, A), (F_2, A), (F_3, A), (F_4, A), (F_9, A), (F_{10}, A), (F_{11}, A), (F_{12}, A), (F_{16}, A)$.

2. Soft pre separation axioms

Definition 2.1. *Let (X, A, τ) be a soft topological space over X and $\tilde{x}_e,\ \tilde{y}_f \tilde{\in} (X, A)$ such that $\tilde{x}_e \neq \tilde{y}_f$. Then (X, A, τ) is said to be soft pre T_0 space if there exist soft preopen sets (F, A) and (G, A) such that $\tilde{x}_e \tilde{\in} (F, A)$ and $\tilde{y}_f \widetilde{\notin} (F, A)$ or $\tilde{y}_f \tilde{\in} (G, A)$ and $\tilde{x}_e \widetilde{\notin} (G, A)$.*

Remark 2.1. *A discrete soft topological space is a soft pre T_0 space, since every soft singleton set is a soft pre open set.*

Theorem 2.1. *A soft topological space (X, A, τ) is soft pre T_0 space if and only if soft pre closures of any two distinct soft singletons are different.*

Proof. Let the soft topological space (X, A, τ) be a soft pre T_0 space and $(\tilde{x}_e, A)$ and $(\tilde{y}_f, A)$ be two distinct soft singletons.
Then $(\tilde{x}_e, A) \widetilde{\subseteq} (U, A) \widetilde{\subseteq} (\tilde{y}_f, A)^c$, where (U, A) is a soft preopen set.
$\Rightarrow (\tilde{y}_f, A) \widetilde{\subseteq} \widetilde{spcl}(\tilde{y}_f, A) \widetilde{\subseteq} (U, A)^c$.
$\Rightarrow (\tilde{x}_e, A) \widetilde{\nsubseteq} \widetilde{spcl}(\tilde{y}_f, A)$ but $(\tilde{x}_e, A) \widetilde{\subseteq} \widetilde{spcl}(\tilde{x}_e, A)$
$\Rightarrow \widetilde{spcl}(\tilde{x}_e, A) \neq \widetilde{spcl}(\tilde{y}_f, A)$
Conversely $(\tilde{x}_e, A)$ and $(\tilde{y}_f, A)$ be any two soft singletons with different soft preclosures. Then $(\widetilde{spcl}(\tilde{y}_f, A))^c$ and $(\widetilde{spcl}(\tilde{x}_e, A))^c$ are distinct soft preopen sets containing $(\tilde{x}_e, A)$ and $(\tilde{y}_f, A)$ respectively. □

Theorem 2.2. *A soft subspace of a soft pre T_0 space is soft pre T_0.*

Proof. Let (X, A, τ) be a soft pre T_0 space. Let (Y, A) be a soft subspace of (X, A, τ). Any two distinct points in (Y, A) are distinct in (X, A). They have distinct soft pre neighbourhood in (Y, A) under subspace soft topology. □

Definition 2.2. *Let (X, A, τ) be a soft topological space over X and $\tilde{x}_e,\ \tilde{y}_e \in (X, A)$ such that $\tilde{x}_e \neq \tilde{y}_e$. Then (X, A, τ) is said to be soft pre T_1 space if there exist soft pre open sets (F, A) and (G, A) such that $\tilde{x}_e \in (F, A)$ and $\tilde{y}_e \widetilde{\notin} (F, A)$ and $\tilde{y}_e \tilde{\in} (G, A)$ and $\tilde{x}_e \widetilde{\notin} (G, A)$.*

Example 2. *In Example 1, if $\tau = \{\tilde{\Phi}_A, \tilde{X}_A, (F_2, A), (F_{15}, A)\}$, then (F_2, A) and (F_{15}, A) are soft preopen sets such that $a_{e_2} \tilde{\in} (F_2, A)$, $b_{e_1} \tilde{\notin} (F_2, A)$, $b_{e_1} \tilde{\in} (F_{15}, A)$ and $a_{e_2} \widetilde{\notin} (F_{15}, A)$.*

Remark 2.2. *Obviously every soft pre T_1 space is soft pre T_0 space but the converse is not true.*

Example 3. *In Example 1, if $\tau = \{\tilde{\Phi}_A, \tilde{X}_A, (F_2, A), (F_7, A)\}$, then (X, A, τ) is a soft pre T_0 space but not soft pre T_1 space.*

Theorem 2.3. *A soft subspace of a soft pre T_1 space is soft pre T_1.*

Proof. Obvious. □

Theorem 2.4. *Let (X, A, τ) be a soft topological space over X. If every soft point of a soft topological space (X, A, τ) is soft preclosed, then (X, A, τ) is a soft pre T_1 space.*

Proof. If every soft point of soft topological space (X, A, τ) is soft preclosed, then their compliments are soft preopen sets satisfying the required condition. □

Definition 2.3. *Let (X, A, τ) be a soft topological over X and $\tilde{x}_e,\ \tilde{y}_f \tilde{\in} (X, A)$ such that $\tilde{x}_e \neq \tilde{y}_f$ Then a soft topological space (X, A, τ) is said to be soft pre T_2 space if there exists soft preopen sets (F,A) and (G,A) such that $\tilde{x}_e \tilde{\in} (F, A)$ and $\tilde{y}_f \tilde{\in} (G, A)$ and $(F, A) \tilde{\cap} (G, A) = (\Phi, A)$.*

Example 4. *In Example 1, if $\tau = \{\tilde{\Phi}, \tilde{X}_A, (F_6, A), (F_{11}, A)\}$, then (X, A, τ) is a soft pre T_2 space.*

Remark 2.3. *Obviously every soft pre T_2 space is soft pre T_1 space but the converse is not true.*

Theorem 2.5. *A soft subspace of a soft pre T_2 space is soft pre T_2.*

Proof. Obvious. □

Theorem 2.6. *Let (X, A, τ) be a soft topological space over X and $\tilde{x}_e \tilde{\in} (X, A)$. If (X, A) is a soft pre T_2 space then $(\tilde{x}_e, A) = \tilde{\cap}\ (F, A)$ for each soft preopen set (F, A) with $\tilde{x}_e \tilde{\in} (F, A)$.*

Proof. Let $\tilde{x}_e, \tilde{z}_e \tilde{\in} \tilde{\cap}\ (F, A)$. Since (X, A, τ) is soft pre T_2 space there exists soft preopen sets (H, A) and (G, A) such that $\tilde{x}_e \tilde{\in} (H, A)$ and $\tilde{z}_e \tilde{\in} (G, A)$ with $(H, A) \tilde{\cap} (G, A) = (\Phi, A)$, implies that $\tilde{z}_g \tilde{\notin} (H, A) \Rightarrow \tilde{z}_g \tilde{\notin} \tilde{\cap}\ (F, A)$, which is the contradiction. □

Definition 2.4. *Let (X, A, τ) be a soft topological space over X, and let (G, A) be a soft preclosed set in (X, A) and $\tilde{x}_e \tilde{\in} (X, A)$ such that $\tilde{x}_e \tilde{\notin} (G, A)$. If there exist soft preopen sets (F_1, A) and (F_2, A) such that $\tilde{x}_e \tilde{\in} (F_1, A)$ and $(G, A) \tilde{\subseteq} (F_2, A)$ and $(F_1, A) \tilde{\cap} (F_2, A) = (\Phi, A)$ then $(X, A,\ \tau)$ is called a soft preregular space.*

Proposition 2.1. *Let (X, A, τ) be a soft topological space over X. If every soft preopen set of (X, A) is soft preclosed, then (X, A) is soft preregular space.*

Proof. Let (F, A) be a soft preopen set in (X, A, τ) and $\tilde{x}_e \tilde{\in} (X, A)$ such that $\tilde{x}_e \tilde{\notin} (F, A)$. Then (F, A) and $(F, A)^c$ are soft preopen sets. Also $(F, A) \tilde{\subseteq} (F, A)$ and $\tilde{x}_e \tilde{\in} (F, A)^c$ but $(F, A) \tilde{\cap} (F, A)^c = (\Phi, A)$. Therefore (X, A, τ) is a soft preregular space. □

Remark 2.4. *Every discrete soft topological space is soft preregular.*

Definition 2.5. *Let (X, A, τ) be soft topological space over X. Then (X, A, τ) is said to be soft pre T_3 space if it is soft preregular and soft pre T_1 space.*

Definition 2.6. *A soft topological space (X, A, τ) is said to be a soft prenormal space if for every pair of disjoint soft preclosed sets (F, A) and (G, A) there exists two disjoint soft preopen sets (F_1, A) and (F_2, A) such that $(F, A) \tilde{\subseteq} (F_1, A)$ and $(G, A) \tilde{\subseteq} (F_2, A)$.*

Definition 2.7. *A soft prenormal pre T_1 space is called a soft pre T_4 space.*

Remark 2.5. *Every soft pre T_4 space is soft pre T_3 space, every soft pre T_3 space is soft pre T_2 space, every soft pre T_2 space is soft pre T_1 space and every soft pre T_1 space is soft pre T_0 space.*

Definition 2.8. *A soft topological space (X, A, τ) is said to be soft pre R_0 space if and only if for each soft preopen set (G, A), $\tilde{x}_e \tilde{\in} (G, A)$ implies $\tilde{s}pcl(\tilde{x}_e) \tilde{\subseteq} (G, A)$.*

Definition 2.9. *A soft topological space (X, A, τ) is said to be a soft pre R_1 space if and only if for $\tilde{x}_e,\ \tilde{y}_f \tilde{\in} (X, A)$ with $\tilde{s}pcl(\tilde{x}_e) \neq \tilde{s}pcl(\tilde{y}_f)$ there exist disjoint soft preopen sets (F, A) and (G, A) such that $\tilde{s}pcl(\tilde{x}_e) \tilde{\subseteq} (F, A)$ and $\tilde{s}pcl(\tilde{y}_f) \tilde{\subseteq} (G, A)$.*

Theorem 2.7. *A soft topological space (X, A, τ) is soft preR_0 if and only if for every soft preclosed set (F, A) and $\tilde{x}_e \notin (F, A)$ there exists a soft preopen set (U, A) such that $(F, A) \tilde{\subseteq} (U, A)$ and $\tilde{x}_e \notin (U, A)$.*

Proof. Let (X, A, τ) be a soft pre R_0 and $(F, A) \tilde{\subseteq} (X, A)$ be soft preclosed set not containing the point $\tilde{x}_e \tilde{\in} (X, A)$. Then $(X, A) - (F, A)$ is soft preopen and $\tilde{x}_e \tilde{\in} (X, A) - (F, A)$. Since (X, A) is a soft pre R_0, $\tilde{s}pcl(\tilde{x}_e) \tilde{\subseteq} (X, A) - (F, A)$. Then it follows that $(F, A) \tilde{\subseteq} (X, A) - \tilde{s}pcl(\tilde{x}_e)$. Let $(U, A) = (X, A) - \tilde{s}pcl(\tilde{x}_e)$. Then (U, A) is soft preopen set such that $(F, A) \tilde{\subseteq} (U, A)$ and $\tilde{x}_e \tilde{\notin} (U, A)$. Conversely, let $\tilde{x}_e \tilde{\in} (U, A)$ where (U, A) is a soft preopen set in (X, A). Then $(X, A) - (U, A)$ is a soft preclosd set and $\tilde{x}_e \tilde{\notin} (X, A) - (U, A)$. Then by hypothesis, there is a soft preopen set (W, A) such that $(X, A) - (U, A) \tilde{\subseteq} (W, A)$ and $\tilde{x}_e \tilde{\in} (W, A)$.

Now $(X, A) - (W, A) \tilde{\subseteq} (U, A)$ and $\tilde{x}_e \tilde{\in} (X, A) - (W, A)$. Now $(X, A) - (W, A)$ is soft preclosed. Hence $\tilde{s}pcl(\tilde{x}_e) \tilde{\subseteq} (X, A) - (W, A) \tilde{\subseteq} (U, A)$.

Therefore (X, A) is soft pre R_0. □

Theorem 2.8. *A soft topological space (X, A, τ) is soft pre T_1 if and only if it is soft preT_0 and soft pre R_0 space.*

Proof. Let (X, A, τ) be a soft pre T_1 space. Then by definition, and as every soft pre T_1 space is soft pre R_0 it is clear that (X, A, τ) is soft preT_0 and soft pre R_0 space. Conversely, Let us assume that the soft topological space (X, A, τ) is both soft preT_0 and soft pre R_0. To show that (X, A, τ) is a soft pre T_1 space. Let $\tilde{x}_e,\ \tilde{y}_f \tilde{\in} (X, A, \tau)$ be any pair of distinct points. Since (X, A, τ) is soft pre T_0 , there exists soft preopen set (G, A) such that $\tilde{x}_e \tilde{\in} (G, A)$ and $\tilde{y}_e \tilde{\notin} (G, A)$ or there exists a soft preopen set (H, A) such that $\tilde{y}_e \tilde{\in} (H, A)$ and $\tilde{x}_e \tilde{\notin} (H, A)$. Suppose $\tilde{x}_e \tilde{\in} (G, A)$ and $\tilde{y}_f \tilde{\notin} (G, A)$. As $\tilde{x}_e \tilde{\in} (G, A)$ implies $\tilde{s}pcl(\tilde{x}_e) \tilde{\subseteq} (G, A)$. $\tilde{y}_f \tilde{\notin} (G, A),\ \tilde{y}_f \tilde{\notin} \tilde{s}pcl(\tilde{x}_e)$.

Hence $\tilde{y}_f \tilde{\in} (H, A) = (X, A) - \tilde{s}pcl(\tilde{x}_e)$ and it is clear that $\tilde{x}_e \tilde{\notin} (H, A)$. Hence, it follows that there exists soft preopen sets (G, A) and (H, A) containing $\tilde{x}_e$ and $\tilde{y}_f$ respectively such that $\tilde{x}_e \tilde{\in} (H, A)$ and $\tilde{y}_f \tilde{\notin} (G, A)$.

This implies that (X, A, τ) is soft pre T_1. □

Remark 2.6. *Every soft pre R_1 space is soft pre R_0.*

Remark 2.7. *Every soft pre T_2 space is soft preT_0.*

Theorem 2.9. *The soft topological space (X, A, τ) is soft pre T_2 if and only if it is soft preR_1 and soft pre T_0.*

Proof. Let (X, A, τ) be soft pre T_2. Let $\tilde{x}_e,\ \tilde{y}_f \tilde{\in}\ (X, A, \tau)$ then there exist disjoint soft preopen sets (F, A) and (G, A) such that $\tilde{x}_e \tilde{\in} (F, A)$ and $\tilde{y}_f \tilde{\in} (G, A)$. which implies (X, A, τ) is soft pre T_0. Also $\tilde{y}_f \tilde{\in} (X, A) - \tilde{s}pcl(\tilde{x}_e)$ and $\tilde{x}_e \tilde{\in} (X, A) - \tilde{s}pcl(\tilde{y}_f)$ are disjoint soft preopen sets. Moreover $\tilde{s}pcl(\tilde{y}_f) \tilde{\subseteq} (X, A) - \tilde{s}pcl(\tilde{x}_e)$ and $\tilde{s}pcl(\tilde{x}_e) \tilde{\subseteq} (X, A) - \tilde{s}pcl(\tilde{y}_f)$, which implies (X, A, τ) is soft pre R_1. Conversely, let (X, A, τ) is soft pre T_0. This implies $\tilde{s}pcl(\tilde{x}_e) \neq \tilde{s}pcl(\tilde{y}_f)$ where $\tilde{x}_e \neq \tilde{y}_f$. Since (X, A) is soft pre R_1, there exist soft preopen sets (F, A) and (G, A) such that $\tilde{s}pcl(\tilde{x}_e) \tilde{\subseteq} (F, A)$ and $\tilde{s}pcl(\tilde{y}_f) \tilde{\subseteq} (G, A)$. That is $\tilde{x}_e \tilde{\in} (F, A)$ and $\tilde{y}_f \tilde{\in} (G, A)$ where $(F, A) \tilde{\cap} (G, A) = (\Phi, A)$. Hence(X, A) is soft pre T_2. □

3. SOFT PRE COMPACT SPACES

Compactness help to study about a space just with a finite number of open sets. In this section soft compactness is dealt with.

Definition 3.1. *A collection $\{(F_\alpha, A)\}_{\alpha \in J}$ of soft pre open sets in (X, A) is said to be soft pre open cover of (X, A), if $(X, A) = \tilde{\cup}_{\alpha \in J} \{(F_\alpha, A)\}$.*

Definition 3.2. *A soft topological space (X, A, τ) is said to be soft precompact, if every soft pre open covering of (X, A) contains a finite sub collection that also cover (X, A). A subset (F, A) of (X, A) is said to be soft precompact, if every covering of (F, A) by soft preopen sets in (X, A) contains a finite subcover.*

Theorem 3.1.

(i) *A soft topological space (X, A, τ) is soft precompact ⇒ soft compact.*

(ii) *Any finite soft topological space is soft precompact.*

Proof.

(i) Let $\{(F_\alpha, A)\}_{\alpha\in J}$ be a soft open cover for (X, A). Then each (F_α, A) is soft preopen. Since (X, A) is soft precompact, this soft open cover has a finite sub cover. Therefore (X, A, τ) is soft compact.

(ii) The proof is obvious, since the soft topological space is finite.

□

Example 5. *Let (X, A, τ) be an infinite indiscrete soft topological space. In this space all subsets are soft preopen. Obviously it is soft compact. But $\{\tilde{x}_e\}_{\tilde{x}_e\in(X,A)}$ is a soft pre open cover which has no finite subcover. So it is not soft pre compact. Hence soft compactness need not imply soft precompactness.*

Theorem 3.2. *A soft preclosed subset of a soft precompact space is soft pre compact.*

Proof. Let (F, A) be a soft preclosed subset of a soft precompact space (X, A, τ) and $(G_\alpha, A)_{\alpha\in J}$ be a soft preopen cover for (F, A).

Then $\{(G_\alpha, A)_{\alpha\in J}, ((X, A) - (F, A))\}$ is a soft preopen cover for (X, A). Since (X, A) is soft precompact, there exists $\alpha_1, \alpha_2, ...\alpha_n \in J$ such that $(X, A) = (G\alpha_1, A)\ \tilde{\cup}...\tilde{\cup}\ (G\alpha_n, A)\ \tilde{\cup}\ \{(X, A) - (F, A)\}$. Therefore (F, A) $\tilde{\subseteq}\ (G\alpha_1, A)\ \tilde{\cup}...\tilde{\cup}\ (G\alpha_n, A)$ which proves (F, A) is soft precompact. □

Remark 3.1. *The converse of the above theorem need not be true.*

Example 6. *Let (X, A, τ) be as in Example 1, (F_2, A) is a soft preopen set and soft precompact but its not soft preclosed.*

Theorem 3.3. *A soft topological space (X, A, τ) is soft precompact if and only if for every collection τ' of soft preclosed sets in (X, A) having finite intersection property $\tilde{\cap}_{(F,A)\in\tau}(F, A)$ of all elements of τ' is non empty.*

Proof. Let (X, A, τ) be soft precompact and τ' be a collection of soft pre closed sets with finite intersection property. Suppose $\tilde{\cap}_{(F,A)\in\tau'}(F, A) = (\Phi, A)$ then $\tilde{\cup}_{(F,A)\in\tau'}\{(X, A) - (F, A)\} = (X, A)$. Therefore $(X, A) - (F, A)$ is soft preopen cover for (X, A). Then there exist $(F_1, A), (F_2, A), ...(F_n, A) \in \tau'$ such that $\tilde{\cup}_{i=1}^{n}\{(X, A) - (F_i, A)\} = (X, A)$. Therefore $\tilde{\cap}_{i=1}^{n}(F_i, A) = (\Phi, A)$ which is a contradiction. Therefore $\tilde{\cap}_{(F,A)\in\tau'}(F, A) \neq (\Phi, A)$.

Conversely, assume the hypothesis given in the statment. To prove (X, A) is soft precompact. Let $\{(F_\alpha, A)\}_{\alpha\in J}$ be a soft preopen cover for (X, A). Then

$\tilde{\cup}_{\alpha\in J}(F_\alpha, A) = (X, A)$, implies that $\tilde{\cap}_{\alpha\in J}((X, A) - (F_\alpha, A)) = (\Phi, A)$ By hypothesis, there exists $\alpha_1, \alpha_2, ...\alpha_n$ such that $\tilde{\cap}_{i=i}^{n}((X, A) - (F_{\alpha_i}, A)) = (\Phi, A)$ $\Rightarrow \tilde{\cup}_{i=1}^{n}(F_{\alpha_i}, A) = (X, A)$. $\Rightarrow$ (X, A) is soft precompact. □

Definition 3.3. *Let $\tilde{f} : (X, A, \tau) \to (Y, B, \tau')$ be a function, then $\tilde{f}$ said to be (i) Soft pre irresolute if $\tilde{f^{-1}}(F, B)$ is soft preopen in (X, A, τ) whenever (F, B) is soft preopen in (Y, A, χ'). (ii) Soft pre resolute if $\tilde{f_{pu}}(G, A)$ is soft preopen in (Y, A, τ') whenever (G, A) is soft preopen in (X, A, τ).*

Theorem 3.4. *Let (X, A, τ) and (Y, B, τ') be two topological spaces and $\tilde{f}$: $(X, A, \tau) \to (Y, B, \tau')$ be a bijection, then*

- (i) *$\tilde{f}$ is soft pre continuous and (X, A, τ) is soft precompact $\Rightarrow$ (Y, B, τ') is soft compact.*
- (ii) *$\tilde{f}$ is soft preirresoulte and (X, A, τ) is soft precompact $\Rightarrow$ (Y, B, τ') is soft pre compact.*
- (iii) *$\tilde{f}$ is soft continuous and (X, A, τ) is soft precompact $\Rightarrow$ (Y, B, τ') is soft compact.*
- (iv) *$\tilde{f}$ is soft preopen and (Y, B, τ') is soft precompact $\Rightarrow$ (X, A, τ) is soft compact.*
- (v) *$\tilde{f}$ is soft open and (Y, B, τ') is soft precompact $\Rightarrow$ (X, A, τ) is soft compact.*
- (vi) *$\tilde{f}$ is preresolute and (Y, B, τ') is soft precompact $\Rightarrow$ (X, A, τ) is soft precompact.*

Proof.

- (i) Let $\{(F_\alpha, B)\}_{\alpha\in J}$ be a soft open cover for (Y, B, τ'). Therefore $(Y, B, \tau') = \tilde{\cup}(F_\alpha, B)$. Therefore $(X, A, \tau) = \tilde{f}^{-1}(Y, B, \tau') = \tilde{\cup}\tilde{f}^{-1}(F_\alpha, B)$. Then $\left\{\tilde{f}^{-1}(F_\alpha, B)\right\}_{\alpha\in J}$ is a soft preopen cover for (X, A, τ). Since (X, A, τ) is soft precompact, then there exist $\alpha_1, \alpha_2, ...\alpha_n$ such that $(X, A, \tau) = \tilde{\cup}\tilde{f^{-1}}(F_{\alpha_i}, B)$, $(Y, B, \tau') = \tilde{f}(X, A, \tau) = \cup(F_{\alpha i}, B)$. Therefore (Y, B, τ') is soft compact.
- (ii) $\tilde{f}$ is soft preirresoulte $\Rightarrow$ $\tilde{f^{-1}}(F, B)$ is soft preopen whenever (F, B) soft pre open. (X, A, τ) is soft precompact implies every soft preopen cover has a finite sub cover. Let $\{(F_\alpha, B)\}_{\alpha\in J}$ be a soft preopen cover for (Y, B, τ'). Since $\tilde{f}$ is soft preirresolute, $\tilde{f}^{-1}(F_\alpha, B)$ is soft preopen. Also $\tilde{\cup}(F_\alpha, B) = (Y, B, \tau')$, $\tilde{f}^{-1}(F_\alpha, B) = (X, A, \tau)$
  $\Rightarrow \tilde{f}^{-1}(F_\alpha, B)$ is soft preopen cover for (X, A, τ).

$\Rightarrow \tilde{f}^{-1}(F_\alpha, B)$ has a finite sub collection to cover (X, A, τ).

$\Rightarrow (F_\alpha, B)$ has a finite sub collection to cover (Y, B, τ').

Therefore (Y, B, τ') is soft precompact.

(iii) $\tilde{f}$ is continuous $\Rightarrow$ $\tilde{f}^{-1}(F_\alpha, B)$ is soft open, whenever (F_α, B) is soft open. (X, A, τ) is soft precompact.

$\Rightarrow \tilde{f}^{-1}(F_\alpha, B)$ has a finite sub collecetion. (Since, every soft open is soft preopen). There is finite sub collection of (F_α, B) to cover (Y, B, τ').

$\Rightarrow (Y, B, \tau')$ is soft compact.

(iv) Let $\{(G_\alpha, A)\}_{\alpha\in J}$ be a soft open cover for (X, A, τ) then, $\tilde{f}(G_\alpha, A)$ is a soft preopen cover for (Y, B, τ'). Therefore,(Y, B, τ') is soft precompact, there exist finite sub cover of (Y, B, τ').

$\Rightarrow (X, A, \tau)$ has a finite sub cover of soft open sets.

Therefore (X, A, τ) is soft compact.

(v) Let $\{(G_\alpha, A)\}_{\alpha\in J}$ be a soft open cover for (X, A, τ) then, $\tilde{f}(G_\alpha, A)$ is a soft open cover for (Y, B, τ'). Since, every soft open sets in soft preopen sets. $\tilde{f}(G_\alpha, A)$ has soft preopen cover for (Y, B, τ'). Since (Y, B, τ') is soft pre compact there is finite sub cover. (X, A, τ) has finite sub cover for soft open sets. Therefore, (X, A, τ) is soft compact.

(vi) Let $\{(G_\alpha, A)\}_{\alpha\in J}$ be a soft preopen cover of (X, A, τ) then $\tilde{f}$ is soft preresolute, $\tilde{f}(G_\alpha, A)$ is soft preopen in (Y, B, τ'). Since, (Y, B, τ') soft precompact, it has a finite subcover. $\Rightarrow (X, A, \tau)$ has finite sub cover of soft preopen sets. Therefore, (X, A, τ) is soft precompact.

□

Theorem 3.5. *Every soft closed subspace of a soft precompact space is soft precompact.*

Proof. Let $\tau : \{(F_\alpha, A)_{\alpha\in J}\}$ be a soft precover for (F, A). If (F, A) is soft preclosed. Then $(X, A) - (F, A)$ is soft preopen. Then $\tau = \tilde{\cup}((X, A) - (F, A))$ is soft preopen cover of (X, A, τ). (X, A, τ) is soft precompact, which implies it has finite subcover of soft preopen sets. Hence (F, A) has a finite subcover of soft pre open sets. So it is soft pre compact. □

Theorem 3.6. *Let (X, A, τ) be a soft Hausdorff space. If (F, A) is soft precompact on (X, A, τ), then (F, A) is soft preclosed.*

Proof. **Claim:** $(F, A)^c$ is soft preopen.

Let $\tilde{x}_e \tilde{\in} (F,A)^c$. So, for each $\tilde{x}_e \tilde{\in} (F,A)^c = (X,A) - (F,A)$ and $\tilde{x}_e \neq (F,A)$ then for all $\tilde{y}_f \tilde{\in} (F,A)$ $\tilde{x}_e \neq \tilde{y}_f$. Since (X,A,τ) is soft Hausdroff space there exist $(G,B)_{\tilde{y}_f}, (H,C)_{\tilde{y}_f} \in \tau$ such that $\tilde{x}_e \tilde{\in} (G,B)_{\tilde{y}_f}$, $\tilde{y}_f \tilde{\in} (H,C)_{\tilde{y}_f}$ and $(G,B)_{\tilde{y}_f} \tilde{\cap} (H,C)_{\tilde{y}_f} = (\Phi, A)$. Then $(F,A) \tilde{\subseteq} (H,C)_{\tilde{y}_f}$. The family $\tau = \{(H,C)_{\tilde{y}_f} : \tilde{y}_f \tilde{\in} (F,A)\}$ is a soft open cover of (F,A). Since (F,A) is soft precompact, (F,A) has a finite subcover, and so $(F,A) \tilde{\subseteq} \tilde{\cup}_{i=1}^{n}(H,C)_{\tilde{y}_f}$. Then $\tilde{\cup}_{i=1}^{n}(H,C)_{\tilde{y}_e}$ and $\tilde{\cap}_{i=1}^{n}(G,B)_{\tilde{y}_e} = (\Phi, A)$. Since $\tilde{x}_e \tilde{\in} (G,B)_{\tilde{y}_f}$, then

$$\tilde{x}_e \tilde{\in} (G,B)_{\tilde{y}_e} \tilde{\subseteq} \tilde{\cup}_{i=1}^{n}(H,C)_{\tilde{y}_e} \tilde{\subseteq} (F,A)^c.$$

Hence $(F,A)^c$ is soft preopen. Therefore (F,A) is soft preclosed. □

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